

Grade 9

**CONTENT BOOKLETS:
TARGETED SUPPORT**

Term 1



A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). The FSS helped the DBE trial the NECT Maths, Science and language learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Provincialisation Programme. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

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TOPIC 1: WHOLE NUMBERS

INTRODUCTION

- This unit runs for 4.5 hours.
- It forms part of the Content Area 'Number, Operations and Relationships' which counts for 15% in the final exam.
- The unit covers a consolidation of number knowledge and calculation techniques for whole numbers, developed in Grade 8. Proportion is covered in more detail involving Direct and Indirect Proportion. Compound Interest is introduced in the Finance section.
- It is important to note that competency in finding multiples and factors, and prime factorisation of whole numbers, remains important for developing competency in factorising Algebraic Expressions and solving Algebraic Equations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Mental Calculations • Ordering and Comparing Whole Numbers • Properties of Whole Numbers [recognise division by zero property] • Calculations using Whole Numbers • Calculation techniques • Multiples and Factors [prime factors] • Problem Solving [increasing or decreasing in a given ratio] • Hire purchase and exchange rates included in financial Mathematics 	<ul style="list-style-type: none"> • Properties of Whole Numbers • Calculations using Whole Numbers • Calculation techniques • Multiples and Factors • Problem Solving • Commission, rentals and compound interest included in financial mathematics 	<ul style="list-style-type: none"> • Although Whole Numbers are not done again as a stand-alone topic in the FET phase, a knowledge of how to work with them is important to gaining a better understanding of other areas in mathematics. • Finance is covered in the FET phase: Use of simple and compound growth formulae to solve problems including hire purchase, inflation and depreciation



GLOSSARY OF TERMS

Term	Explanation / Diagram
Natural numbers	The set of numbers starting at 1. Also known as counting numbers. Symbol N
Whole numbers	The natural numbers including the number zero. Symbol N_0
Integers	The number set ranging from $-\infty$ to ∞ , but does not have decimals or fractions included. Symbol Z
Rational numbers	The exact value of a rational number is known. Rational numbers include all integers as well as all the fractions between them. All numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and b is not zero. Symbol Q
Irrational numbers	All numbers usually represented with a symbol such as π and can only be represented in number form as a never ending number. These numbers cannot be represented as rational numbers. Irrational numbers (for example $\sqrt{2}$) cannot be expressed as a rational number. Symbol Q'
Real numbers	All rational and irrational numbers together make up the real number system. Symbol R
Non - real numbers	These are the complex numbers that are non-real or imaginary. Complex numbers = real + non-real
Prime numbers	A number that has only two factors 1 and itself. 1 is not a prime number as it only has a single factor.
Ladder method/ prime factorization	A process of dividing a number into its prime number factor components.
Highest common factor (HCF)	The factor shared by two or more numbers that is of the highest value.
Lowest common multiple (LCM)	The smallest possible number into which two or more numbers could be divided.
Fraction	A measure of sharing or division. There are several types of fractions and these will be discussed in later lessons.
Powers of 10	Decimal number system. The number system we use
Terminating decimals	A decimal that ends after a specified number of digits.
Recurring decimals	A decimal that continues to repeat itself with no end.
Ratio	A comparison of 2 items with the same unit. Represented as a:b

Term	Explanation / Diagram
Rate	A comparison of two different measurements to result in a single comparison. km/h would be a comparison between the km travelled in a single measure of time [hour]
Direct proportion	When two sets are in proportion using a constant multiplier (so that an increase in the one matches an increase in the other) Also referred to as direct variation.
Inverse proportion	When two sets are in proportion and the increase in one matches the decrease in the other. Also referred to as indirect variation.
Exchange rate	A statement of how a value in one system may be given as a equivalent value in another system.
Profit/loss	A measure of the gain/loss made in a transaction or enterprise.
VAT	Value Added Tax is paid on goods or services. In South Africa VAT is 14% of the price.
Simple interest	Interest on an investment is earned or interest on a loan amount is charged with only the principal amount taken into consideration.
Hire purchase	When an item is purchased but only a deposit is paid and then the item is paid off over a specified period of time. The interest charged is simple interest
Deposit	A sum of money paid as a first instalment on an item with the understanding that the balance will be paid at a later stage.
Compound interest	Interest is calculated not only on the principal amount but also will include any accumulated interest that may have been levied previously.



SUMMARY OF KEY CONCEPTS

Ensure learners are confident in working with multiples and factors. This should include finding the highest common factor and the lowest common multiple.

Some revision work involving ratio, rate and proportion should also be covered.

Direct and Inverse Proportion

1. Direct Proportion is a relationship between two variables in which one is a constant multiple of the other. When one variable changes the other also changes in proportion to the first.



Example:

One chocolate costs R5

∴ Three chocolates will cost R15

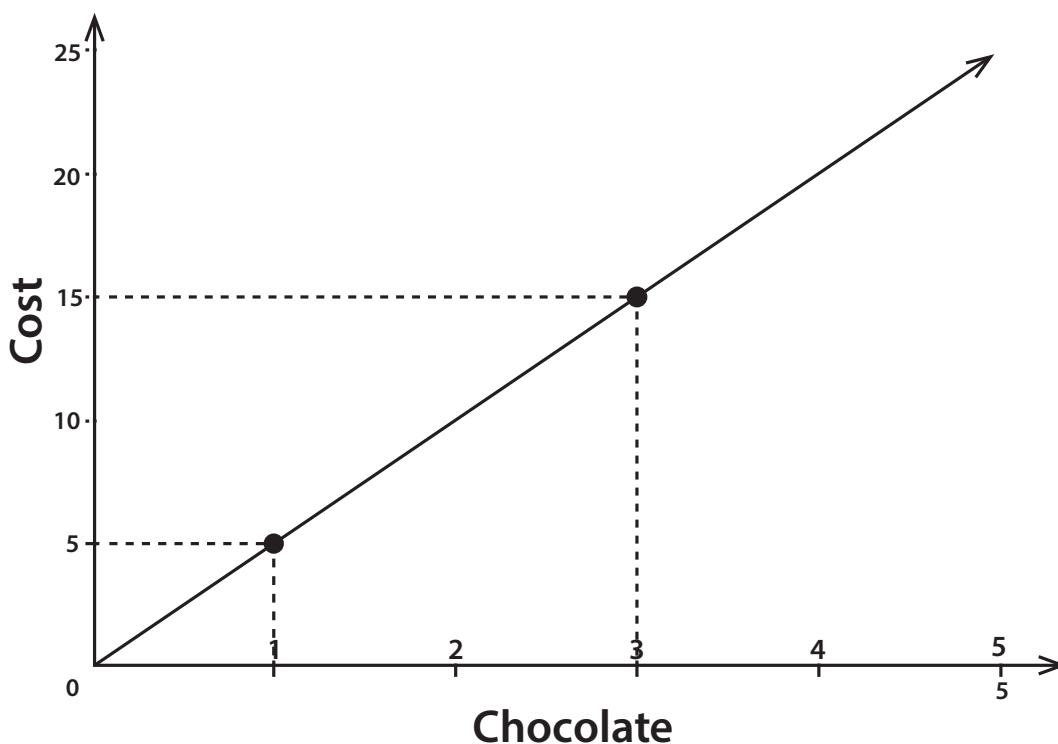
(To get from one to three involved multiplying by 3. Therefore, to find the cost of 3 chocolates, the cost also had to be multiplied by 3.)



Note: As the number of chocolate increases, the cost increases.

A straight line graph represents Direct Proportion.

There is a constant quotient between the variables.



2. Indirect (Inverse) Proportion is a relationship between two variables in which the product is a constant. When one variable increases the other decreases in proportion so that the product remains the same.



Example:

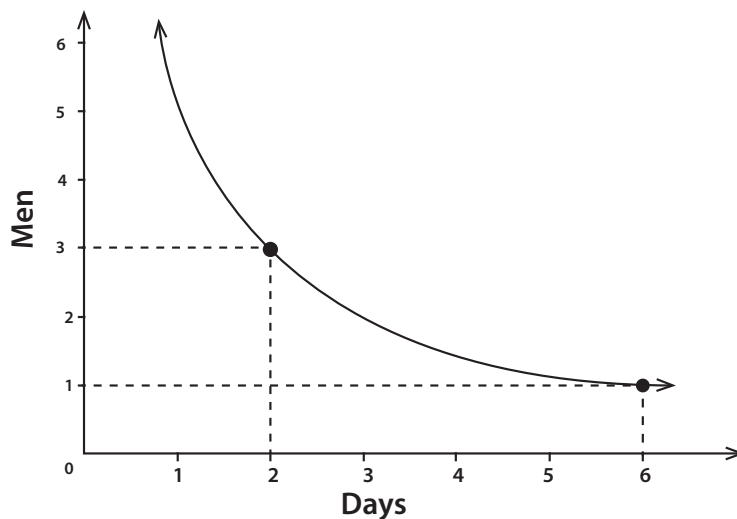
One man takes 6 hours to do a certain job (Note: $1 \times 6 = 6$ \therefore 6 is the constant product)

To find how long 3 men would take to do the same job, find what is multiplied by 3 to get the constant product of 6 ($6 \div 3 = 2$). 3 men will take two hours.



Note: As the number of men increases, the number of hours decrease.

A curved graph represents Indirect Proportion.



Finance

1. Percentages

Percentages are most often used to calculate discounts or a deposit required when buying on Hire Purchase.

2. Simple Interest

There are a number of formulae that can be used when calculating Simple Interest. The two main ones are:

a. $SI = p \times i \times n$

This calculates the actual amount of interest charged.

p = principal amount (starting amount)

i = interest rate (a percentage but needs to be changed to a decimal fraction)

n = Number of years (that the loan will take to be paid back)

Topic 1 Whole Numbers

b. $A = P(1 + i.n)$

This calculates the interest but also adds it to the initial amount that you started with so that the overall total is found. (This formula is the one used through to Grade 12)

A = Accumulated amount (Final total)

P = Principal amount

i = interest rate (expressed as a decimal fraction converted from the given percentage)

n = Number of years



c. For example:

Find the interest charged on a loan of R1500 at 12% per annum (p.a.) for 3 years.

$$\begin{aligned}SI &= p \times i \times n \\ &= (1500)(0,12)(3) \\ &= 540\end{aligned}$$

This gives the amount of interest that will be charged. If you wanted to know what would be owed in total you would now need to add that to the original R1500. ($1500 + 540 = 2040$)

If you used the 2nd formula you will see that it will already be added for you.

$$\begin{aligned}A &= P(1 + i.n) \\ &= 1500 (1 + (0,12)(3)) \quad \text{NOTE: A calculator will be essential for this} \\ &= 2040\end{aligned}$$

3. Compound Interest

There is only one formula used when dealing with Compound interest.

$$A = P(1+i)^n$$

A = Accumulated amount (Final total)

P = Principal amount

i = interest rate

n = Number of times interest will be allocated (in Grade 9 it will also be the same as the number of years and interest will always be given 'per annum' – every year – but this will change in later grades)

4. Hire Purchase

a. Simple Interest is always used in Hire Purchase.

b. A person can buy something from a shop on credit by paying a deposit and paying the balance (which will include interest) over a number of months.



For example:

A computer costs R5999. The shop requires a 10% deposit and the rest will be paid in equal monthly instalments over a three year period at an interest rate of 14%p.a. (per annum which means per year). Find the monthly instalments.

First calculate the deposit and subtract it from the original price.

10% of 5999

= 599,90

New amount owing is R5999 – R599,90 = 5399,10

Secondly, calculate the total amount now owing to the shop after the deposit has been taken off.

$$A = P(1+i.n)$$

[P=loan amount; n = number of years; r = interest rate changed from a percentage to a decimal]

$$A = 5399,10[1 + (0,14) (3)]$$

$$= 7666,72$$

Divide this amount owing by the number of months in the years to find the monthly repayment.

$$R7666,72 \div 36 = R212,96$$

5. Exchange rates

These are determined by the economy. An Exchange Rate is a rate which learners have come across before. Exchange Rates change daily.

TOPIC 2: INTEGERS

INTRODUCTION

- This unit runs for 4.5 hours.
- It forms part of the Content Area Numbers, Operations and Relationships which counts for 15% in the final exam.
- The unit covers the basic operations in Integers.
- It is important to note that a clear understanding of the integers is very important for the sections covered later such as Algebraic Expressions, Equations, Finance and Graphing. Learners must know how to work with negative and positive numbers, understanding the challenges that can occur with the various basic operations.
- The purpose of this section is to ensure that the learners are confident and competent in working with Integers.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Counting, ordering and comparing integers • Calculations with Integers [multiplication, division and a combination of all four operations, including squaring and cubing and square roots and cube roots] • Properties of Integers [recognize and use additive and multiplicative inverses] • Problem Solving [involving multiple operations] 	<ul style="list-style-type: none"> • Calculations with Integers • Properties of Integers • Problem Solving 	<ul style="list-style-type: none"> • Although Integers are not done again as a stand-alone topic in the FET phase, a knowledge of how to work with them is important to gaining a better understanding of other areas in mathematics.



GLOSSARY OF TERMS

Term	Explanation / Diagram
Positive integer	Whole numbers greater than 0.
Negative integer	Whole numbers smaller than 0.
Additive inverse	The number you would add to another number to get an answer of 0.
Product	The result of numbers that have been multiplied.
Quotient	When two numbers are divided this is the result.
Square	A number multiplied by itself once, or a number that has two identical factors. For example, in $2 \times 2 = 4$, the multiplication sign \times is only once.
Cube	A number multiplied by itself twice, or a number that has three identical factors. In $2 \times 2 \times 2 = 8$, the multiplication sign \times appears only twice.
BEDMAS/ BODMAS	The order of operations when calculations are done. B – Brackets E – Exponents or O – Of [to the power of] D – Division M – Multiplication A – Addition S – Subtraction
Algebra	A branch of mathematics that deals with generalised calculations by using letters or symbols to replace the numbers.
Like terms	Terms that are identical with respect to the variable they contain. The variable and the exponent to which it is raised must be exactly the same.



SUMMARY OF KEY CONCEPTS

Integers extend the number system to include numbers less than zero. There are no fractions in Integers.

Zero is the centre of the number line that splits the positive integers from the negative integers.

Addition and Subtraction of Integers

1. In addition and subtraction, when there are two signs next to each other, these need to be changed into one sign.

$+ [+]$	$+$
$- [-]$	$+$
$+ [-]$	$-$
$- [+]$	$-$



For example:

$$\begin{aligned} 3 + (-2) \\ = 3 - 2 \\ = 1 \end{aligned}$$

$$\begin{aligned} 2 + (+1) \\ = 2 + 1 \\ = 3 \end{aligned}$$

$$\begin{aligned} 4 - (-3) \\ = 4 + 3 \\ = 7 \end{aligned}$$

2. Grade 9 learners should be more confident to tackle more complicated questions. Examples involving all four operations as well as squaring, cubing and finding roots can be found at the end of the notes.

Multiplication and Division of Integers

1. In multiplication and Division a similar rule applies as to addition and subtraction but now the signs will not be next to each other. Each integer will have a particular sign.

$[+][+]$	$\frac{+}{+}$	$+$
$[-][-]$	$\frac{-}{-}$	$+$
$[+][-]$	$\frac{+}{-}$	$-$
$[-][+]$	$\frac{-}{+}$	$-$



For example:

$$\begin{aligned} (+10)(+2) \\ = +20 \end{aligned}$$

$$\begin{aligned} (-5)(+4) \\ = -20 \end{aligned}$$

$$\begin{aligned} (+3)(-2) \\ = -6 \end{aligned}$$

$$\begin{aligned} (-2)(-6) \\ = +12 \end{aligned}$$

$$\begin{aligned} (-10) \div (+5) \\ = -5 \end{aligned}$$

$$\begin{aligned} (-20) \div (-4) \\ = 5 \end{aligned}$$

$$\begin{aligned} (+18) \div (+3) \\ = +6 \end{aligned}$$

$$\begin{aligned} (-24) \div (-8) \\ = +3 \end{aligned}$$

Four Operations of Integers combined



For example:

$$\begin{aligned} \frac{-27}{3} + (-4)(-2) - (-2-4) + (-3) - (-1) \\ = -9 + 8 - (-6) - 3 + 1 \\ = -9 + 8 + 6 - 3 + 1 \\ = 3 \end{aligned}$$

Squares and Cubes of Integers

Remember that squaring a number means multiplying the number by itself.



For example:

$$5^2 = 5 \times 5 = 25$$

As you know, many integers have a negative sign.

When squaring a negative number, the answer will always be positive.



For example:

$$(-3)^2 = (-3) \times (-3) = 9$$

BUT, be aware that: $-3^2 = -(3 \times 3) = -9$. In this case the ‘-’ (negative) is not being squared.

Cubing a number means to multiply it by itself twice. It has three equal factors.



For example:

$$2^3 = 2 \times 2 \times 2 = 8$$

$$(-3)^3 = (-3) \times (-3) \times (-3) = -27$$

Square Roots and Cube Roots of Integers

This is the opposite operation of squaring and cubing. It is like un-doing the operation of squaring or cubing.

Since $5 \times 5 = 25$, then $\sqrt{25} = 5$

Similarly, $2 \times 2 \times 2 = 8$, then $\sqrt[3]{8} = 2$

The square root of a negative number is not possible and is a non-real number. However, it is possible to find the cube root of a negative number.

$$\sqrt{-49} = \text{non-real} \qquad \sqrt[3]{-27} = -3 \quad (\text{because } (-3)(-3)(-3) = -27)$$

Some Extra Examples for Grade 9 Learners

NOTE: Although Integers is taught before Algebraic Expressions it may be a good idea to bring in some Algebra at this stage as the learners have already covered a number of concepts in Grade 8. This will help learners to not only consolidate their Integer work but also prepare for Algebra at the same time. (d) below is an example of the kind of question that will be useful – as can be seen it incorporates the use of Integers well.

- a. At 6am the temperature is -3°C . During the morning it rises by 15°C . At midday it becomes cloudy and the temperature drops again by 5°C . Later in the afternoon it rises by 4°C and finally falls by another 12°C during the night. What is the temperature the next morning?
- $$-3^{\circ}\text{C} + 15^{\circ}\text{C} - 5^{\circ}\text{C} + 4^{\circ}\text{C} - 12^{\circ}\text{C} = -1^{\circ}\text{C}$$



Teaching Tip: Discuss with learners the words rise, fall and drop and why each one would either be addition or subtraction. Use a number line if necessary.

- b. South Africa has some of the deepest mines in the world. One of the shafts on Western Deep Levels mine is 4 253m below the surface. The surface is 1 523m above sea level.
- How far is the mine below sea level?
 - Mount Everest, the highest mountain in the world, is 8 848m above sea level. How high is this mountain above the shaft mentioned above?
 - The lowest point on the earth's surface is the Mariana's Trench in the middle of the Pacific Ocean. It is 11 033m below sea level. How much deeper is the trench than the mine shaft?

How much higher is Mount Everest than Mariana's Trench?



Teaching Tip: Encourage learners to draw a diagram marking off sea level in the centre and marking off the mine shaft and Mariana's Trench below that and mount Everest above it.

Solutions: (i) 2730m (ii) 11578m (iii) 8303m (iv) 19881m

c. $(3 - (-1))^3 - (-4 - 2) + 0 \times 7$



Teaching Tip: When there are brackets within brackets remind learners to always work from the inside out.

$$\begin{aligned} &= (3 + 1)^3 - (-6) + 0 \\ &= 4^3 + 6 + 0 \\ &= 64 + 6 + 0 \\ &= 70 \end{aligned}$$

d. Simplify the following expression then find the numerical value by substituting "a" with -2 and "b" with -4.

$$\begin{aligned} &\frac{-(3ab^2)(4a^4) + 5a^2(a^3b^2)}{14a^6b} \\ &= \frac{-12a^5b^2 + 5a^5b^2}{14a^6b} \\ &= \frac{-7a^5b^2}{14a^6b} \\ &= \frac{-b}{2a} \\ &= \frac{-(-4)}{2(-2)} \\ &= -1 \end{aligned}$$

It is suggested that you look for more questions similar to the above four depending on the time available.

TOPIC 3: COMMON FRACTIONS

INTRODUCTION

- This unit runs for 4.5 hours.
- It forms part of the Content Area 'Numbers, Operations and Relationships' which counts for 15% in the final exam.
- The unit covers all the basic operations on Common Fractions.
- It is important to note that this section is a recap of some basic skills that were taught in the intermediate phase. It is very important to remind the learners of this section and to make sure they understand these concepts really well as they form the foundation for skills to be applied later in Algebra and this is also important in sections such as Trigonometry and Geometry.
- The purpose of this section is for Grade 9 learners to consolidate number knowledge and calculation techniques for Common Fractions, developed in Grade 8. This will enable them to apply these skills when doing Algebraic Fractions in a later section

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Describing and ordering fractions • Calculations with fractions • Solving problems • Percentages • Equivalent forms 	<ul style="list-style-type: none"> • Ordering fractions • Calculations with fractions • Solving problems • Percentages • Equivalent forms 	<ul style="list-style-type: none"> • Although Common Fractions are not done again as a stand-alone topic in the FET phase, a knowledge of how to work with them is important to gaining a better understanding of other areas in mathematics, including [but not limited to] Algebra and Probability



GLOSSARY OF TERMS

Term	Explanation / Diagram
Common fraction	Written as two whole numbers, one above the other, separated by a horizontal line or dash.
Numerator	The number that is on the top of a fraction
Denominator	The number at the bottom of a fraction, this number is the divisor in the fraction. The denominator may never be zero as this makes a fraction undefined.
Proper fraction	A fraction where the numerator is smaller than the denominator.
Improper fraction	A fraction where the numerator is greater than the denominator.
Lowest common denominator (LCD)	This is the smallest number into which all the denominators of the fractions can be divided. It is therefore the lowest common multiple of each of the fractions' denominators.
Reciprocal	This is the multiplicative inverse of any number. If the product of two numbers is 1 then they are reciprocals of each other. e.g. $9 \times \frac{1}{9} = 1$ means that 9 and $\frac{1}{9}$ are reciprocals of each other.
Square root	This is the factor of the number that can be multiplied by itself to result in the original number. It is the inverse operation to squaring.
Cube root	The factor of a number that can be multiplied by itself three times to result in the original number. It is the inverse operation to cubing.
Equivalent fractions	Fractions are equivalent if they reduce to the same simplified fraction. Equivalent fractions are equal in value.
Mixed fraction	An improper fraction written partly as a whole number and partly as a proper fraction.
Algebraic fraction	A fraction with variables e.g. $\frac{5x}{4y}$



SUMMARY OF KEY CONCEPTS

Addition and Subtraction of Common fractions

1. When adding or subtracting fractions, the Lowest Common Denominator (LCD) must be found. This is done in order to form equivalent fractions.



2. Mixed numbers need to be changed into improper fractions before finding the LCD.

For example:

$$\begin{aligned} \text{a. } & \frac{1}{2} + \frac{1}{3} \\ &= \frac{3+2}{6} \\ &= \frac{5}{6} \end{aligned}$$

Note: LCD = 6. The lowest common multiple of 2 and 3

$$\begin{aligned} \text{b. } & 2\frac{2}{5} - 1\frac{3}{4} \\ &= \frac{12}{5} - \frac{7}{4} \\ &= \frac{48-35}{20} \\ &= \frac{13}{20} \end{aligned}$$

Note: Even if this answer was an improper fraction there is no need to change it back into a mixed number. There is a need, however, to ensure that it is in its simplest form.

Multiplication and Division of Common Fractions.

1. No LCD is required to multiply or divide.
2. Numerators get multiplied with numerators and denominators get multiplied with denominators.
3. It is best to simplify first before multiplying. This requires checking if any numerator in the question has a common factor with any denominator in the question.
4. If this step does not happen and multiplying begins immediately, the numbers to work with will be larger and the answer will need to be simplified at the end.
5. For division, the divide sign must be changed into a multiplication sign then the fraction immediately after the changed sign must be reciprocated (turned upside down).
6. Any mixed numbers must be turned into improper fractions.



7. For example:

$$\begin{aligned} \text{a. } & \frac{2}{5} \times \frac{15}{8} \\ &= \frac{1}{1} \times \frac{3}{4} \\ &= \frac{3}{4} \end{aligned}$$

Note: 2 and 8 have a Highest Common Factor (HCF) of 2, so this can be divided into both. Similarly, 5 and 15 have a HCF of 5.

$$\begin{aligned} \text{b. } & \frac{4}{7} \div \frac{12}{21} \\ &= \frac{4}{7} \times \frac{21}{12} \\ &= \frac{1}{1} \times \frac{3}{3} \\ &= \frac{1}{1} \times \frac{1}{1} = 1 \end{aligned}$$

Note: more simplifying can occur at this stage.

$$\begin{aligned} \text{c. } & 3 \frac{3}{4} \div \frac{9}{10} \\ &= \frac{15}{4} \times \frac{10}{9} \\ &= \frac{5}{2} \times \frac{5}{3} \\ &= \frac{25}{6} \end{aligned}$$

Note: HCF of 9 and 15 is 3. HCF of 4 and 10 is 2.

Fractions as a percentage

1. It is important to remember that a percentage is actually a fraction out of 100.
2. If a common fraction is being changed to a percentage, the denominator needs to be changed to 100.



For example: $22\% = \frac{22}{100} = \frac{11}{50}$

TOPIC 4: DECIMAL FRACTIONS

INTRODUCTION

- This unit runs for 4.5 hours.
- It forms part of the Content Area 'Numbers, Operations and Relationships' which counts for 15% in the final exam.
- The unit covers calculations with Decimal Fractions as well as some techniques to help learners with these calculations.
- It is important to note that this section is an important recap of some basic skills that were taught in the intermediate phase.
- The purpose of this section in Grade 9, learners work with decimal fractions mostly as coefficients in algebraic expressions and equations. They are expected to be competent in performing multiple operations using decimal fractions and mixed numbers, applying properties of rational numbers appropriately. They are also expected to recognize and use equivalent forms for decimal fractions appropriately in calculations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Ordering and comparing decimal fractions • Calculations with decimal fractions • Calculation techniques • Solving problems • Equivalent forms 	<ul style="list-style-type: none"> • Ordering and comparing decimal fractions • Calculation techniques • Solving problems • Equivalent forms 	<ul style="list-style-type: none"> • Although Decimal fractions are not done again as a stand-alone topic in the FET phase, a knowledge of how to work with them is important to gaining a better understanding of other areas in mathematics, including [but not limited to] Finance and Measurement



GLOSSARY OF TERMS

Term	Explanation / Diagram
Decimal fraction	A decimal number is a number with a decimal point in it, like these: 1.56 3.14. The number to the left of the decimal is an ordinary whole number. The first number to the right of the decimal is the number of tenths [$\frac{1}{10}$'s]. The second is the number of hundredths [$\frac{1}{100}$'s] and so on.
Decimal place	The position of a number after the comma.
Place value	The value a digit has due to its place or position in the number
Rounding off	Reducing the number of decimal places according to the instruction or the convention that we are working with. To round off correctly look at the first digit that will not be used in the final answer; if it is 5 or greater then we round up; if it is 4 or less we round down.
Tip and Times	Inverting the fraction after a divide sign so that we can simplify by changing the division into a multiplication sign.



SUMMARY OF KEY CONCEPTS

Addition and Subtraction of Decimal Fractions

1. The simplest way to add or subtract decimals is in columns (vertically). The decimal signs must be exactly underneath each other so that the place values match. (Tens underneath Tens, hundredths underneath hundredths and so on). If need be, zeroes can be placed at the end to make the decimal fractions the same 'length'.



For example:

a. $3,2 + 1,45$

$$\begin{array}{r} 3,20 \\ + 1,45 \\ \hline 4,65 \end{array}$$

b. $4,291 + 5,64$ (carrying over required)

$$\begin{array}{r} 4,291 \\ + 5,640 \\ \hline 9,931 \end{array}$$

c. $5,12 - 2,452$ (borrowing is required)

$$\begin{array}{r} 5,120 \\ - 2,452 \\ \hline 2,668 \end{array}$$

Multiplication of Decimal Fractions

1. Multiplying by powers of 10

This is very easy to do - for every 10 you are multiplying by, the digits move one place to the left.

Tip:

(Try and imagine the decimal being fixed or locked into position)

Remember to check that your number has become larger (as it should when you are multiplying)



Examples:

$$3,21 \times 10 = 32,1$$

$$4,192 \times 100 = 419,2$$

2. Multiplication of decimals by integers

The method of multiplying in columns (vertically) as was done in primary school is used here.

While you are in the process of multiplying, you can ignore any decimal signs. Once the multiplication is complete, count the total number of digits after the decimal sign in the decimal fraction – this is how many places there should be in the answer. Insert the decimal sign to make this correct.



Example:

$$4,62 \times 3$$

$$\begin{array}{r} 462 \\ \times \quad 3 \\ \hline 1386 \end{array}$$

$\therefore 4,62 \times 3 = 13,86$ (two decimal places in the question \therefore two decimal places in the answer)

3. Multiplication of decimal fractions by decimal fractions

The method from primary school is used here as well.

While you are in the process of multiplying, you can ignore any decimal signs. Once the multiplication is complete, count the total number of digits after the decimal sign in both decimal fractions – this is how many places there should be in the answer. Insert the decimal sign to make this correct.



Example:

$$2,43 \times 0,6$$

$$\begin{array}{r} 243 \\ \times \quad 6 \\ \hline 1458 \end{array}$$

As there are 3 digits after the decimal signs in total (2 in 2,43 and 1 in 0,6), there needs to be 3 in the final answer.

$$\therefore 2,43 \times 0,6 = 1,458$$



Teaching Tip: Encourage learners to circle all digits that come after the decimal sign(s) before they start the multiplication and even write down on the side how many that totals so they know/remember to insert the decimal sign in the answer.

Topic 4 Decimal Fractions

Division of Decimal Fractions

1. Dividing by powers of 10

This is very easy to do - for every 10 you are dividing by, the digits move one place to the right. (Try and imagine the decimal being fixed or locked into position). Remember to check that your number has become smaller (as it should when you are dividing)



Examples:

$$34,21 \div 10 = 3,421$$

$$34,21 \div 100 = 0,3421$$

$$34,21 \div 1000 = 0,03421$$

2. Dividing decimals by Integers

Dividing using short or long division, keeping the decimal in the same position for the answer (quotient) as it is in the question.

Add zeroes to the end of the decimal if necessary until there is no remainder.



Examples:

$$48,24 \div 3$$

$$\begin{array}{r} 3 \overline{)48,24} \\ \underline{16,08} \end{array}$$

3. Dividing decimals by decimals

In order to do this, we need to multiply both decimals in the question by the same number of 10's until we are dividing by an integer, then we can follow the steps above.

Examples:

$$0,62 \div 0,2$$

(In order to make sure we are dividing by an integer, we need to multiply by 10. Both the divisor and the dividend must be multiplied by 10)

$$= 6,2 \div 2$$

$$= 3,1$$

Squaring, Cubing and Rooting Decimal Fractions.

1. Squares and Cubes

If a decimal fraction is squared the answer will have twice as many decimal places as the original decimal fraction.

If a decimal fraction is cubed the answer will have three times as many decimal places as the original decimal fraction.



Examples:

a. $(0,2)^2 = 0,04$ $(2^2 = 4)$

b. $(0,2)^3 = 0,008$ $(2^3 = 8)$

So, a short way to square or cube a decimal fraction is to:

- Ignore the decimal (for the time being)
- Square/cube the number you can see other than zeroes
- Place the decimal sign in the answer according to the twice as many (squaring) or three times as many (cubing) rule.



For example:

a. $(0,03)^2$

- Square the 3 $(3^2 = 9)$
- Number of decimal places required in answer: 4 (twice as many as the original 2)
 $\therefore (0,03)^2 = 0,0009$

b. $(0,12)^3$

- $12^3 = 1728$
- Number of decimal places required in answer: 6 (three times as many as 2 decimal places in the question)
 $\therefore (0,12)^3 = 0,001728$

You could also change the decimal fractions into common fractions, then square or cube them.

2. Square Roots and Cube Roots

The previous rule of the answer having twice as many or three times as many is now reversed.

If you square root a decimal fraction it will have half as many decimal places.

If you cube root a decimal fraction it will have a third as many decimal places.

Topic 4 Decimal Fractions



Examples:

$$\sqrt{0,25}$$

$$= 0,5$$

$$\sqrt[3]{0,000001}$$

$$= 0,01$$

$$(\sqrt{25} = 5)$$

(need one decimal place – half as much as the original two decimal places)

($\sqrt[3]{1} = 1$ and we need 2 decimal places – a third of the original 6)

Converting Recurring Decimals to Common Fractions

All decimal fractions that are rational (terminating or recurring) can be written as common fractions.



1. Terminating Decimals

For example:

a. $0,3$

$$= \frac{3}{10}$$

(1 digit to the right so that the denominator will be 10)

b. $4,3$

$$= 4\frac{3}{10}$$

(1 digit to the right so that the denominator will be 10, but watch out for the whole number)

c. $0,16$

$$= \frac{16}{100}$$

$$= \frac{4}{25}$$

(2 digits to the right so that the denominator will be 100)

(check if simplification is possible)

2. Recurring Decimals

To do this some knowledge of Algebra and Equations is required.



For example:

a. 0,44444.....

Let $x = 0.44444 \dots$	
$\therefore 10x = 4.44444 \dots$	Multiply each side by 10.100.1000 according to the number of decimals recurring. 1 recurring decimal [x10] 2 recurring decimals [x100] etc
$10x - x = 4.44444 \dots - 0.44444 \dots$	Subtract the Left Hand Side from the two previous steps to equal the subtraction of the Right Hand Sides from the two previous steps.
$\therefore 9x = 4$	Complete the subtraction process
$\therefore x = \frac{4}{9}$	Solve for x

b. 0,121212....

Let $x = 0,121212 \dots$

$\therefore 100x = 12,121212 \dots$

$100x - x = 12,121212 \dots - 0,121212 \dots$

$\therefore 99x = 12$

$\therefore x = \frac{12}{99}$

$\therefore x = \frac{4}{33}$

TOPIC 5: EXPONENTS

INTRODUCTION

- This unit runs for 5 hours.
- It forms part of the Content Area 'Numbers, Operations and Relationships' which counts for 15% in the final exam.
- The unit covers more practice with the four laws of Exponents already covered in Grade 8. The topic is also extended to include Integers as Exponents and the solving of equations involving Exponents.
- It is important to note that a clear understanding of exponents and the laws governing Exponents is very important for the sections covered later such as Algebraic Expressions, Equations, Finance and Graphing. Learners must know how to work with negative and positive exponents, understanding the challenges that can occur with the various basic operations. It is very important to ensure that each section is mastered before moving onto the next as this is a section that requires continual referral to previous knowledge.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Mental Calculations • Comparing and representing numbers in exponential form [whole numbers, integers and scientific notation] • Calculations using numbers in exponential form [establish and use the general laws of exponents] • Calculate squares, cubes, square roots and cube roots of exponential numbers • Problem solving 	<ul style="list-style-type: none"> • Mental Calculations • Comparing and representing numbers in exponential form [extend scientific notation to include negative exponents] • Calculations using numbers in exponential form [extend the general laws to include integer exponents] • Problem solving 	<ul style="list-style-type: none"> • Simplify expressions using the laws of exponents for rational exponents • Establish between which two integers a simple surd lies • Round real numbers to an appropriate degree of accuracy



GLOSSARY OF TERMS

Term	Explanation / Diagram
Expanded Form	Writing a number as a product of its prime factors listing each factor separately in the multiplication calculation.
Exponential Form	This is when we write a number as a power. Usually it will have a prime number as a base and then an exponent[index] [e.g $8 = 2^3$]
Exponent or Index	The superscript digit that is written above the base and indicates the number of times the base is repeated in a multiplication calculation.
Power	
Prime Factors	The factors that make up the number that are prime numbers. These are numbers that only have 1 and themselves as factors.
Scientific Notation	A number written as the product of a number between 1 and 10 AND a power of 10 Example: 3.42×10^4 [This represents the number 3 420 000 – 3 million four hundred and twenty thousand] Really small numbers can also be written this way which eliminates excessive zeros.



SUMMARY OF KEY CONCEPTS

Laws of Exponents

Note: the rules are shown here in their simplest form. Refer to the Grade 8 content booklet to show how these laws were derived.

1. Law 1



When multiplying powers of the same base, the exponents are added.

For example:

a. $x^4 \times x^2 = x^{4+2} = x^6$

b. $5^3 \times 5^6$
 $= (5 \times 5 \times 5) \times (5 \times 5 \times 5 \times 5 \times 5)$
 $= 5^{3+6}$ [here the exponents are added]
 $= 5^9$

2. Law 2



When dividing powers of the same base, the exponents are subtracted.

For example:

a. $x^{10} \div x^2 = x^{10-2} = x^8$

b. $4^4 \div 4^2$
 $= (4 \times 4 \times 4 \times 4) \div (4 \times 4)$
 $= 4^{4-2}$ [here the exponents are subtracted]
 $= 4^2$

3. Law 3



When a power is raised to another power, the exponents are multiplied

For example:

a. $(x^2)^3 = x^{2 \times 3} = x^6$

b. $(3^3)^5 = 3^{15}$

4. Law 4



When more than one base is raised to a power, the exponent is given to each base

For example:

a. $(abc)^2 = a^2b^2c^2$

b. $(3a^2b^3)^4 = 3^4a^8b^{12} = 81a^8b^{12}$ NOTE: This example also uses Law 3.



Teaching Tip: All exponent rules are just a shortcut for working with exponents. If the learners are ever unsure of a rule, encourage them to write the question out in full. This is often useful to seeing the correct answer.



For example:

$$(3a^2b^3)^4 = (3a^2b^3) \times (3a^2b^3) \times (3a^2b^3) \times (3a^2b^3)$$

$$= (3 \times a \times a \times b \times b \times b) \times (3 \times a \times a \times b \times b \times b) \times (3 \times a \times a \times b \times b \times b) \times (3 \times a \times a \times b \times b \times b)$$

Now all the same bases can be counted and brought together in exponential form

$$= 3^4a^8b^{12} = 81a^8b^{12}$$

This is not recommended all the time – as you can see it is quite a long process but it does help to show why the laws apply as they do.

Negative Exponents

- Answers to exponent questions should not be written with a negative exponent.
- A negative exponent shows that the base it is connected to is on the wrong side of the fraction line. In other words, if it is in the denominator it actually means the base should be in the numerator.



For example:

a. $a^{-2} = \frac{1}{a^2}$

b. $2^{-1} = \frac{1}{2}$

c. $\frac{x^5y^3z^{-1}}{x^{-2}y^2}$

$$= \frac{x^5x^2}{y^2y^3z^1}$$

$$= \frac{x^7}{y^5z}$$

NOTE: Some bases have positive exponents and others have negative exponents. Only the bases with negative exponents

need to be dealt with – each one needs to be ‘moved’ to the other side of the fraction in order to make the exponent

positive. Once all bases are in the correct places, simplify where possible.

Topic 5 Exponents



Teaching Tip: Using the laws of exponents you can show the learners WHY the negative exponents can become positive exponents simply by 'moving' them.

Law 2 will help show this.

Simplify: $\frac{x^2}{x^5}$

Using law 2: $\frac{x^2}{x^5} = x^{2-5} = x^{-3}$

Using the longer method of simplifying: $\frac{x^2}{x^5} = \frac{x x}{x x x x x} = \frac{1}{x^3}$ (cancelling of x 's)

This demonstration should prove that $x^{-3} = \frac{1}{x^3}$

Zero as an Exponent

1. Any base raised to the power zero is equal to one. The only exception to this rule is zero to the power of zero which is undefined.



For example:

- a. $a^0 = 1$
- b. $5^0 = 1$
- c. $(x^5y)^0 = 1$



Teaching Tip: Using the laws of exponents you can show the learners WHY a base to the power zero will always equal one.

Law 2 will help show this.

Simplify: $\frac{x^5}{x^5}$

Using law 2: $\frac{x^5}{x^5} = x^{5-5} = x^0$

Using the longer method of simplifying: $\frac{x^5}{x^5} = \frac{x x x x x}{x x x x x} = 1$ (cancelling of x 's)

This demonstration should prove that $x^0 = 1$

Another way to help learners remember this rule about a power zero is to say that 'power zero means that there are none of those bases left after simplifying has taken place'.

So y^0 means that all y 's must have been cancelled out and now there are no y 's left at all.



Note regarding some common misconceptions Once learners have dealt with exponents they often make the following errors. As a teacher, it is important that you keep a look out for these and correct them as necessary. It would be a good idea to show these misconceptions on the board and get the learners to write them in their books. If this is done, ensure it is written under a very clear heading stating that the following are NOT true.

Incorrect answer/ misconception	Explanation of misconception	Correct answer
$3^2 = 3 \times 2 = 6$	Learners are multiplying the base with the exponent instead of using the exponent to show how many times to multiply the base with itself	$3^2 = 3 \times 3 = 9$
$-2 = \frac{1}{2}$	Once learners have heard of/learnt the negative exponent rule, they think anything negative can be made positive by 'moving it to the other side of the fraction'.	$-2 = -2$

Exponential Equations

- These are equations in which the unknown is an exponent.
- The main rule to remember when solving for exponents is that it is essential to have the SAME base on each side of the equation.



For example:

- If $3^x = 3^4$, it should be easy to see that $x = 4$
- If $3^x = 27$, the first step is to ensure that the base on each side is the same.

27 needs to be broken down into 3 to the power of...

$$\therefore 3^x = 3^3$$

$$\therefore x = 3$$

It becomes a little more complicated. The negative exponent rule is sometime required.

$$\text{c. } 2^x = \frac{1}{8}$$

First change the 8 to a base 2.

$$2^x = \frac{1}{2^3}$$

Apply the negative exponent rule backwards as the base needs to be at the top.

$$2^x = 2^{-3}$$

$$\therefore x = -3$$

Topic 5 Exponents

d. $3 \cdot 2^{x+1} = 96$

First work to remove the $\times 3$ on the Left hand Side (LHS).
Divide by 3 on both sides.

$$2^{x+1} = 32$$

Break 32 down into a base 2

$$2^{x+1} = 2^5$$

Now that the bases are the same, show that the exponents must be the same.

$$\therefore x+1 = 5$$

$$\therefore x = 4$$

Scientific Notation

1. Scientific Notation is used to write very small or very large numbers in exponential form.
2. A number between one and ten multiplied by a power of 10 makes up Scientific Notation.



For example:

$$2,5 \times 10^4 \text{ or } 4,651 \times 10^{-7}$$

3. To change large numbers into scientific notation, follow these steps:
 - Place a decimal after the first digit (to create a number between one and 10). Remember that this means you have divided the number by a certain numbers of 10's.
 - Count how many places the decimal has been 'moved'. Remember that in a large number that doesn't seem to have a decimal it is at the back. ($260 = 260,0$)
 - The number of places the decimal has moved becomes the power of 10. This represents how many powers of 10 have been divided.



For example:

Write 765 000 000 (765 million) in scientific notation.

7,65 (place the decimal to create a number between one and ten)

Decimal was moved 8 places (to the left)

$$\therefore 765\,000\,000 = 7,65 \times 10^8$$

4. To change small numbers into scientific notation, follow these steps:

- Place a decimal to create a number between one and 10. Remember this is the amount of 10's you have multiplied by.
- Count how many places the decimal has been 'moved'. With a small number note that the movement will be to the right.
- The number of places the decimal has moved becomes the power of 10 – but the exponent will be negative due to the movement in the opposite direction making the number smaller. This is because the negative exponent really means that division has occurred.



For example:

Write 0,000000564 in scientific notation.

5,64 (place the decimal to create a number between one and ten)

Decimal was moved 7 places (to the right)

$$\therefore 0,000000564 = 5,64 \times 10^{-7}$$



Teaching tip: Ask learners to write the answer again then use the laws of exponents to simplify (so no negative exponent). This will show them that division needs to occur to get back to the original number.

TOPIC 6: NUMERIC AND GEOMETRIC PATTERNS

INTRODUCTION

- This unit runs for 4.5 hours.
- It forms part of the Content Area 'Patterns, Functions and Algebra' which counts for 35% in the final exam.
- The unit covers the same work as covered in Grade 8. Use the time to ensure learners are fully confident when working with all types of patterns and questions. If learners are coping well with the section before the allocated time is up, extend them a little in order to assist them in managing Grade 10 Patterns better.
- It is important to note that this section is the introduction to understanding Functions that becomes a major section in the FET phase.
- The purpose of having an understanding of patterns is to aid in developing mental skills. In order to recognize patterns an understanding of critical thinking and logic are required and these are clearly important skills to develop. Patterns can provide a clear understanding of mathematical relationships. This can be seen in the form of multiplication tables. 2×2 , 2×4 , 2×6 are just one example of the relationship pattern found in multiplication.
- Understanding patterns helps provide the basis for understanding algebra. This is because a major component of solving algebra problems involves analysing which is related to the understanding of patterns. Without being able to recognize the appearance of patterns the ability to be good at Algebra will be limited.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Investigate and extend patterns [numeric, geometric] represented visually or in tables • Algebraic representation of patterns • Describe and justify the general rules in own words and in algebraic language 	<ul style="list-style-type: none"> • Investigate and extend patterns [numeric, geometric] represented visually or in tables • Algebraic representation of patterns • Describe and justify the general rules in own words and in algebraic language 	<ul style="list-style-type: none"> • Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term being therefore linear • Quadratic patterns and series of patterns are covered towards the end of the FET phase



GLOSSARY OF TERMS

Term	Explanation / Diagram
Consecutive	One after the other.
Numeric Sequence	A sequence of numbers that follow a particular pattern.
Constant difference	Same number being added or subtracted to form a sequence.
Geometric Sequence	A sequence of numbers with a constant ratio between consecutive terms.
Constant Ratio	The number used to multiply one term to get to the next term in a geometric sequence. If division occurs, reciprocate and turn it into a multiplication. [Example: $\div 5 = \times \frac{1}{5}$]
Rule	Explanation of how a pattern is formed.
Formula	Instructions in an algebraic form. Will have variables in it.
Term	A number in a given sequence. Example: In the sequence: 5 ; 0 ; -5 ; -10 , all four numbers represent terms and each one of them are in a particular position.
Variable	A letter representing a quantity which is able to have different values.
Position	The place in the sequence held by one of the terms. Example: In the sequence: 2 ; 4 ; 6 ; 8... 6 is in the 3rd position.



SUMMARY OF KEY CONCEPTS

Describe and extend linear number patterns

1. All of these patterns have a common difference (d) between each term. This is a number that is either added or subtracted each time to find the next term.

2. Learners must be able to describe what is happening in a simple pattern.



Example:

3 9 15 21.....

The pattern is formed by adding 6

3. Learners must be able to extend the pattern. The above pattern would continue: 27
33 39....

Describe and extend geometric patterns

1. These patterns have common ratio. This is a number that is multiplied each time to find the next term. It could also be a fraction which may seem like division but is essentially the same thing.

2. Learners must be able to describe what is happening in a simple pattern.



Example:

2 6 18.....

The pattern is formed by multiplying by 3.

3. Learners must be able to extend the pattern. The above pattern would continue: 54
162 486.....

Describe and extend other numeric patterns



1. Learners can be expected to extend and describe patterns that are neither linear or geometric.

For example:

a. 1 ; 8 ; 27 ; 64....

Learners would be expected to recognise that these are the cube numbers and be able to extend the pattern.

b. 1 ; 1 ; 2 ; 3 ; 5 ; 8.....

This one is more difficult. From the third term onwards, the terms are found by adding the two previous terms. The next two terms would then be: 13 ; 21....

Using tables and rules to extend patterns



A rule is usually written as a formula.

For example: $y = -3x + 5$ or $T_n = 3n - 2$

This rule or formula can then be used to complete a table in which values are given. These values are substituted to find the numbers that form the pattern.



For example: Use the rule $y = -2x - 1$ to complete the table:

x	1	2	3	4	5
y					

$$\begin{array}{ccccc}
 y = -2x - 1 & y = -2x - 1 & y = -2x - 1 & y = -2x - 1 & y = -2x - 1 \\
 = -2(1) - 1 & = -2(2) - 1 & = -2(3) - 1 & = -2(4) - 1 & = -2(5) - 1 \\
 = -3 & = -5 & = -7 & = -9 & = -11
 \end{array}$$

x	1	2	3	4	5
y	-3	-5	-7	-9	-11

It is important to note the '-2' in the rule here and that the common difference is also '-2' in the number pattern formed.

Topic 6 Numeric and Geometric Patterns

Find the rule to describe linear patterns

Linear patterns have a constant difference. This constant difference is essential to finding the rule of a pattern.



For example:

Consider the pattern 4 ; 9 ; 14

The common difference is +5

Consider the pattern 12 ; 2 ; -8....

The common difference is -10

Steps to follow to find the rule of a linear pattern:

- Find the constant difference
- Using the idea that you know what the first term is, multiply the constant difference by 1 (representing the first term) and find what still needs to be done to get the first term
- Check the rule works for term 2 and term 3.



For example:

- a. Find the rule for the pattern 7 ; 13 ; 19 ; 25.....

Common difference is +6

$$T_1 = 6(1) + 1 \quad (\text{this will give the 7, the 1st term})$$

$$T_2 = 6(2) + 1 \quad (\text{this will give the 13, the 2nd term})$$

$$T_3 = 6(3) + 1 \quad (\text{this will give the 19, the 3rd term})$$

∴ the rule is to multiply by 6 and add 1.

This can be written as $T_n = 6n + 1$ where 'n' can represent any position in the pattern.

- b. Find the rule and hence the 15th term of the following pattern:

16 ; 9 ; 2 ; -5....

Constant difference: -7

$$T_1 = -7(1) + 23 \quad (\text{this will give the 16, the 1st term})$$

$$T_2 = -7(2) + 23 \quad (\text{this will give the 9, the 2nd term})$$

$$T_3 = -7(3) + 23 \quad (\text{this will give the 2, the 3rd term})$$

∴ the rule is to multiply by -7 and add 23.

The rule is: $T_n = -7n + 23$

To find the term in the 15th position, substitute 15 for 'n'

$$\begin{aligned} T_{15} &= -7(15) + 23 \\ &= -105 + 23 \\ &= -82 \end{aligned}$$

The 15th term is -82.

Some Extra Examples for Grade 9 Learners

This section covers the same work in Grade 9 as was covered in Grade 8. Below are some extra examples that can be done to challenge the learners and also prepare them better for this section in the FET phase.

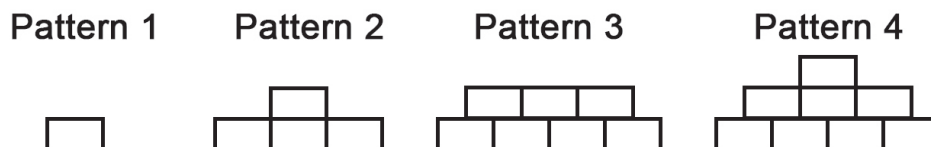
- a. Consider the sequence: $\frac{2}{3}; \frac{5}{8}; \frac{8}{13}; \frac{11}{18} \dots$
- i. Determine the nth term (the rule).
 - ii. Find the 10th term.



Teaching Tip: This may look like a very difficult question at first, particularly as fractions are involved. However, if the numerators are considered on their own (2 ; 5 ; 8 ; 11...) and the denominators are considered on their own (3 ; 8 ; 13 ; 18....) it will be much easier to work with. Finding the rule for each of these patterns – and they are both linear with common differences of 3 and 5 respectively – and putting the two rules together in fraction form will solve the problem.

Solutions: (i) $\frac{3n - 1}{5n - 2}$ (ii) $\frac{3(10) - 1}{5(10) - 2} = \frac{29}{48}$

- b. Consider the following pattern formed by bricks being laid:



- i. List the number of bricks in each of the patterns shown above.
- ii. How many bricks will be in the next pattern?
- iii. Find the general term which represents this pattern.
- iv. Use the rule to find how many bricks will be required for the 12th pattern.
- v. Which pattern will need 52 bricks?

Topic 6 Numeric and Geometric Patterns



Teaching Tip: When learners are given a question where the pattern is drawn/made up of diagrams, encourage them to count whatever it is being used. In this case the number of bricks. A pattern could be formed by using matchsticks to make triangles and the number of triangles could be increasing as the pattern extends. In this case, the number of matches could be the pattern or the number of triangles could. Both could even be considered in the question.

Solutions: (i) 1; 4; 7; 10 (ii) 13 (iii) $T_n = 3n - 2$ (iv) 34 (v) $52 = 3n - 2 \therefore n = 18$

- c. A school hall has many chairs that often need to be stacked away.



Each chair that is added to this stack makes it 8cm taller. One chair is 55cm tall. Use your knowledge of patterns to find how high a stack of chairs will be that has 8 chairs in it. How many chairs would there be if the stack was 127cm high?

Use the following table to help you.

Number of chairs	1	2	3	4	5
Height (cm)					

Solution:

Number of chairs	1	2	3	4	5
Height (cm)	55	63	71	79	87

Common Difference: 8

$$T_n = 8n + 47$$

8 chairs forms a height of 111cm ; There will be 10 chairs in a stack that is 127cm high.

TOPIC 7: FUNCTIONS AND RELATIONSHIPS

INTRODUCTION

- This unit runs for 4 hours.
- It forms part of the Content Area Patterns, Functions and Algebra which counts for 35% in the final exam.
- The unit covers all work covered in Grade 8. In Term 3 when the topic is extended further, learners will be required to use their knowledge to draw functions on a Cartesian Plane.
- It is important to note that this section is very important as it is the introduction to Linear Functions as well as the foundation for Functions to be covered later during the FET phase.
- The purpose of this topic is for learners to consolidate work with input and output values done in Grade 8. They should continue to find input or output values in flow diagrams, tables, formulae and equations.
- The concept of a function is important in almost every branch of mathematics.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Input and Output values using flow diagrams, tables, formulae and equations • Equivalent Forms [verbally, in flow diagrams, in tables, by formulae and by equations] 	<ul style="list-style-type: none"> • Input and Output values using flow diagrams, tables, formulae and equations • Equivalent Forms [verbally, in flow diagrams, in tables, by formulae, by equations and by graphs on the Cartesian plane] 	<ul style="list-style-type: none"> • Work with relationships between variables [numerically, graphically and verbally] • Functions must include linear, quadratic, exponential and trigonometric • Generate graphs with vertical shifts, vertical stretches and reflections about the x-axis • Problem solving



GLOSSARY OF TERMS

Term	Explanation / Diagram
Input	The initial value put into a function
Output	The resulting value from a function
Rule	The process through which you change the input number into the output number
Flow Chart/ Diagram	A drawing intended to make clear the order in which operations must be done so as to produce desired results.
Dependant variable (Output)	This is the resulting value after the process has been applied to the input value. It is dependent on the process because as the process changes the result will also then change.
Independent variable (Input)	This is the value put into any function or rule. It is the given or specifically selected value that needs to be put into the process to change.
Cartesian plane	The plane in which graphs are drawn. Consisting of an x-axis [horizontal axis] and a y-axis [vertical axis]. This creates a plane to graphically represent functions and relationships.
X-axis or Horizontal axis	The Horizontal component of the Cartesian plane. Basically a horizontal number line that has the zero in the middle, and has the positive integers to the right and negative integers to the left of the zero.
Y-axis or Vertical axis	The Vertical component of the Cartesian plane. Basically a vertical number line that has the zero in the middle, and has the positive integers above and negative integers below the zero.
Co-ordinate	Gives the position of a specific point in two-dimensional space. The first number is the Abscissa [x-value] and the second is the ordinate [y-value]
Literal Equation	An equation where variables are used to represent constants in a set of rules that are constantly used just with different input variables.
Distributive Law	A method used to show how two operations can be used in relation to each other.



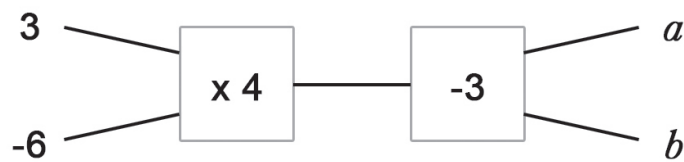
SUMMARY OF KEY CONCEPTS

Finding output when given the input and the rule

We can use flow diagrams to find missing values.



Example:



We can use flow diagrams to find missing values.

To find 'a': Start with 3, multiply by 4 and then subtract 3 ($a = 9$)

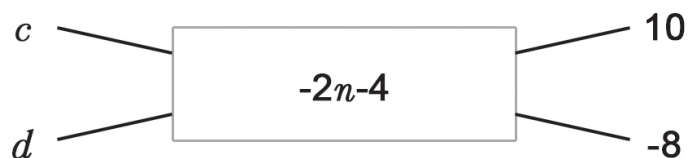
To find 'b': Start with -6, multiply by 4 and then subtract ($b = -27$)

Finding input when given the output and the rule

Since we are working backwards, we need to work with INVERSE operations in order to 'undo' the expression.



Example:



First consider the meaning of $-2n - 4$: Multiply by -2, then subtract 4

To find 'c': Using inverse operations:

Start with 10, add 4 (inverse operation to subtraction)
then divide by -2 (inverse operation to multiplication) ($c = -7$)

To find 'd': using inverse operations:

Start with -8, add 4
then divide by -2 ($d = 2$)

Encourage learners to check their answers by working forwards again.

Topic 7 Functions and Relationships

Finding the rule when given input and output values

To find the rule, the input and output numbers both need to be considered. Knowledge of number patterns will be useful here.

There are two types of rules to expect:



1. To get from the input to the output only one operation is performed.

For example:

a.

a	3	5	7	9
b	6	10	14	18

To get from the input (a) to the output (b), we need to multiply by 2
The rule is: $a \times 2 = b$ (This is better to write as $2a = b$)

b.

p	-1	0	1	2
q	4	5	6	7

To get from the input (p) to the output (q), we need to add 5
The rule is: $p + 5 = q$



2. To get from the input to the output two operations have been performed.

For example:

a.

f	1	2	3	4
g	4	7	10	13

Steps to follow to find rule:

- Look at the output numbers and find the common difference (in this case 3)
- This tells us that the first part of the rule is 'times 3'.
- Look at the input numbers and times them by 3. What still needs to be done to get to the output? (in this case add 1)
- Therefore, the rule is: 'times 3, add 1'.
- Rule: $f \times 3 + 1 = g$, which is better written as: $3f + 1 = g$

b.

x	-1	0	1	2
y	-3	2	7	12

Common difference of the output values: +5 (\therefore first part of the rule is: $\times 5$)

Input numbers $\times 5$: -5 ; 0 ; 5 ; 10

Operation still needed to get from these values to output values: + 2

$\therefore \times 5$ and + 2 (multiply by 5 and add 2)

$$x \times 5 + 2$$

$$\therefore 5x + 2 = y$$

Flow diagrams, tables, formulae and equations can all be used to represent PATTERNS.

TOPIC 8: ALGEBRAIC EXPRESSIONS

INTRODUCTION

- This unit runs for 4.5 hours.
- It forms part of the Content Area 'Patterns, Functions and Algebra' which counts for 35% in the final exam.
- The unit covers all work covered in Grade 8. Finding the product of two Binomials and squaring a Binomial is new for Term 1.
- It is important to note that Algebraic Expressions is covered again in Term 3 when further new work is introduced.
- The purpose of this section is to form the foundation for further studies in Mathematical Sciences. Algebra becomes the largest contributing section in Mathematics and helps provide structure for the study of Trigonometry and many other sections of the FET syllabus. Learners should be encouraged to practice as often as possible at home.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Add and subtract like terms in expressions • Multiply integers and monomials by: monomials, binomials and trinomials • Divide monomials, binomials and polynomials by integers or monomials • Simplify algebraic expressions • Find squares, square roots, cubes and cube roots of single algebraic terms • Determine the numerical value of an expression using substitution 	<ul style="list-style-type: none"> • Find the product of two binomials • Square a binomial • Factorise algebraic expressions by finding a common factor, difference of two squares and trinomials • Simplify algebraic fractions by using factorisation 	<ul style="list-style-type: none"> • Manipulate algebraic expressions by: • Multiplying a binomial by a trinomial • Factorising trinomials • Factorising the difference and sums of two cubes • Factorising by grouping • Simplifying, adding and subtracting algebraic fractions



GLOSSARY OF TERMS

Term	Explanation / Diagram
Expression	A mathematical formula which can include variables [letters], constants and operations. Example: $2b + 3c$
Product	The answer to a multiplication question. The product of six and four is 24. $6 \times 4 = 24$.
Sum	The answer to an addition question. The sum of 6 and 4 is 10. $6 + 4 = 10$
Difference	The answer to a subtraction question. The difference between 15 and 8 is 7. $15 - 8 = 7$
Quotient	The answer to a division question. The quotient of 24 and 8 is 3. $24 \div 8 = 3$
Term	Part of an algebraic expression. Terms are separated by '+' or '-' signs.
Coefficient	A number or symbol [including its sign] multiplied with a variable in a term. Example: 3 is the coefficient of y in $3y$ $-2a$ is the coefficient of b in the term $-2ab$
Variable	Letters of the alphabet which could represent different values Example: In the expression $m + 2$, m is a variable and could be replaced by a number in order to calculate the answer when m is equal to that specific number. Variables can change values.
Constant	A number making up a term on its own in an expression Example: $a + 3b - 10$: -10 is the constant. Constants cannot change value.
Substitution	Replacing a variable [letter of the alphabet] with a number to perform a calculation. Example: If $b = 3$, the $b + 2 = 3 + 2 = 5$
Monomial	One term expression.
Binomial	Two term expression.
Trinomial	Three term expression.
Polynomial	More than one term expression [two or more].
Like Term	Terms that have exactly the same variables. Example: $2a$ and $4a$ are like terms and can be added or subtracted $3abc$ and $10abc$ are like terms and can be added or subtracted
Unlike term	Terms that do not have the same variables. Example: $3a$ and $2b$ are unlike terms and cannot be added or subtracted x and y are unlike terms and cannot be added or subtracted
Exponent	In the example a^2 , '2' [or squared] is the exponent. It is the number or variable written at the top of the base in smaller font.
FOIL	Acronym short for: First, Outer, Inner, Last. A method for multiplying a binomial by a binomial.



SUMMARY OF KEY CONCEPTS

Revision of Grade 8 Algebra

It is essential that all the basics covered in grade 8 are revised. Learners will struggle with any new concepts in Grade 9 if they are not fully confident with concepts from Grade 8.

Multiplying a Binomial with a Binomial

1. A binomial is a two term expression. For example: $a + 5$

When multiplying a binomial with another binomial, EACH term in one binomial must be multiplied by EACH term in another binomial.



For example: $(a + 5)(a + 2)$

The ' a ' in the first binomial must be multiplied by both the ' a ' and the '+2' in the second binomial.

The '+5' in the first binomial must be multiplied by both the ' a ' and the '+2' in the second binomial.

FOUR multiplications need to be done.

FOIL is used to help remember the four products that need to be found.

F – First (the first term in one binomial with the first term in the second binomial)

O– Outer (the outer two terms)

I – Inner (the inner two terms)

L – Last (the last term in one binomial with the last term in the second binomial)

This can be shown as follows:



$$\begin{aligned} & (a + 5)(a + 2) \\ &= a^2 + 2a + 5a + 10 \\ &= a^2 + 7a + 10 \end{aligned}$$

Teaching Tip: Sometimes it helps the learners to let them imagine the terms inside each bracket represent players on a team. Each player in one team (the terms in the first bracket) must play against each player in the other team (the terms in the second bracket). This may assist them in remembering all the connections needed to find the products, particularly later on (in Grade 10) when they are required to multiply a binomial by a polynomial and FOIL no longer covers all the work needed to be done.

a. $(x + 4)(x - 5)$

$$= x^2 - 5x + 4x - 20$$

$$= x^2 - x - 20$$

Note the signs. Remember that 'the sign to the left of it belongs to it'.

b. $(y - 3)(y - 6)$

$$= y^2 - 6y - 3y + 18$$

$$= y^2 - 9y + 18$$

c. $(x + 3)(x - 3)$

$$= x^2 + 3x - 3x - 9$$

$$= x^2 - 9$$



Note: This is an important concept for later in the year when learners start factorising. When the terms are exactly the same except for the sign, the middle terms cancel each other out, being like terms of opposite signs. This needs to be pointed out to learners.

Encourage learners to look out for this in the question and when they recognise it to only multiply the first and last terms to find the answer.

Squaring a Binomial

2. Remember that squaring something means to multiply it by itself.

3. This concept is basically the same as multiplying a binomial with a binomial. The only difference being that the two binomials are exactly the same.



For example:

a. $(a + 4)^2$

$$= (a + 4)(a + 4)$$

$$= a^2 + 4a + 4a + 16$$

$$= a^2 + 8a + 16$$

b. $(x - 5)^2$

$$= (x - 5)(x - 5)$$

$$= x^2 - 5x - 5x + 25$$

$$= x^2 - 10x + 25$$



Teaching Tip: Point out the importance to learners of writing the bracket out twice and using FOIL. Learners tend to try a shortcut and just square both terms. They need reminding that squaring across a '+' or '-' sign is NOT mathematically possible.

$$(a + 4)^2 \neq a^2 + 16$$

Topic 8 Algebraic Expressions

Shortcut for squaring a Binomial



Note: It is recommended that this method is only shown to learners if they are extremely good at using the method of FOIL and it has become very easy for them.

Steps to squaring a binomial the short way:

- Square the first term
- Multiply the two terms (remembering to be careful of signs) and DOUBLE it.
- Square the last term



For example:

a. $(x + 7)^2$

First term squared: x^2

Two terms multiplied: $7x$ then doubled: $14x$

Second term squared: 49

$$\therefore (x + 7)^2 = x^2 + 14x + 49$$

b. $(a - 5)^2 = a^2 - 10a + 25$

TOPIC 9: ALGEBRAIC EQUATIONS

INTRODUCTION

- This unit runs for 4 hours.
- It forms part of the Content Area 'Patterns, Functions and Algebra' which counts for 35% in the final exam.
- The unit covers a recap of the grade 8 work and extends to include the solving of Equations with fractions and Quadratic Equations (equations that have two solutions).
- It is important to note that this section is vital as it forms the foundations for further studies in Mathematical Sciences. Algebra becomes the largest contributing section in Mathematics and helps provide structure for the study of Trigonometry, statistics, geometry and many other sections of the FET syllabus.
- The purpose of learning to solve Equations is also to help with Problem Solving which is the basis of all mathematics.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 8	GRADE 9	GRADE 10/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Use substitution in equations to generate tables of ordered pairs • Use additive and multiplicative inverses to solve equations • Use the laws of exponents to solve equations 	<ul style="list-style-type: none"> • Use factorisation to solve equations • Solve equations of the form: a product of factors = 0 	Solve: <ul style="list-style-type: none"> • Linear equations • Quadratic equations • Literal equations • Exponential equations • Linear inequalities • System of linear equations • Word problems



GLOSSARY OF TERMS

Term	Explanation / Diagram
Equation	A mathematical statement with an equal sign that includes a variable Example: $3x-5=20$
Expression	An algebraic statement consisting of terms with variables and constants. There is no equal sign Example: $3a+2b$
Formula	A formula is used to calculate a specific type of answer and has variables that represent a certain kind of value Example: Area = $l \times b$. This formula finds area of a rectangle and only measurements can replace the l and b
Variable	Letters of the alphabet which could represent different values Example: In the expression $m + 2$, m is a variable and could be replaced by a number in order to calculate the answer when m is equal to that specific number. Variables can change values
Like Terms	Terms that have exactly the same variables Example: $2a$ and $4a$ are like terms and can be added or subtracted $3abc$ and $10abc$ are like terms and can be added or subtracted
Inverse Operation	The opposite operation that will 'undo' an operation that has been performed. Addition and subtraction are the inverse operation of each other. Multiplication and Division are the inverse operation of each other.



SUMMARY OF KEY CONCEPTS

Revision of Grade 8 Algebra

It is essential that all the basics covered in Grade 8 are revised. Learners will struggle with any new concepts in Grade 9 if they are not fully confident with concepts from Grade 8.

In order to do this, a learner needs to:

1. Understand BODMAS and how to 'undo' operations (as was done in the Functions and Relationships section while looking for input value when you had the output value)
2. Be able to collect like terms (as was done in the Algebra section)

The most important rule to keep in mind when dealing with equations:

Whatever you do to one side of an equation you must ALWAYS do to the other side to keep it balanced.



1. Solving Equations Algebraically

For example:

a. $-2x + 4 = 10$

The aim: to get x on its own and find its value

Consider the order of operations to be performed on x then 'undo' them by using inverse operations.

First we need to 'undo' $+ 4$ (so we will $- 4$ on both sides)

$$-2x + 4 - 4 = 10 - 4$$

$$-2x = 6$$

Then we need to 'undo' $\times -2$ (so we will $\div - 2$ on both sides)

$$\frac{-2x}{-2} = \frac{6}{-2}$$

$$x = -3$$

An answer can be checked by substituting it back into the equation and checking if the LHS = RHS

$$\text{LHS} = -2x + 4 \qquad \text{RHS} = 10$$

$$= -2(-3) + 4$$

$$= 6 + 4$$

$$= 10 = \text{RHS}$$

$$\text{LHS} = \text{RHS} \quad \therefore x = -3 \text{ is correct}$$

Topic 9 Algebraic Equations

b. $4 - 4x = 8 + 2x$

$$-4x - 2x = 8 - 4$$

$$-6x = 4$$

$$x = \frac{-2}{3}$$

c. $5(x - 1) = 2(x + 3)$

$$5x - 5 = 2x + 6$$

$$5x - 2x = 6 + 5$$

$$3x = 11$$

$$x = \frac{11}{3}$$

d. $a - 4 = \frac{2}{3}a + 4$

$$3a - 12 = 2a + 12$$

$$3a - 2a = 12 + 12$$

$$a = 24$$

The four examples above represent all the types that the learners should be confident doing before starting the Grade 9 work.

NOTE: Summary Notes and examples on all of the above can be found in more detail in the Grade 8 notes. The Grade 9 textbook should also have a revision exercise of grade 8 work at the beginning of this topic. If this is not the case in the textbook your school is using, then it would be advisable to do the revision topic from the end of the topic on equations from a Grade 8 book before starting the new concepts for Grade 9.

Equations with fractions

- When solving equations with fractions in them, it is possible to remove all fractions by multiplying all terms by the lowest common denominator. This was shown in (d) above.
- It is important to note that this CANNOT be done on an expression. This is explained in more detail after the examples.
- Steps to solving equations with fractions:
 - Find the Lowest Common Multiple of the denominators (commonly known as the Lowest Common Denominator (LCD)).
 - Multiply EACH term (across the entire equation) with the LCD.
 - There should be no fractions at all after the previous step.
 - Solve as usual.



For example:

a. Solve: $\frac{x}{4} - \frac{2x}{3} = 5$ LCD= 12

$$\frac{12}{1} \times \frac{x}{4} - \frac{12}{1} \times \frac{2x}{3} = 5 \times 12$$

$$3x - 8x = 60$$

$$-5x = 60$$

$$x = -12$$

Note that there are no fractions after the step where the LCD was used to multiply all terms.

b. Solve: $\frac{x+3}{2} + \frac{x-2}{3} = x-2$ LCD=6

$$\frac{6}{1} \times \frac{x+3}{2} + \frac{6}{1} \times \frac{x-2}{3} = 6 \times x - 6 \times 2$$

$$3(x+3) + 2(x-2) = 6x - 12$$

$$3x + 9 + 2x - 4 = 6x - 12$$

$$5x + 5 = 6x - 12$$

$$5x - 6x = -12 - 5$$

$$-x = -17$$

$$x = 17$$

The above example will now be used to make clear the point made in (2) above.

Topic 9 Algebraic Equations

4. If a question is about adding or subtracting fractions (not solving for a variable) it is NOT possible to remove the fractions. It is the equal sign in the middle of an equation that allows us to multiply and remove fractions as long as whatever is done to one term will be done to all terms in order to keep the equation balanced.

Simplify: $\frac{x+3}{2} + \frac{x-2}{3} + x - 2$ Note carefully the similarities and differences.

The instruction is 'simplify' not 'solve'

There is no equal sign in the question.

In this type of question the LCD is still required but it is done in order to be able to add and subtract these unlike terms by making them all like terms.

LCD = 6 The common denominator of '6' will now be used to make all denominators '6'.

$$\begin{aligned} & \frac{3(x+3) + 2(x-2) + 6x - 12}{6} \\ &= \frac{3x + 9 + 2x - 4 + 6x - 12}{6} \\ &= \frac{11x - 7}{6} \end{aligned}$$

Note that the denominator remains throughout all the steps as well as in the answer.

Common misconception:

Learners often get confused about when they can 'get rid' of the fractions and when they must keep them. It needs to be pointed out constantly to them that solving an equation and therefore starting a question with an equal sign in it allows them to remove fractions, Simplifying an expression does NOT allow the removal of fractions.

Equations involving the Product of Factors (Quadratic Equations)



1. If the product of two values is zero, one of those values must equal zero.

For example:

If $a \times b = 0$, then either $a = 0$ or $b = 0$.

2. In the following example, the idea from above is used:

$$x(x + 2) = 0$$

Note the product (the bracket indicates multiplication) of two values (x and $(x+2)$) is equal to zero. The only way this is possible is if one of those values is zero.

$$\therefore x = 0 \quad \text{OR} \quad x + 2 = 0$$

The second part does not yet show a final solution so needs to be completed using a knowledge of solving equations

$$\therefore x = 0 \quad \text{OR} \quad x = -2$$

Note that both of these values can be substituted back into the equation to test if either one of them will work:

$$x(x + 2) = 0$$

If $x = 0$:

$$\begin{aligned} \text{LHS} &= 0(0+2) \\ &= 0(2) \\ &= 0 = \text{RHS} \end{aligned}$$

If $x = -2$:

$$\begin{aligned} \text{LHS} &= -2(-2+2) \\ &= -2(0) \\ &= 0 = \text{RHS} \end{aligned}$$

This shows that both solutions will work.

Topic 9 Algebraic Equations

3. The next example deals with the product of two binomials equalling zero.



For example:

$$(x + 3)(x - 4) = 0$$

$$\therefore x + 3 = 0 \quad \text{OR} \quad x - 4 = 0$$

$$\therefore x = -3 \quad \text{OR} \quad x = 4$$



Teaching Tip: Learners may want to multiply out at the beginning of these questions as they have just completed the Algebraic Expressions topic where they multiplied expressions using the distributive law and FOIL to multiply a binomial with a binomial. Be aware of this and remind them that when they see the product of two factors equal to zero, they should immediately be thinking that each factor must equal zero.

Later on in the year when learners start learning to factorise, this area of equations will be revisited.