NECT

GRADE 10 - 12

MATHEMATICS

TERM 1 & 2 2019

TRAINER'S GUIDE

CONTENTS

Workshop Objectives

By the end of this training session, participants will:

1. Be aware of the programme for this training session
2. Be informed of the NECT Programme 1 updates
3. Have improved Term 1 & 2 pedagogical content knowledge.
4. Be fully oriented to the Trainer's Guide that will be used to train teachers on this programme
5. Be motivated to improve their personal facilitation skills
6. Be motivated to improve teaching and learning in their district

Before the Training

1. **Be prepared to model excellence in training and facilitation.**
2. Prepare the venue as best as possible, to ensure that participants are comfortable, that they can all see the presenter, and that the setup is conducive for discussion.
3. Be prepared to show the slide show and videos. Deal with technical issues before the training.
4. Be fully prepared, have all your materials laid out in an orderly fashion.
5. Display the objectives of the workshop and go through these with participants.
6. Display an 'agenda' – a chart listing every activity that will be completed, together with the planned time allocation.
7. At the end of every training day, reflect on the objectives and agenda, and tick off what has been achieved that day.
8. DISPLAY ALL RELEVANT RESOURCE THAT HAVE BEEN PRODUCED BY THE NECT FOR CLASSROOMS, I.E.: POSTERS; RESOURCE PACK ITEMS; ETC. (Make an effort to properly prepare these items to present them in a way that models good practice and will protect resources from wear and tear.)

Tone of the Training

1. Remember that you are training TRAINERS and TEACHERS. Please ensure that you address participants correctly.
2. Be polite, patient and RESPECTFUL always. This is possibly the most important aspect of being a trainer.
    - Participants will generally be open to you and to the programme if they are treated with respect.
    - Arrive early and be prepared – for every session!
    - Greet participants by name whenever possible and ensure that names are pronounced correctly.
    - Do not be dismissive of a participant's concern. If you do not have time, or if you know that the issue will be addressed later in the session, create a PARKING LOT. Write down the query and stick it in the parking lot to be addressed later.
    - Do not be dismissive of participants' knowledge, skills and experience. As much as possible, allow participants to contribute to discussions.
3. Remember that humour is always a good strategy – try to add some fun to the training, in a way that does not make anyone uncomfortable.
4. Please remember to use icebreakers and energisers when required – it is important to keep the mood and energy of the training positive.

NECT

GRADES 10 – 12 MATHEMATICS

TERM 1 & 2 2019 TRAINING PROGRAMME

	TIME	ACTIVITY	TRAINER WORKSHOP	TEACHER WORKSHOP
1	30 minutes	Welcome, housekeeping and updates Introductions, reflections and agenda		
2	30 minutes	Pre-training Activity		
3	30 minutes	Orientation to materials and feedback from Term 3 & 4		
4	1 hour	Conceptual understanding in a learning centred classroom – the basics and principles		
5	2 hours	Patterns: Grade 10 - 12		
6	2 hours 30 minutes	Trigonometry: Grade 10 & 12		
7	2 hours	Functions and Inverses: Grade 12		
8a	30 minutes	Preparation for lesson demonstrations		
8b	1 hour	Preparation for lesson demonstrations		
9	2 hours 30 minutes	Lesson demonstrations and feedback		
10	1 hour	Assessment		
11	1 hour	Orientation to the trainer's guide		
12	1 hour	Training of teachers: planning session		
13	30 minutes	Post test		
14	30 minutes	Final questions and answers Closure and evaluation		

What you will need for this training:

ITEM	QUANTITY	CHECK
Flipchart stand and paper	1	
Kokis	10	
Blank A4 paper	100	
Laptop, data-projector and speakers	1	
USB with all materials and videos	1	
Attendance register	1	
Prestik	5	
Evaluation Forms	1 per participant	
Grade 10-12 Term 1 & 2 Lesson Plans	1 per participant	
Grade 10-12 Term 1 & 2 Trackers	1 per participant	
Grade 10-12 Term 1 & 2 Resource Packs and Assessments	1 per participant	
Grade 10-12 Posters	1 per participant	
Grade 10-12 Mathematics Training Handout Term 1 & 2 2019	1 per participant	

1	30 minutes	WELCOME, HOUSEKEEPING AND UPDATES INTRODUCTIONS, REFLECTIONS AND AGENDA	Facilitator:	What you will need: • Ensure that there is a sign outside your training room

1. Settle the group in plenary.
2. Welcome participants and complete the introductions.
3. Start the day with a short message, reading or prayer if appropriate.
4. Share the relevant housekeeping notes, to ensure that participants are clear about the toilet and catering arrangements.
5. Present any relevant updates or share interesting and successful data or stories.

INTRODUCTIONS, REFLECTIONS AND AGENDA

1. Settle participants so that you have their attention.
2. If there are any new members of the group, or if you are new to the group, briefly do a round of introductions.
3. Next, tell participants that you would like to take some time to get them to reflect on their own experience of the implementation of the training and programme.
4. Make sure each participant has a piece of A4 paper.
5. Ask participants to fold the paper into 4.
6. Next, ask them to do the following:
    a. **In the first square**, **they must write:** their name, position, school or district.
    b. **In the second square**, **they must write:** one thing about the programme that is being successfully implemented in schools. Ask them to please write some details about this, even a short narrative to explain what is happening.
    c. **In the third square**, **they must write:** Something that is still problematic, that the programme has not managed to address. Ask them to write some detail about this, even a short narrative to explain what is happening.
    d. **In the fourth square, they must write:** Anything further that they still want from the NECT. Please point out that this cannot be resources.

Draw this diagram on flipchart paper to help participants remember what to do:

Name **Position** **School or District**	**One thing that is working well in schools:**
One thing that is still a problem in schools:	**One thing I think the NECT should do for my subject:**

2	**30 minutes**	**PRE-TRAINING ACTIVITY**	**Facilitator: MQA**	**What you will need:** • Copies of pre-test

1. Work together to hand out copies of the pre-training activity to participants.
2. Ask participants to not look at the activity yet.
3. Briefly explain the purpose of the pre-training activity, which is to measure the success of the training, not to measure the scores of individuals.
4. Briefly explain the text conditions, i.e.: to work independently and in silence, for a period of 30 minutes. Ask participants who finish before time to please cover their work and wait quietly for others.
5. As participants complete the pre-training activity, walk around and offer practical assistance if needed.
6. Once time is up, help to collect and collate pre-training activities in an orderly fashion.

3	30 minutes	ORIENTATION TO THE LEARNING PROGRAMME AND MATERIALS	Facilitator:	What you will need: • Gr 12 Lesson plan book • Gr 12 resource book • Posters

1. Settle participants so that you have their attention.
2. Ask, by a show of hands, how many trainers have worked in schools where the lesson plans were used in Term 3 & 4.
3. Listen to participant's comments. Thank them for their contributions. Take note of any issues that could be addressed.
4. Tell participants that as there are new people here today who have not yet been introduced to the materials we are going to have a look at them in more detail now.
5. Tell participants that the programme developed by the NECT provides targeted support by providing:
    - Lesson Plans
    - Planner & Tracker
    - Resource Pack (with assessments and memoranda) and Posters
6. Ask participants to open their **Grade 12 Lesson Plan Booklet.**
7. Briefly show the introduction with a message from the NECT as well as an orientation to the programme.
8. Explain to participants that next, you will orientate them to the programme using the Grade 12 topic: Sequences & Series, as an example.
9. Ask participants to reread the orientation section again after the workshop, so that any questions that they still have may be answered the next day.
10. Ask participants to turn to the topic, **Sequences and Series** in the booklet. Go through the headings as follows:

TOPIC OVERVIEW (A)

1. The topic overview starts by locating where this topic is within the term (Sequences and Series is Topic 1 in Term 1). It gives further information with regards to the number of weeks taken as well as how many lessons it has been broken into.
2. Explain that the number of lessons has been calculated based on the CAPS requirement of 4.5 hours per week for Mathematics.
3. The topic has then been broken down according to sub-topics. Different schools run on different lesson times whereas the textbooks mostly run according to sub-topics.

4. This is a key issue with regards to planning. Teachers will need to ensure they plan according to their own lesson times at school to know how many school lessons should be used to cover each sub-topic.
5. The overview also has a few points regarding what the topic mark allocation in the final examination is, as well as a one or two key points regarding the topic.
6. In this topic, note that mention is made of the importance of patterns in Mathematics because mathematics is especially useful when it helps you predict, and number patterns are all about prediction.
7. The overview also includes a table showing the breakdown of each of the lessons. This includes the sub-topic and the time allocated to each lesson.

SEQUENTIAL TABLE (B)

1. This table shows the prior knowledge required for this topic, the current knowledge which will be covered this year. In the grade 10 and 11 books, there is also a column to show how the topic will progress in future years.
2. It is important that teachers spend some time ensuring the learners are confident in concepts from previous years. In the topics where time allows, a revision lesson has been incorporated into the lesson plans. Where time is an issue, teachers need to make another plan to help learners feel prepared to tackle the new concepts in the topic.
3. Teachers should also have a careful look at the 'Looking forward' column especially if they don't teach that grade. Knowing what learners are being prepared for can make a difference to how it is taught.

DIAGNOSTIC REPORTS (C)

1. Every year, after the NSC examination has been written and marked, a detailed diagnostic report is compiled. An analysis of learner responses ensures teachers have access to information that can add value to their teaching.
2. Key issues mentioned here are those that relate directly to the grade concerned. For this reason, there may be less information in the Grade 10 and 11 books as many of the points in these reports are directly related to Grade 12 issues.

ASSESSMENT OF THE TOPIC (D)

1. This section reminds the teacher what is required during the term concerned and what is included in the resource booklet.
2. It also mentions a point or two related to the format that questions are more likely to take in the given topic.

MATHEMATICAL VOCABULARY (E)

1. Each word related to the topic is explained in this section.
2. Teachers should ensure they are fully familiar with the vocabulary before teaching the topic.
3. Teachers must also make every effort to use the vocabulary throughout the teaching of the topic.
4. Learners should have access to the vocabulary; it should be displayed in the classroom where possible and learners should also have it in their books.

THE LESSON PLANS

1. Show participants that each lesson plan follows the same structure.
2. The main topic's CAPS reference and the lesson's objectives are listed first.
3. Some guidance regarding the management of the classroom is given. This includes suggestions of what to have written on the board before learners arrive to save time. As the lesson will run for more than one school lesson, teachers should always try and have something already on the board when learners arrive.
4. The learner practice table comes next. Take the time to explain this with care:
    - The different approved textbooks do not necessarily teach a topic in the same order.
    - These textbooks may have a different number of exercises relating to the various aspects of the topic.
    - For this reason, the decision was taken to put the list of possible exercises at the beginning of the lesson, in the Learner Practice Table.
    - It is up to the teacher to be well prepared and to note where in the lesson plan he/she will stop to give the learners time to practise what has been taught.
    - If there is only one exercise available, this is usually done at the end of the entire lesson.
    - Alternately, even if there is only one exercise, part of it may be done at a certain point in the lesson, and the remainder of the exercise may only be finished at the end of the lesson.
5. If the textbook used at a school has no exercise or too few questions, teachers must be prepared to source some questions from elsewhere. Past assessments or another textbook may be used for this purpose.
6. Teachers must ensure that learners get all the practice that they need.

CONCEPTUAL DEVELOPMENT

1. This section covers the actual teaching of the lesson.
2. It begins with an introduction which is a brief overview of what will be covered in the lesson, why it is important to the topic, and any other pertinent points.
3. The direct instruction contains the key content and, as far as possible, specific ways to get this information across to learners.
4. In general, any given lesson will follow a similar pattern from day to day:
    - First, the teacher explains the concept (there are many ways that this could occur)
    - Second, learners take down information, notes and examples
    - Third, the learners spend some time practising the new concepts
5. The work, notes and examples that learners write in their books will need to be complete and accurate enough for learners to use their books when preparing for assessments.
6. Finally, each lesson has a suggestion or two for additional reading, or links to a video to view.
7. Teachers should be encouraged to always read more, to watch the videos, and to be at least one step ahead of the learners. This has the advantage of the teacher feeling confident and prepared.
8. The one or two items recommended should not be the limit as to what the teacher watches or reads – a good teacher is always willing to find more ways to improve their own knowledge.

PLANNER & TRACKER

1. Ask participants to open their **Grade 12 Planner & Tracker.**
2. Tell participants that the P & T is a tool to assist them in their planning.
3. It will also help ensure that teachers stay on track with regards to time.
4. It will also be used to document the teacher's progress and reflection on any of the lessons. This could include what went well, what could be improved on, or how learners responded to a lesson. All these thoughts will be helpful next time the lesson is taught.
5. Ask participants to turn to the first page of the tracker. Note that each lesson from the booklet is written down the left-hand side.
6. On the right-hand side, the first column is '*Time*' and states the length of the lesson.
7. The CAPS reference appears in the next column.
8. Thereafter there is an area where the date that each lesson is completed can be filled in. The class' name can be filled in at the top and the date recorded across from the lesson.
9. This should be completed on a weekly basis. If the teacher has fallen behind, plans should be made as soon as possible to catch up.

RESOURCE PACK (INCLUDING ASSESSMENTS)

1. Ask participants to put their Planner & Tracker aside and look at the **Resource Pack** supplied.
2. This pack is generally made up of resources that could either be photocopied for learners (where possible) or diagrams that may be a little complicated to draw on the chalkboard.
3. Show participants that the only resource for Series and Sequences is the assignment. Allow a few minutes for participants to page through the resource pack and note the type of resource that is in the booklet.
4. Point out that the lesson is written at the top of the page, so it is easy to link to the lesson. It is also referenced in the lesson, so teachers know at the beginning of the lesson that there is a resource that will/can be used.
5. Ask participants to turn to the assessments at the end of the pack (page 43 onwards).
6. Point out that there are two tests for Grade 12 even though only one is required. This ensures that all the work can be assessed in Term 1.
7. The assessment includes the memorandum.

POSTERS

1. Finally, show participants the posters which you have displayed on the wall.
2. One of the posters is on one of the topics that we will be looking at in more detail in the training – Trigonometry – and each poster covers all the basic requirements covered throughout the FET.
3. This makes it useful in any mathematics classroom where at least one senior grade is taught.
4. Teachers should display this poster in their classroom.

CONCLUSION

1. Tell participants that we have come to the end of our look at the new materials available to FET mathematics teachers.
2. If time permits, allow participants a few minutes to browse through the materials on their own.
3. Ask if anyone has questions and answer them the best you can.
4. Ask participants if they think teachers will find this programme useful.
5. Thank participants for their attention.

4	1 hour	CONCEPTUAL UNDERSTANDING IN A LEARNING CENTRED CLASSROOM – THE BASICS AND PRINCIPLES	Facilitator:	What you will need: • Training handout

INTRODUCTION

1. No term captures completely all the aspects of knowledge, expertise, skills, understanding and competence in mathematics. We adopt (along with the modern trend in mathematics education globally) the term *mathematical proficiency* when we refer to what is necessary to master mathematics. Our understanding of mathematical proficiency (adapted from the original concept of the five strands of proficiency as coined by Kilpatrick, Swafford and Findell in their work *Adding it Up,* 2001) includes:
    a. Conceptual understanding – comprehension of mathematical concepts, operations and relations;
    b. Procedural fluency – the skill to carry out procedures flexibly, accurately, efficiently and appropriately;
    c. Strategic competence – the ability to formulate, represent and solve mathematical problems;
    d. Mathematical reasoning – the capacity for logical thinking, reflection, explanation and justification;
    e. The learning centred classroom – a mathematics learning environment where teachers and learners are both set to focus on the conceptual understanding of mathematics, and not solely on the teacher or the learner.
2. Up to now, much attention was spent on procedural fluency. The focus of our work in Term 1 & 2 this year is on conceptual understanding and the learning centred classroom – two concepts that need to be blended, as it is suggested that conceptual understanding is created in a learning centred classroom.
3. Ask participants to turn to page 4 in the training handout. As each of the following concepts are discussed, participants should make notes in their handout. Space is provided for writing one's own understanding of a concept before it is discussed as a group. More notes can then be added as required.

CONCEPTUAL UNDERSTANDING

A **concept** is a 'big idea' – a principle or notion that is enduring, the significance of which goes beyond particular origins, subject matter or place in time. Concepts represent the vehicle for learner inquiry and provide the means by which they can explore the essence of mathematics.

Concept Formation is an inductive teaching strategy that helps students form a clear understanding of a concept (or idea) through studying a small set of examples of the concept. Concepts are the “furniture” of our minds. ... For something to be an example of a concept, it must contain all these critical characteristics.
In contrast with the deductive method, inductive instruction makes use of the learner “noticing”. Instead of explaining a given concept and following this explanation with examples, the teacher presents students with many examples showing how the concept is used.

Conceptual understanding is knowing more than isolated facts and methods.
The successful learner understands mathematical ideas and has the ability to transfer their knowledge into new situations and apply it to new contexts. This deep conceptual understanding is a key principle for school mathematics.
Here are 5 tips to help develop conceptual understanding in mathematics:

1. Belief
Do you believe that ALL students in your classroom can access mathematical ideas without being told what to do and how to do it?

2. Sense Making
Consider asking your learners the following on a regular basis: "How/Why does your solution make sense to you?" This will give you insight into understanding what learners believe is correct and why they believe this and give you as the teacher the opportunity to respond with more effective questions. Moreover, in order to have those conversations, the task they were responding to had to be a rich, inquiry-based task. This means no teaching with shortcuts.

3. Scaffolding
Allow learners to take the information in, in smaller steps. Use easy numbers in the beginning so that manipulation with them is easy and the focus can be on the concept instead of the manipulation. This way learners won’t get overwhelmed too quickly and switch off before they have even got started. More challenging numbers can be brought in gradually.

4. Time
Conceptual ideas are not built in one day or even two. They are developed after repeated exposure to a particular mathematical idea in various contexts. Learners must struggle and resolve that struggle to internalize a concept. This process takes time. In the classroom, this means a learning objective may remain consistent for a week or two as the ideas are developed.

5. Multiple Representations

When developing conceptual understanding, it's imperative to give learners freedom of choice in how they might potentially respond. Narrowing to one representation too often causes learners to try to find the one correct path towards a solution, rather than thinking expansively and for themselves.

Learners (and parents!) like shortcuts. This makes teaching conceptually challenging. It will be worth it for both learner and teacher if the teacher is willing to persist.

Facilitating concept formation is a teaching task for which there is not always direct guidance in the curriculum document. The required outcomes of the teaching and learning are set out systematically and learners are expected to use their understanding to solve problems, but exactly *how* conceptual understanding comes about and is cultivated, remains unclear. In this training we shall focus on the learning centred classroom as the key to concept formation.

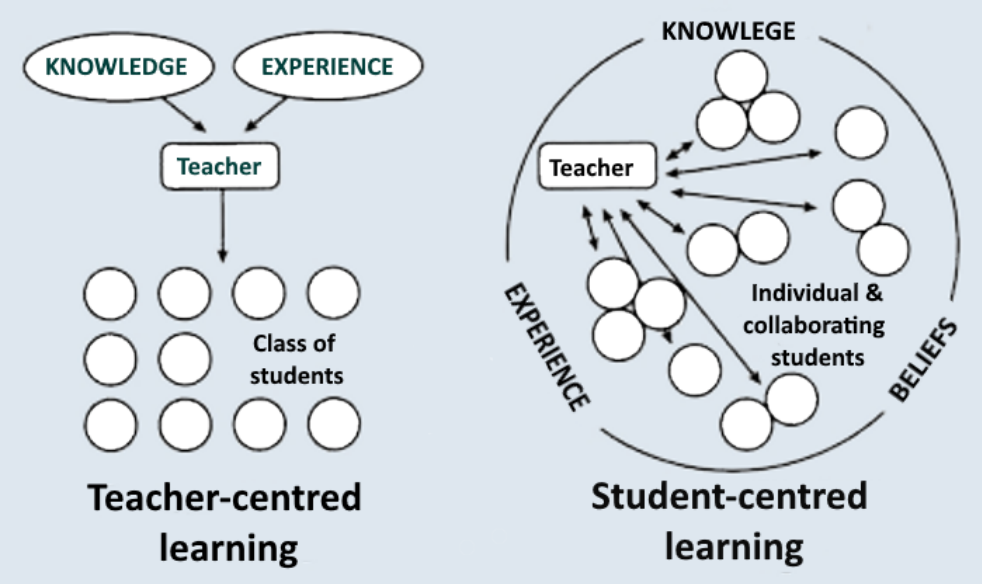
DIFFERENT ORIENTATIONS TOWARDS TEACHING

Three main orientations towards teaching include a) the teacher centred classroom, b) the learner centred classroom and c) the learning centred classroom.

a) In the **teacher centred classroom** the teacher is the primary owner of knowledge and the main source of energy. He/she transfers knowledge as the more active partner and learners receive the knowledge as the more passive partners. The typical sequence in such a class is that the teacher will teach the concept (for example the properties of quadrilaterals), write on the board and/or hand out notes then give exercises to the learners to complete on their own. The teacher may even at this stage work at their desk while learners work quietly. It is not difficult to retain discipline in such a classroom because there is absolute structure. The teacher is in full control and learners are not allowed to voice opinions unless requested. The teacher can continue with her own work such as marking and administration while learners are doing the exercises. The shape best suited to depict this type of classroom is a pyramid, where the teacher is at the top, acting, and learners are at the bottom, reacting to the teachings and instructions. It is a top-down approach and politically resembles an autocracy where decisions and initiatives are mainly taken by the one at the top.

b) In the **learner centred classroom** learners are the primary owners of knowledge and the main source of energy. They create knowledge by investigation and exploration. The teacher acknowledges contributions as valid and should maintain structure at the lowest possible level to allow learners to explore freely. It is hard to retain discipline in such a classroom because there is virtually no structure, the teacher is not in control and learners may voice their opinions at any time. In the extreme practice of learner centred

education no method or strategy can be enforced and there is no curriculum but what learners construct as knowledge. Learners' rights reign supreme and all their choices and decisions must be accommodated. The typical sequence in such a classroom is that learners will explore, come up with various findings and all get acknowledged. Class discussions may follow where some may be persuaded to other insights. No methods or strategies are prescribed, although the teacher may suggest some, but learners have the freedom of choice whether to accept them or not.



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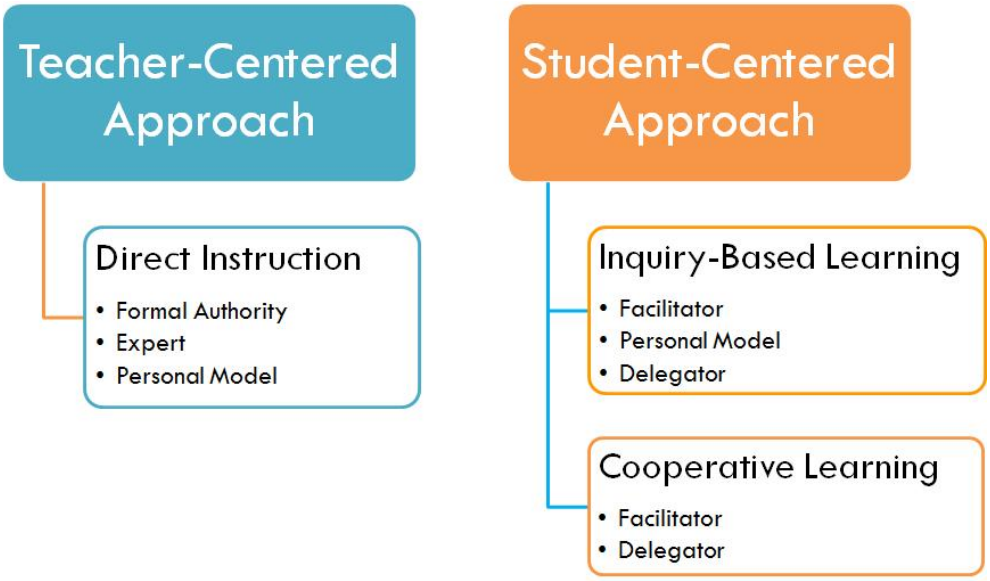


https://www.plaz-tech.com/technology-in-the-classroom-making-the-shift-from-teacher-centered-to-student-centered-approach/

c) In the **learning centred classroom** there is a distinct shift of focus away from people in the centre of the approach (subjective) to the mathematics concept as the core of all actions and activities (objective). All role players work towards the learning, understanding and mastering of a mathematical idea, no matter whether today the learner plays the larger part or tomorrow the teacher takes the larger part, as long as learning takes place and mathematical proficiency is reached.

The learning centred classroom has a few characteristics and requirements:

- It is **specific**: the focus must be precise, and it needs to be clear what the mathematical concept is that has to be developed.
- It is **intentional**: the purpose of the day must be clear to the teacher and to the learners and learners need to know what the outcomes are towards which they are working.
- It is **semi-structured**: because it is intentional and specific, the situation is structured and planned but the activities have the minimum structure to invite learners' innovative thinking.
- It is **meaningful**: there are no random activities that distract from the formation of the specific concept that is intended to be developed.
- It is **reciprocal**: there are different lines of connection – individual learners interact with the mathematical object, teacher and learner(s) interact, learners interact amongst themselves.
- It is **realistic**: the activities planned out for the formation of the concept originate, as far as possible, from the real world known to learners.
- It is **transcended**: once the concept has been formed, learners are practising the transcendence of the idea to other, related situations, both set up for them and by themselves.
- It is **assessed**: the process of concept development is continuously assessed to keep track of the level of understanding – informally by the teacher, self-assessment and peer assessment.
- It is **reasonable**: during the process and for all actions and decisions learners need to be able to explain *why.* In other words, be able to give reasons for doing what they do to develop logic and step-by-step thinking.
- It is **solidified:** once the concept is formed and understood it can be said it is still fluid. Now it needs to be practised over and over to ensure procedural fluency and ease of application.

NOTE

i. Concept formation can go wider and deeper than the curriculum and is not bound by absolute curriculum stipulations. Teachers must be able to discern the deep concept underpinning the topic in the curriculum. When the concept has been established, and there is clear understanding, the applications can be done within the confines of the curriculum.

ii. The same basic concept formation exercise can be done across a phase and is not necessarily bound to a specific grade, because the concept is underpinning both simple and complex applications – the

concept does not change – its applications in different situations/contexts change, the exercises based on the concept are more complex and the level of abstraction is continuously raised in higher grades.

iii. The creation of the learning space must be developmentally appropriate, firstly as far as the cognitive development of learners are concerned, and secondly as far as the social dynamics of the specific age group is concerned, especially considering their gender preferences. The creation of a totally new concept can be supported with physical manipulatives, however learners in the Intermediate Phase are already in the semi-concrete phase of cognitive development, and understand representations of the concrete, as well as symbolic representations like number symbols. Learners in the Senior Phase can most certainly handle the symbolic representation of reality and they are moving towards the semi-abstract phase of cognitive development.

iv. It stays a point of discussion whether to group learners in same-ability groupings or in mixed ability groupings, since both have their advantages and disadvantages. The main concern in mixed ability grouping is that the stronger learner will dominate the weaker learner and the weaker learner, though slower, never reaches their own authentic understanding of the concept.

v. As will become clear from the example below, the formation of a concept is not something that is far away or only remotely related to mathematics. It uses mathematical terminology and processes, and when it is done there is not a huge distance to overcome to relate it to the exercises following.

vi. Clear and simple, step by step instruction is an important part of concept development; This allows the learners to focus on the concept and not to battle confusion and uncertainty about what is expected of them.

According to Daniel Willingham, successful mathematics learning requires three different abilities that must be developed and woven together. These are:

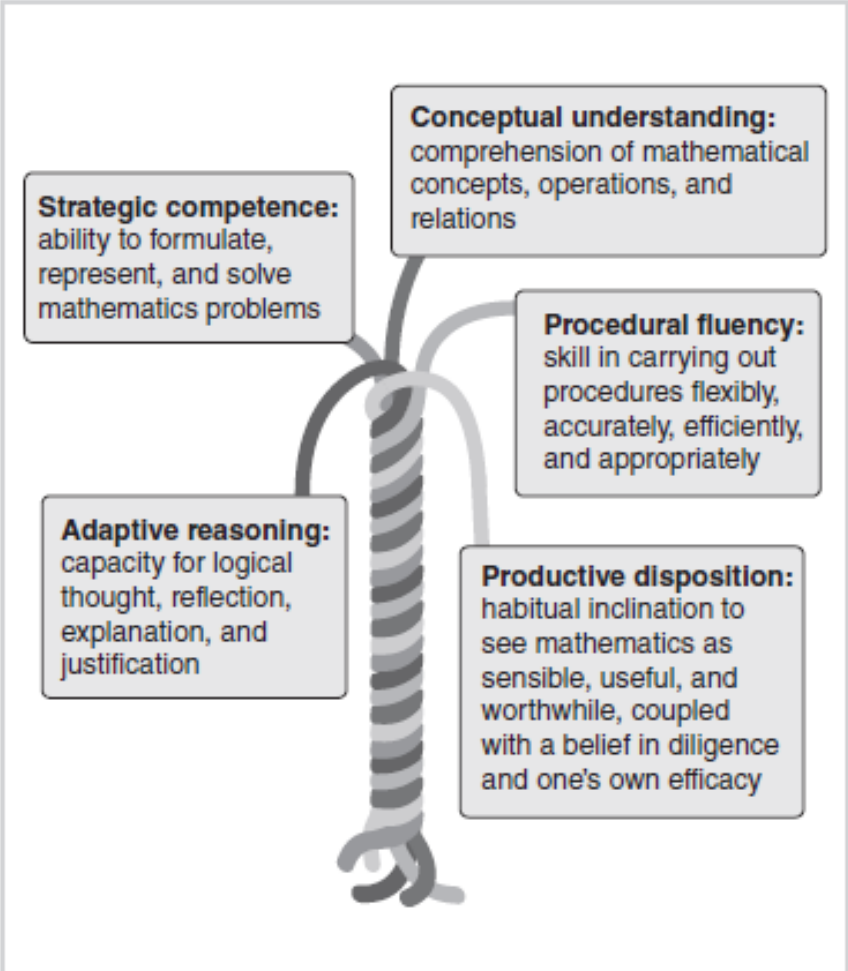
- control of facts
- control of processes
- conceptual understanding

He also notes that instruction for understanding must be different than the learning of basic skills and facts. Learning a new concept is dependent on what the learner already knows. As a learner advances through the Senior phase and into the FET phase, new concepts will increasingly depend on conceptual knowledge.

For example, understanding algebraic equations depends on the right conceptual understanding of the equal sign. If learners fail to gain conceptual understanding, it will become more and more difficult to catch up, as new conceptual knowledge depends on the old. Learners will become more and more likely to simply memorize algorithms and apply them without understanding.

To achieve conceptual understanding requires carefully thought out strategies that will ask of the learner to bring their past experience into present work and then use these to learn a new concept by connecting their experience to understanding.

In the first topic for Grade 10 in term 1 (Algebraic expressions) there is an investigation on the difference of two squares. Learners should already know how to factorise these from Grade 9. But do they understand the concept? This investigation leads them to visualising the factorising of the difference of two squares.
Time permitting participants can look at the investigation in the resource booklet.



Source: Reprinted with permission from Kilpatrick, J., Swafford, J., & Findell, B. (Eds.), *Adding It Up: Helping Children Learn Mathematics.* Copyright 2001 by the National Academy of Sciences. Courtesy of the National Academies Press, Washington, D.C.

5	2 hours	**NUMBER PATTERNS (Grades 10 – 12):** • **Linear Patterns** • **Quadratic Patterns** • **Series; sigma notation; sum to infinity**	**Facilitator:**	**What you will need:** • A3 & A4 paper • Handout • Six activities on card

1. Settle participants and tell them that we are going to look at the main concepts throughout the FET now.
2. Working with number patterns leads directly to the concept of functions in mathematics: a formal description of the relationships among different quantities.
3. Recognising number patterns is also an important problem-solving skill. If you see a pattern when you look systematically at specific examples, you can use that pattern to generalize what you see into a broader solution to a problem.
4. Investigating patterns allows students to experience the excitement and satisfaction of mathematical discovery. Working through investigations encourages learners to become risk-takers, inquirers and critical thinkers. The ability to inquire is invaluable and contributes to lifelong learning.

Grade 10 - Linear patterns

1. One point made in the diagnostic reports is that learners do not always know the difference between the position and the value of the term. This needs to be addressed from Grade 10 and right through to Grade 12.
2. In Grade 10, learners work with linear patterns again which they have already encountered in Grade 8 and 9 (and in fact before that in a more informal way).
3. Sadly, patterns are often rushed through by some teachers in the earlier grades (Intermediate and Senior phase). If a teacher doesn't know the importance of topics and how they will be used later, they often make their own lives easier by doing them briefly which does not lend itself to a learner arriving in Grade 10 with good conceptual understanding.
4. In the lesson plan book the start of the number patterns lesson in Grade 10 starts by asking learners to find the next three terms of several patterns and find the common difference which could be addition or subtraction.
5. Today we are going to look at some other ideas that could be used in a classroom where the teacher is concerned that there is not a good conceptual understanding of linear patterns.
6. Divide the group into six equally sized smaller groups.
7. Have the following six activities written on a card. Give one card and one piece of A3 paper to each group. The group should answer the question on the A3 sheet, so it will be easily visible to the whole group when it is complete. Ask participants to write the question on the sheet too and to use the whole sheet.

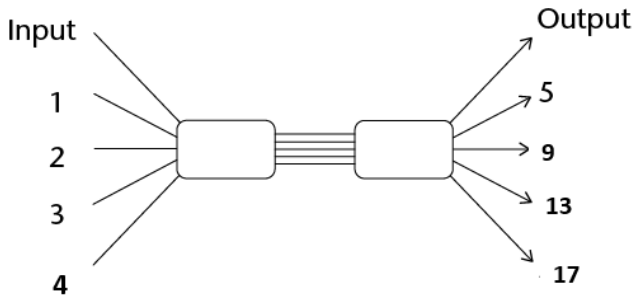
Using the function $y = 4x + 1$, complete the following table and plot the points on a cartesian plane.

x	1	2	3	4
y				

The following pattern is made with matchsticks:

Find the general term to represent the number of matches used in each pattern.

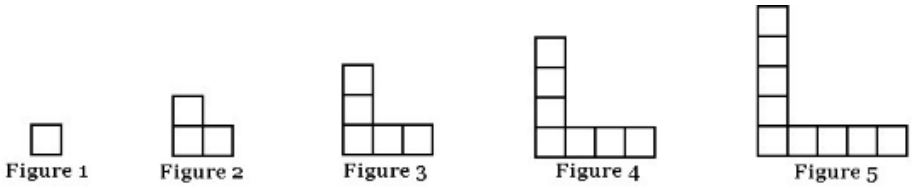
Find the rule in the following flow chart and make up a word problem to match the values given.



Using the function $y = 2x - 1$, complete the following table and plot the points on a cartesian plane.

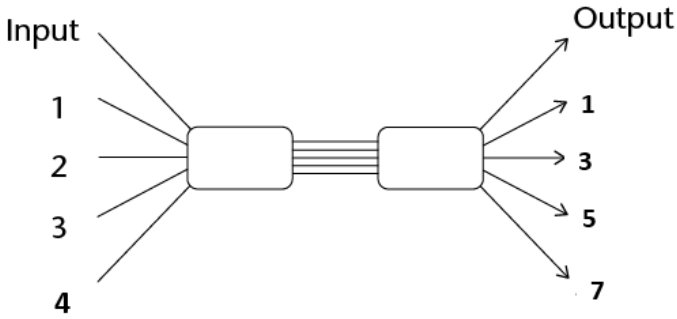
x	1	2	3	4
y				

The following pattern is made from squares:



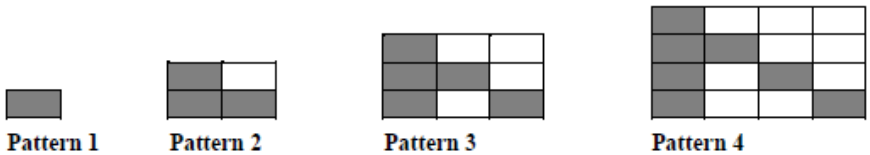
Find the general term to represent the number of squares used in each pattern.

Find the rule in the following flow chart and make up a word problem to match the values given.



8. Once participants have had enough time to complete the question allocated ask a volunteer from each group to come to the front and tell the rest of the group their question and how they went about answering it.
9. Once all six solutions are stuck up ask participants what they notice about the solutions.
    - Do any solutions have anything in common?
10. This is a key issue to showing that learners have a conceptual understanding of patterns – a linear pattern is a linear function which in turn could be posed as a word problem or a flow diagram.
    - Substitution is an important part of all these concepts – it is required to find coordinates to draw a function or to find a term in a certain position in the sequence
    - The solving of equations and an understanding of inverse operations is also important – they are required when finding the x −value if given the y −value for a function and they are also used to find the position of a term in a given sequence.
11. In the classroom, more time could be spent on this valuable exercise.
12. To complete the look at Grade 10 number patterns we are going to do an example that covers all the aspects required at this level.
13. Remind teachers at their training to discuss 'position' with learners. Point out that a position can only ever be positive and a natural number. It is not possible to have a term in a negative position or in a fractional position. It helps to clarify this point by using ordinal numbers to indicate position and cardinal numbers to indicate the value of the term, for example "*the value of the 3rd term in the match stick pattern is 13.*
    The diagnostic reports state that learners are often confused between a term and the position. They should spend some time on these definitions and ask directed questions using patterns being worked on to ensure that learners understand the difference. Use the patterns just worked with in the activity to demonstrate. For example, ask, *in pattern 1, what term is in the 3rd position? In pattern 2, what is the position of the term 7?* Discuss how in the flow diagram that position is the input and the term is the output and in functions the x −coordinate is position and the y −coordinate is the term.
14. Ask participants to turn to the example in their handout (page 10) and complete it with participants now.

Dark tiles (D) and light tiles (L) are used to create patterns on a floor. The first four patterns are shown below. For the patterns that follow the tiles are arranged in a similar manner.



a) How many dark tiles were used in pattern 5?

b) How many light tiles were used in pattern 6?

c) Write down the general term (D_n) for the number of dark floor tiles used in each pattern.

d) Write down the general term (L_n) for the number of light floor tiles used in each pattern.

e) Which pattern will have exactly 64 light floor tiles?

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Solutions:

a) 9 dark tiles

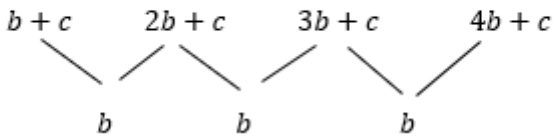
b) 25 light tiles

Before finding the general term of this linear pattern spend some time discussing that the common difference is the coefficient of n.

Write $T_n = bn + c$ on the board. Substitute $n = 1$ to find the first term. When this has been done, find the 2nd, 3rd and 4th terms by substituting $n = 2$; $n = 3$; $n = 4$

The terms are: $b + c$; $2b + c$; $3b + c$ and $4b + c$

List them and find the common/constant difference:



Using this information, we can conclude that: $T_1 = b + c$ and the constant difference is b

This knowledge can now be used to find the general term of the pattern:

c) 1 ; 3 ; 5 …

$T_1 = b + c$

$1 = b + c$

But b is the constant difference which is 2

$\therefore 1 = 2 + c$

$\therefore -1 = c$ $\quad\quad$ $\therefore \ D_n = 2n - 1$

d)

Pattern	1	2	3	4	5	6	…	9
Light tiles	0	1	4	9	16	25	…	64

I see the set of square numbers $\{0^2, 1^2, 2^2, 3^2, 4^2, 5^2 \ldots\}$

$L_n = (n-1)^2$

$e)\ L_n = (n-1)^2$

$64 = (n-1)^2$

$64 = n^2 - 2n + 1$

$0 = n^2 - 2n - 63$

$0 = (n-9)(n+7)$

$n = 9 \quad or \quad n = -7$

$\therefore n = 9$

15. Ask participants if they have any questions before doing one final example:
16. Find the missing term in the linear pattern: $15 \quad x \quad 27 \quad y$
17. Tell participants that this type of question requires the understanding that in any linear pattern, $T_2 - T_1 = T_3 - T_2$.

    This represents what the common difference is in any pattern. Hence the solution can be found:

$x - 15 = 27 - x$ $\qquad$ $y - 27 = 27 - x$

$2x = 42$ $\qquad$ $y - 27 = 27 - 21$

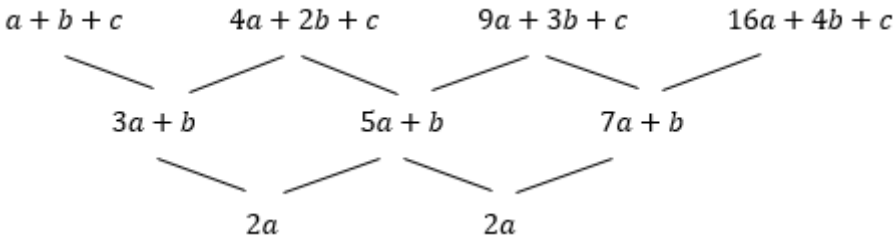
$x = 21$ $\qquad$ $y = 33$

Quadratic Patterns

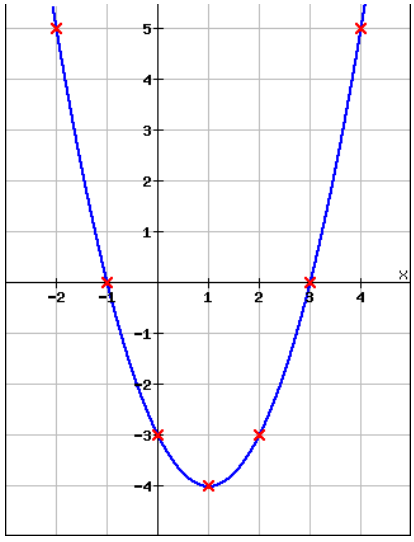
1. After being shown a quadratic sequence like in the case of the light tiles, many teachers teach learners that $2a = 2^{nd}$ difference; $3a + b = 1^{st}$ difference and $a + b + c = 1^{st}$ term and they are told to learn it, without necessarily understanding what it is all about and why.
2. Few are shown why. It is important that learners first experience the idea of quadratic patterns by finding the first difference as they have done in a linear pattern then going on to find the 2nd difference.
3. A key concept often overlooked (or not pointed out) is that the 1st line of differences IS a linear pattern. It often seems so obvious that it isn't mentioned and when someone does notice it, learners seem genuinely surprised. This again points out the lack of conceptual knowledge and being able to transfer the knowledge they do have to something a little different. They know that a linear pattern has a common difference but now they are working with quadratic patterns and that is their focus.
4. In the lesson plans quite a bit of time is spent on leading up to the fact that $T_n = an^2 + bn + c$ is the general form of a quadratic number pattern. For now, we are going to accept that this is the case.
5. Ask participants to find Terms 1, 2 3 and 4 in terms of a, b and c.

$T_n = an^2 + bn + c$ $T_1 = a(1)^2 + b(1) + c$ $T_1 = a + b + c$	$T_n = an^2 + bn + c$ $T_2 = a(2)^2 + b(2) + c$ $T_2 = 4a + 2b + c$
$T_n = an^2 + bn + c$ $T_3 = a(3)^2 + b(3) + c$ $T_3 = 9a + 3b + c$	$T_n = an^2 + bn + c$ $T_4 = a(4)^2 + b(4) + c$ $T_4 = 16a + 4b + c$

6. Use these to find the first line of differences and the 2nd line of differences.



7. Highlight the first term, the first difference and the 2nd difference. Discuss now why $2a$ = 2nd difference; $3a + b$ = 1st difference and $a + b + c$ = 1st term.
8. Use the diagram (page 11 in the handout) below and ask participants to answer the questions that follow:

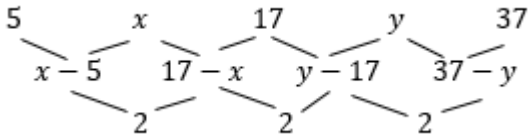


9. Ask participants to write down all the coordinates of the points shown. Once that has been done, ask them to circle the y −coordinates and write those values in a separate list.
    - This is a quadratic sequence, because it has a common second difference. What is this difference?
    - Determine the general term of the sequence.
    - Find the 10th term
    - How does the previous answer link back to the function represented?

10. Teachers could point out to learners that by using their knowledge of quadratic patterns they have even been able to find the equation of a given quadratic function that they haven't even learned about yet!
11. Do one more final example before moving on to Grade 12 work.
    Find the missing terms in the following quadratic pattern if 2 is the constant second difference:

$$5 \quad x \quad 17 \quad y \quad 37$$

12. Again, it is essential that learners understand the process to find the differences and use their knowledge that the second difference should be the same if it is a quadratic number pattern.
    Solution:



$\therefore 17 - x - (x - 5) = 2$ and $37 - y - (y - 17) = 2$

$17 - x - x + 5 = 2$ $\quad$ $37 - y - y + 17 = 2$

$-2x = -20$ $\quad$ $-2y = -52$

$x = 10$ $\quad$ $y = 26$

13. Ask if there are any questions or comments before moving on to Grade 12 work.

Grade 12

Points made in the diagnostic reports is that Sigma notation is not being interpreted correctly and learners need to analyse the type of sequence they are working with.

Unfortunately, algebraic skills are mentioned yet again. This is a problem which comes up regularly throughout a variety of topics. Teachers need to be reminded to spend time on algebra and not to rush through it.

Geometric Sequences

1. Formal work with geometric sequences is new to learners in Grade 12. For this reason, it is worth spending some time on. As with the previous patterns covered, we will look at the link with functions.
2. A geometric sequence is formed when one term is multiplied by a number to get each of the following terms. This number is called a common ratio. To find the ratio, knowledge of inverse operations is used. Division will be used to find the common ratio.
3. The important idea behind any geometric sequence is that $\frac{T_2}{T_1} = \frac{T_3}{T_2}$
4. Tell participants that we are going to use a geometric sequence to show how the general term is derived.
5. Consider the sequence: $4 ; 12 ; 36 ; \ldots$ We can see that the first term is 4.

$T_1 = 4 = \boldsymbol{a}$ x $\mathbf{3^0}$	(4 x 1)
$T_2 = 12 = \boldsymbol{a} \times \mathbf{3^1}$	(4×3)
$T_3 = 36 = \boldsymbol{a} \times \mathbf{3^2}$	$(4 \times 3 \times 3)$
$T_4 = 108 = \boldsymbol{a} \times \mathbf{3^3}$	$(4 \times 3 \times 3 \times 3)$

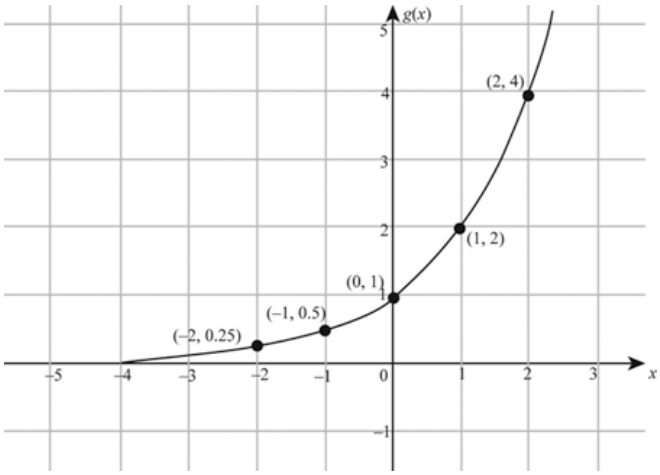
6. Ask participants to take note of the exponent. Learners need to (preferably) notice the rule for themselves. It is always one less than the position of the term. Term 3 could have also been written as:

$$T_3 = \boldsymbol{a} \times \mathbf{3^{3-1}}$$

7. It is this idea that can lead learners to finding the general term for themselves.

$$T_{\boldsymbol{n}} = a.r^{n-1}$$

8. Finding the general term for a geometric pattern comes fairly easily to learners especially as the information required is supplied on the formula sheet.
9. Use the diagram below (page 11 in the handout) and ask participants to answer the questions that follow:



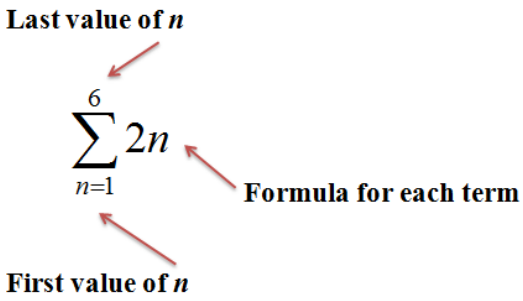
10. Ask participants to write down all the coordinates of the points shown. Once that has been done, ask them to circle the y −coordinates and write those values in a separate list.
    - This is a geometric sequence, because it has a common ratio. What is this ratio?
    - Determine the general term of the sequence.
    - Find the 5th term
    - How does the previous answer link back to the function represented?
11. Learners will move on to functions when this topic is complete. Bringing the basic concept of functions into this topic will assist learners in starting to prepare themselves for the new topic.

Series (Sum of sequences)

12. In general, learners cope fairly well with finding the sum of both linear and geometric sequences because the information required is supplied on the formula sheet. This, however, doesn't imply they have the conceptual understanding of why the formulae work.
13. Ask participants to turn to Lesson 5 point 12 in their Lesson Plan books. There are detailed points on how the sum of an arithmetic sequence formula is derived. Time permitting, it can be discussed at this stage.
14. Similarly, the derivation of the sum of a geometric series is explained in Lesson 6 from point 11 onwards.

Sigma Notation

15. As mentioned earlier, sigma notation seems to be a problem for learners. This implies more time should be spent on it.
16. Once teachers have gone through what the sigma notation is used for and what each of the three parts are made up of, a significant amount of time needs to be spent on changing sigma notation back into a worded question and vice versa. Teachers should also quiz learners regularly on what information is contained in each part.



(Handout page 11)

17. Using the example above (page 11 in the handout), ask participants to expand the information given then find the solution.

    $2(1) + 2(2) + 2(3) + 2(4) + 2(5) + 2(6)$

    (starting at term 1 as shown below the sigma sign and ending at term 6 shown at the top of the sigma notation)

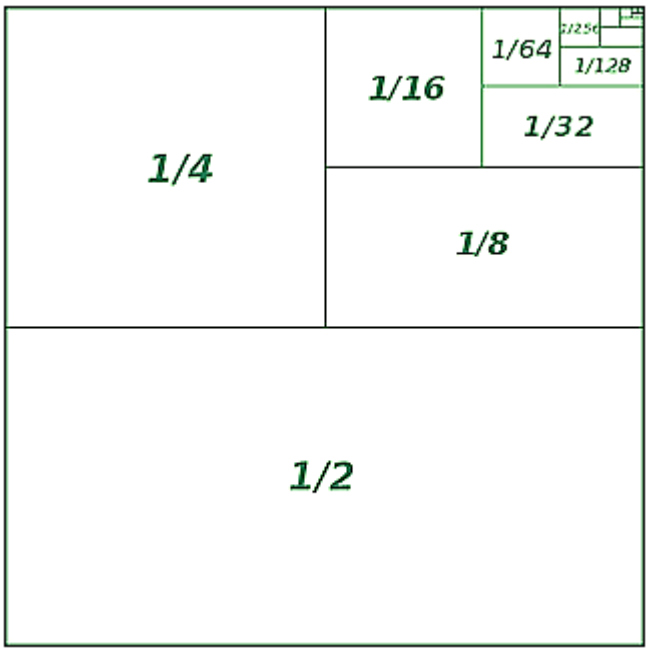
    $= 2 + 4 + 6 + 8 + 10 + 12$

    $= 42$

18. By doing a number of these as well as being given information to change into sigma notation, learners can develop a better understanding of what sigma notation is telling them.

Sum to infinity

19. In the lesson plans, using a geometric sequence and adding on one term at a time was used to assist learners in seeing that the more terms that were added the closer to a certain number the answer got.
20. Once again, functions can assist in the conceptual understanding of the sum to infinity but today we are going to look at another way. A knowledge of limits helps to understand the sum to infinity but as learners have not done Calculus at this stage this will make the abstract explanation more difficult.
21. If we want to find the sum to infinity of a geometric series where the common ratio is greater than one, each term will become larger, for example 4, 12, 36... where the common ratio is 3. We realize that this can go on for ever and the sum of the terms to infinity will be an infinitely large number. Therefore, it does not serve a purpose to calculate it.
22. However, if the common ratio lies between -1 and 1, we can calculate the sum of an infinite geometric series.
23. We will consider the concept using a piece of paper.
24. This offers a more visual understanding. Teachers should be encouraged to use more than one method. Learners have different learning styles and more than one way to explain a concept is always the ideal scenario.
25. Hand one A4 sheet of paper to each participant. Ask them to fold the paper in half and draw a line down the crease. Label ONE side $\frac{1}{2}$. On the unlabelled half, draw a line to cut it in half again. Label one of the new halves (half of a half) $\frac{1}{4}$. On the unlabelled section, draw a line to cut it in half again. Label one of the new halves (half of a half of a half) $\frac{1}{8}$. Continue for as long as possible until the areas are too small to label.
26. The paper should look as follows:

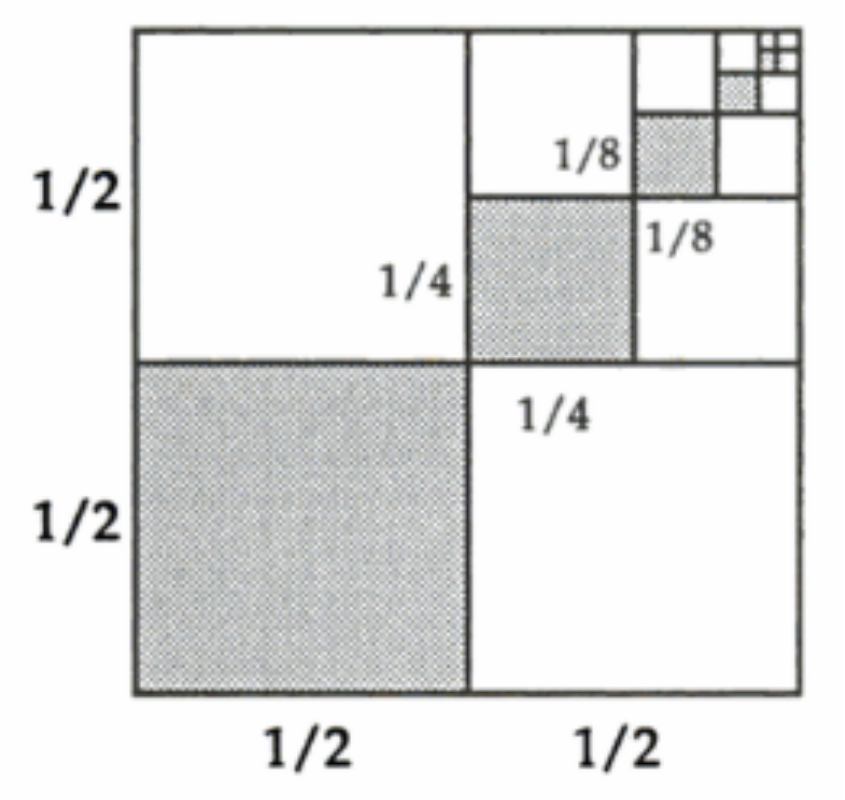


27. Write the following sequence on the board:

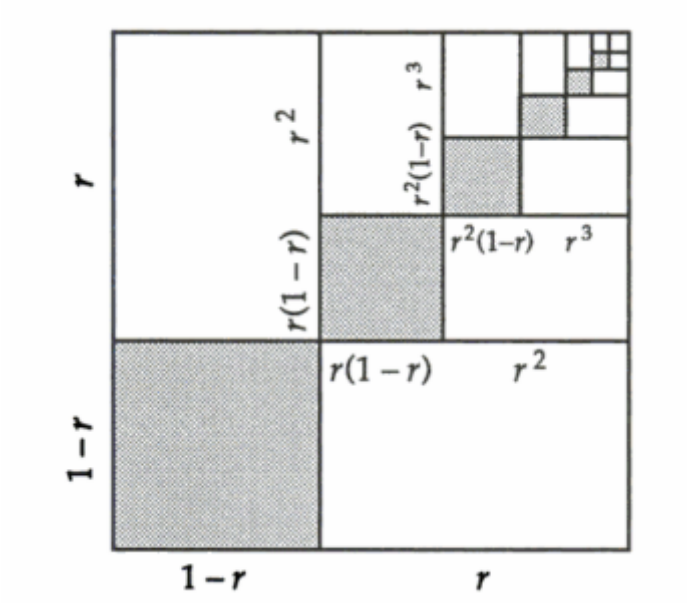
$$\frac{1}{2}\,;\frac{1}{4}\,;\frac{1}{8}\,;\frac{1}{16}\ldots\ldots$$

28. Ask participants which number the sum of this series is approaching, therefore what the sum to infinity of this series would be. The visual aid should be of assistance – calculations should not be necessary, because it becomes clear that the sum of all the fractions are fast approaching the complete area of the paper – one!
29. Once the answer of 1 has been established, include the one term at a time that comes before the first term of the sequence above and discuss the sum to infinity further. Again, the answer should be found from thinking and using the paper as a guide.
30. $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$ (2)

    $2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$ (4)
31. Give one last demonstration using a piece of paper of how the sum to infinity formula is derived. Work with participants to first note how $\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\frac{1}{256}\ldots$ would equal $\frac{1}{3}$.



32. Use this idea to demonstrate using piece of square paper with a length of 1 unit and using r as a measurement on the length of one unit to derive the general sum to infinity formula.



$$(1-r)^2 + r^2(1-r)^2 + r^4(1-r)^2 + \ldots = \frac{(1-r)^2}{(1-r)^2 + 2r(1-r)} = \frac{1-r}{1+r}$$

$$1 + r^2 + r^4 + \ldots = \frac{1}{1-r^2}$$

$$a + ar + ar^2 + \ldots = \frac{a}{1-r}$$

33. Tell participants that we have come to the end of our look at Number Patterns.
34. Thank them for their attention and ask if anyone has any questions.
35. Just for a bit of fun – ask participants to turn to page 33 of their handouts. There they will find two problems involving patterns. These types of patterns have gone viral in the last year or so.
    Ask: can you get the answers??

6	2 hours 30 minutes	**TRIGONOMETRY (Gr 10 & 12):** • **Introduction to Trigonometry** • **Compound and double angles**	**Facilitator:**	**What you will need:** • Whiteboards • Markers • Training handout • Deriving of compound and double angle formulae exercise

1. Start by discussing the importance of starting a new topic in the correct way. It needs to be taught in a way that will promote conceptual understanding.
2. Today we are going to look at an investigation for Grade 10 learners that could assist them in gaining a good conceptual understanding of the basics and have a look at the two new concepts in Grade 12.
3. Discuss the issues according to the NSC diagnostic reports:

   - Incorrect selection and application of reduction formulae
   - Ignoring the instruction 'without the use of a calculator'
   - Algebraic skills (such as using brackets in substitution) let learners down
   - Not recognising compound angles or double angles.

4. Ask participants if they have any questions or comments regarding these points.

Introducing Trigonometry

5. Teachers should show their enthusiasm when teaching a new topic. Learners that find mathematics challenging often feel they are 'never going to do well' because they are faced with the same topics every year. Although trigonometry certainly needs knowledge of previously learned skills, learners can still find the excitement in learning a topic that they have never really heard of until now.
6. Tell participants that we are going to do an investigation that could work well for learners to rasp the idea of why trigonometry 'works'.
7. The introductory lesson in the Lesson Plan booklet covers the same ideas but the teacher works through the concepts with learners whereas the investigation would be more for learners working on their own and discovering more for themselves.
8. Ask participants to find the investigation in their training handout (pages 12 to 17). Tell them they will have 20 minutes to complete it. A ruler will be required.
9. Once time is up, ask participants for comments on the task. Ask if they think this would assist learners in gaining a deeper understanding of trigonometry.
10. Thank participants for their contributions.

Compound angles and Double angles

1. Tell participants that these are the only new concepts covered in Grade 12. Learners need to have a firm grasp of all the concepts from Grade 10 and 11. Without this knowledge, knowing about compound and double angles will be of very little use.
2. For this reason, a 2-hour revision lesson, in which notes are given for learners to have every opportunity to revise and ask questions, is the start of the topic.
3. To introduce compound and double angles, a very basic investigation is given to learners in which calculator work is used to find what combinations of trig functions of angles are equal. This way, learners can 'discover' the rules for themselves.
4. Teachers should note that this investigation should not be used as an assessment. It is merely a tool to introduce the new concepts.
5. Ask participants to turn to the investigation in their handout (pages 18 to 20) and spend 5 – 10 minutes looking through it and discussing it with a partner.
6. Ask if anyone has any questions or comments.
7. Point out that once learners have had a subsequent lesson where they are introduced more formally to the identities, they will be shown how to derive all the formulae by using $\cos(A - B) = \cos A.\cos B + \sin A.\sin B$ as the starting point.
8. Hand out a prepared A3 sheet to every alternate participant with this identity written in the centre and space provided to complete the other compound angle identities as well as the double angle identities. A resource is available in the training handout on page 30.
9. Tell participants to work in pairs to complete the sheet. Allow about 15 minutes.
10. Once enough time has been given for everyone to complete their sheet, go through the exercise to confirm that everyone is happy with the procedure to derive the compound and double angle identities.

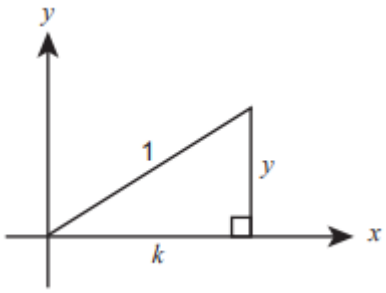
Using compound and double angles in Grade 11 concepts

11. Tell participants that we will do a few examples of previous concepts learned that now require a knowledge of double and compound angles.
12. Tell participants that we shall focus on three types of applications using trigonometric identities, namely *determining values, finding general solutions and proving identities.*

Example 1 (page 21 in the handout)

If $cos\ \theta = \ k$, and $\theta\ \epsilon\ [0^0; 90^0]$, determine the value of $sin\left(\frac{\theta}{2} + \ 30^0\right).cos\left(\frac{\theta}{2} + \ 30^0\right)$ with the aid of a diagram and without the use of a calculator.

Solution:



$cos\ \theta = \frac{k}{1}$

$$y^2 + k^2 = 1^2$$
$$y^2 = 1 - k^2$$
$$y = \sqrt{1 - k^2}$$

$sin\left(\frac{\theta}{2} + 30^0\right).cos\left(\frac{\theta}{2} + 30^0\right)$

$= \frac{1}{2}\left[2\sin\left(\frac{\theta}{2} + 30^0\right).\cos\left(\frac{\theta}{2} + 30^0\right)\right]$

$= \frac{1}{2}[\sin(\theta + 60^o)]$

$= \frac{1}{2}(\sin\theta.\cos 60^o + \cos\theta.\sin 60^o)$

$= \frac{1}{2}\left(\left(\sqrt{1-k^2}\right)\left(\frac{1}{2}\right) + (k)\left(\frac{\sqrt{3}}{2}\right)\right)$

$= \frac{\sqrt{1-k^2}}{4} + \frac{\sqrt{3}k}{4}$

$= \frac{\sqrt{1-k^2} + \sqrt{3}k}{4}$

Example 2: (page 21 in the handout)

Find the general solution of:

$$\cos 2\theta - \cos\theta + 1 = 0$$

Solution:

$$2cos^2\theta - 1 - \cos\theta + 1 = 0$$
$$2cos^2\theta - \cos\theta = 0$$
$$\cos\theta(2\cos\theta - 1) = 0$$
$$\cos\theta = 0 \quad \text{or} \quad 2\cos\theta - 1 = 0$$
$$\cos\theta = \tfrac{1}{2}$$

RA $= 90^o$	RA $= 60^o$
Quad 1:	Quad 1:
$\theta = 90^o + k.360^o$	$\theta = 60^o + k.360^o$
Quad 4:	Quad 4:
$\theta = 360^o - 90^o + k.360^o$	$\theta = 360^o - 60^o + k.360^o$
$\theta = 270^o + k.360^o$	$\theta = 300^o + k.360^o$

$k \epsilon Z$

Example 3: (page 22 in the handout)

Prove the identity:

$$\frac{\cos x + \sin x}{\cos x - \sin x} - \frac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$$

Solution:

$$\text{LHS} = \frac{(\cos x+\sin x)(\cos x+\sin x)-(\cos x-\sin x)(\cos x-\sin x)}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$= \frac{cos^2x+2\sin x\cos x+sin^2x-(cos^2x-2\sin x\cos x+sin^2x)}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$= \frac{cos^2x+2\sin x\cos x+sin^2x-cos^2x+2\sin x\cos x-sin^2x}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$= \frac{4\sin x\cos x}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$= \frac{4\sin x\cos x}{cos^2x-sin^2x}$$

$$= \frac{2.2\sin x\cos x}{\cos 2x}$$

$$= \frac{2\sin 2x}{\cos 2x}$$

$$= 2\tan 2x = \text{RHS}$$

13. Tell participants that we have come to the end of our look at Grade 12 trigonometry. Ask if there are any questions.
14. Thank everyone for their participation.

7	2 hours	**FUNCTIONS AND INVERSES (Grade 12)** • **Introducing the concept of a function and its inverse**	**Facilitator:**	**What you will need:** • Whiteboards/A4 paper • Markers • Training handout

1. To understand an inverse function, learners first need to understand a function. They should be able to define it and describe it.
2. Discuss the issues according to the NSC diagnostic reports:

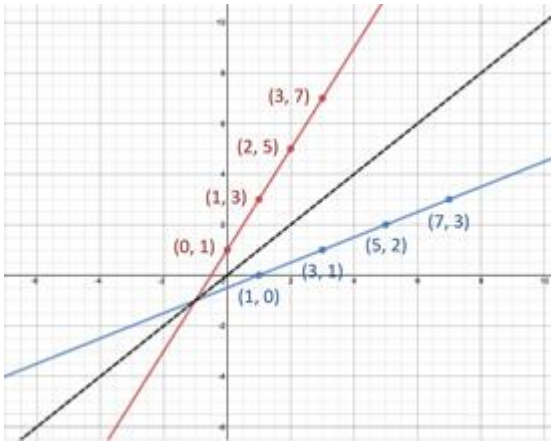
   - Not understanding a restricted domain
   - Not making the link between the domain of a function and the range of its inverse
   - Confusing inverse of a function with derivative of a function

3. Note that the last point will only occur in Term 2 when learners cover Calculus, but it is still worth pointing out at this stage. Teachers need to be aware of the potential problems.
4. Ask participants to use their whiteboards to write down their understanding of a function.
   Ask them to hold them up when complete.
5. Discuss some answers with the group.
6. Definition: A relation for which each element of the domain (x) corresponds to exactly one element of the range (y). For every x −value there is only one possible y −value.
   To test for a function on a graph, use the 'vertical line test' – run a ruler from left to right. If your ruler only ever touches the graph in one spot, it is a function; meaning that for each element of the domain (the x value) there is exactly one, and only one element of the range (y value).
7. Ask participants to use their whiteboards to write down their understanding of domain and range.
   Ask them to hold them up when complete.
8. Discuss some answers with the group.
9. Definitions:
   Domain – All the possible x-values that are covered by the function/graph.
   Range − All the possible y-values that are covered by the function/graph.
10. Ask participants to draw a sketch of a function on their whiteboard and state the domain and range. It need not be a well-known function.
11. Discuss a few different contributions.
12. Ask participants to draw a sketch of a relation that is not a function and state the domain and range.
13. Discuss a few different contributions.

From this point on, it will be important that the Cartesian planes are quite accurate and not roughly drawn.

14. Ask participants to draw a Cartesian plane and the line $y = x$. Next, ask them to plot the points (2 ; 6) and (1 ; -4). Once everyone has completed the sketching, ask them to reflect each point in the line $y = x$.
15. Confirm that the new points plotted are: (6 ; 2) and (-4 ; 1)
16. Note on your own sketch how it is easy to see that it is a reflection by joining each pair of points and show that the line would cross the line $y = x$ at a right angle and the line segments from the point to the line are equal.
17. Ask participants to clear the points but keep the Cartesian plane and line $y = x$. Ask them to draw in the line $y = 2x + 1$. Once this has been done, ask them to draw in a reflection of the line $y = 2x + 1$ in the line $y = x$.

    (Tell participants that when they are doing it with learners in their classrooms, learners could easily fold the paper along the line $y = x$ to check where the new line should go)
18. Ask participants to plot at least three points on the original line $y = 2x + 1$. Using what they know about reflections, plot the corresponding reflections on the other line.



19. Point out that we have drawn a function and its inverse. We have also used this to note the rule that any x −value on the function is now the y −value on the inverse and any y −value on the function is now the x −value on the on the inverse.
20. Use this information to confirm the equation of the 2nd line that was drawn.

$$y = 2x + 1$$

∴ the inverse of the function is:

$$x = 2y + 1$$

In standard form:

$$-2y = -x + 1$$

$$y = \frac{1}{2}x - 2$$

21. Ask participants to clear their boards and draw in the line $y = x$ and the function $y = 2^x$ marking both the y −intecept and horizontal asymptote clearly. Once this has been done, ask them to draw a reflection of the function in the line $y = x$.
22. Point out that we have drawn a function and its inverse. We have also used this to note the rule that any x −value on the function is now the y −value on the inverse and any y −value on the function is now the x −value on the on the inverse.
23. Discuss the intercepts, asymptotes and domain and range of the original function and its inverse.

	Intercept with axes	Asymptote	Domain	Range
Function	$(0; 1)$	$y = 0$	$x \in \mathbb{R}$	$y \in (0; \infty)$
Inverse function	$(1; 0)$	$x = 0$	$x \in (0; \infty)$	$y \in \mathbb{R}$

Point out the relationship with x and y within the functions and their inverses.

24. Use this information to confirm the equation of the 2nd line that was drawn.

$$y = 2^x$$

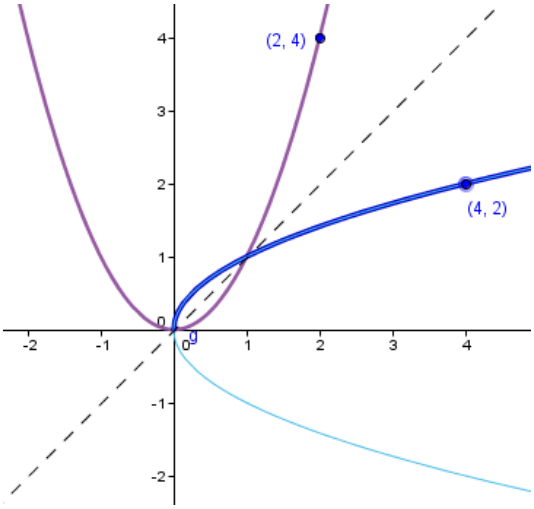
∴ the inverse of the function is:

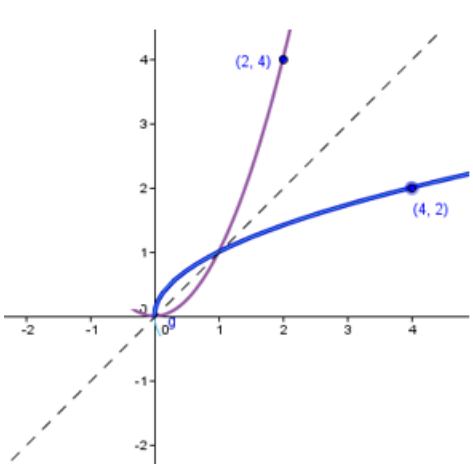
$$x = 2^y$$

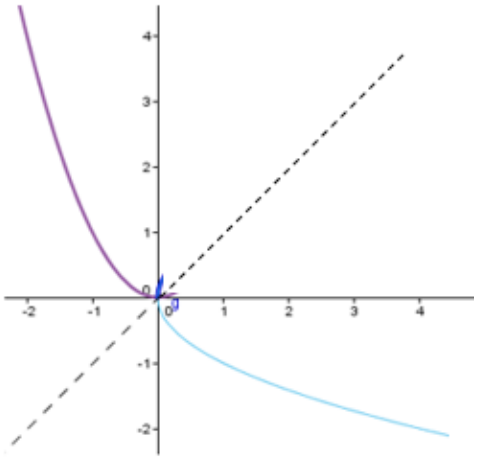
In standard form:

$$y = \log_2 x$$

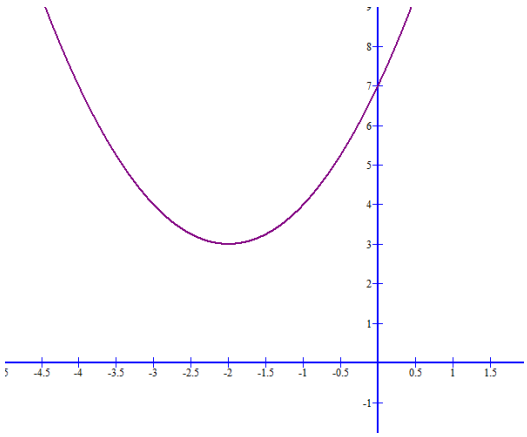
25. Ask participants to clear their boards and draw an accurate Cartesian plane, making the measurements between the markings regular. Draw in the line $y = x$ and the function $y = x^2$; Ask that they mark at least 5 points on the quadratic function.
26. Using the points marked, write down 5 coordinates that will be on the inverse of this function.
27. Plot these points and draw the graph.
28. Ask participants: Is the inverse of the function also a function? (no, there are 2 y −values for every possible x −value except zero, which renders the inverse not a function)
29. Ask: where is the key part of the new graph where we could cut it off in order to ensure it is a function? (the turning point)
30. Use the following diagrams to discuss how the domain of the function can be restricted to ensure that the inverse graph is also a function:
31. Discuss how the suggested restrictions below make sense because the minimum value of the parabola at its vertex is the point where the two symmetrical parts of the parabola meet. If you restrict the domain of the function to the x-values in either half, there would be no instance where an x −value in the graph of the function's inverse has two y −values.







32. Make note and refer to fact that it is the x −coordinate of the turning point that is the key to restricting the domain.
33. Draw one more quadratic function to discuss with participants: $y = (x + 2)^2 + 3$



34. Discuss the domain and range of the original function and of the inverse graph.

	Domain	Range
Function	$x \in \mathbb{R}$	$y \in [3; \infty)$
Inverse	$x \in [3; \infty)$	$y \in \mathbb{R}$

35. Ask participants to draw the inverse graph. Once this has been done, ask them to restrict the domain in two different ways in order that the inverse graph will be a function.

Option 1	Restrict the domain to: $x \geq -2$
Option 2	Restrict the domain to: $x \leq -2$

36. Discuss how this will affect the domain and range of the new inverse function.

Option 1	$x \geq -2$ The range of this function will no longer be $y \in \mathbb{R}$ but will now be: $y \geq -2$
Option 2	$x \leq -2$ The range of this function will no longer be $y \in \mathbb{R}$ but will now be: $y \leq -2$

37. Ask participants if they have any questions or comments or if they would like to discuss some aspects further.
38. Tell participants that we have come to the end of our look at functions and their inverses. Thank participants for their attention and ask if there are any questions.

8	1 hour 30 minutes (30 minutes + 1 hour)	PLANNING: Lesson Demonstrations	Facilitator:	What you will need: • Lesson plan booklets

BRIEFING AND INSTRUCTIONS

1. Tell participants that we are now going to really engage with:
   a. The structure of the lesson plans
   b. The routines and activities in the lesson plans
   c. The content in the lesson plans
2. Explain to participants that in the next session they will be presenting demonstration lessons in groups.
3. Every participant must have his/her lesson plan and resource pack to help them prepare.
   a. Participants should work in groups of 3 or 4 (depending on the size of the group).
   b. Each group must select a lesson in any grade.
   c. The choices are in the handout on page 23.
4. Give each group some flipchart papers and markers for their preparation.
5. Tell the participants that they will now prepare a lesson to present to the rest of the group.
6. Explain that they will have the rest of this session for preparation. They need to work quickly and efficiently.
7. Next, explain that groups will have 20 minutes for the actual presentation, which will be followed by a 5 to 10-minute plenary discussion.
8. Explain that when they present a lesson, they must act as the teacher and address the rest of the group as if they are the class. They must actually teach the lesson to the other participants.
9. Participants must not tell their colleagues HOW they would teach the lesson – the lesson must be taught.
10. Ask all other participants to please play the role of the 'class'.

PRESENTATION REQUIREMENTS

11. Tell participants, that because of the time limitations, presentations must be well prepared, concise and to the point. There is no time for greetings or chat – presenters must get straight into the lesson.
12. The presentation will be stopped after the allocated time – use an alarm on your phone to keep time.
13. The presentations must include:

a. **Introduction** (in which 'learners' are reminded of where we finished off yesterday and what they already know)
b. **Direct Instruction** (in which the new concept in the topic will be taught)

This can also be found on page 23 in the handout.

14. Explain that in their presentations, participants need to note:
    a. When teaching the conceptual development, they should not provide learners with the answers. Learners should be encouraged to think about the answers. They should be challenged.
    b. It is fine for learners to get things wrong – it is important that they are not criticized for trying and that they are shown how to solve problems and come to an understanding.
15. Tell participants that the other participants will comment on their presentations using the tool starting on page 24 in the handout.

LESSON PREPARATION

16. While participants prepare their lessons, move around the venue and assist/ provide guidance wherever applicable or necessary.
17. Ensure that:
    - participants are preparing sufficiently
    - all participants are involved
    - the board work is being neatly prepared
    - the presentations look solid and meaningful
    - that practical work is included
18. Where possible, remind participants that they need to present model lessons that demonstrate their:
    - Concept and content understanding; and
    - Understanding of routines embedded in the lesson plans.

    Remind participants of how much time they have left to prepare.
19. If there is time, move around the venue while participants are preparing and assist / provide guidance wherever applicable or necessary.
20. Suggest a division of labour between group members – someone can prepare the 'board work' on flipchart paper, someone can present the introduction, and someone can do the actual direct instruction. If there are many worked examples to be done, these can also be split amongst participants (i.e.: the 'teacher' can change during the presentation).

21. Ensure that:
    - participants are preparing sufficiently
    - all participants are involved
    - the board work is being neatly prepared
    - the presentations look solid and meaningful
22. Before you close for the day, remind participants that they need to present model lessons that demonstrate HOW TO TEACH A LESSON for **conceptual understanding**.
23. Tell participants that they will have another hour just before the demonstrations in order to consolidate. They must not leave new planning for that time though.
24. Check to see that participants have all the resources they require.

9	2 hours 30 minutes	LESSON DEMONSTRATIONS AND FEEDBACK	Facilitator:	What you will need:

1. After participants have had another hour to complete their preparation, tell participants that you are really looking forward to their presentations.
2. Remind participants of these criteria explained in the briefing yesterday:

> Because of the time limitations, presentations must be well prepared, concise and to the point.
> The presentation will be stopped after the allocated time – use an alarm on your phone to keep time.
> The presentations must include:
>
> a. **Introduction**
> b. **Direct Instruction**

3. Remind participants that their presentations should take 20 minutes.
4. Stop the presentations after the allocated time. You must be strict with the time, otherwise not everybody will have a chance to present.

OBSERVATIONS

5. Tell participants to use the observation tool on page 23 of the handout.
6. Explain that they should use the lesson reflection tool to help them evaluate and give feedback on the lesson demonstrations that they watch.
7. Explain that this tool has been designed for teachers to reflect on their own teaching. For this reason, they will not use the first section on preparation (*). They should however, complete all the other sections.
8. Read through the main headings in the tool with the participants so that they know what to look out for when they observe the lessons.
9. If a group does not manage to do very much within the time, speak to them about time management. Explain that they will not have much more time than this in class to do these presentations. Discuss how the group could speed up.
10. Ask the group to state the grade, topic and subtopic for the lesson that they will present.
11. After each lesson demonstration encourage conversation for critical and constructive feedback.
    Encourage all participants to take part in the feedback session.
    Ensure that all feedback starts with something positive – our approach is to build confidence!

10	1 hour	**ASSESSMENT** • **Types of assessment** • **Assessment for learning** • **Assessment of learning**	**Facilitator:**	**What you will need:** • Training handout • Videos

1. Settle participants and tell them that we are going to look at different types of assessment now and have a discussion on how assessments can be improved in order to make learning more meaningful.
2. Tell participants that we are first going to look at what the CAPS document tells us about assessments. Ask participants to turn to page 27 in the handout where they will find the four types summarised.
3. Before discussing each of these with participants, ask them to take a few minutes to write what they understand each type of assessment is.
4. Once this has been complete, clarify the descriptions as follows, discussing further where necessary.

The types of assessment required according to CAPS

Baseline assessment

- Assessing the baseline skills of learners prior to teaching a topic
- Assists the teacher in planning a lesson and pitching it at the appropriate level
- Results should not be used for promotion purposes.

Diagnostic assessment

- Used to inform the teacher of a learner's problematic areas that could hinder performance
- Formed by two broad areas: (a) content-related challenges (difficulties in comprehending) and (b) psycho-social factors (negative attitudes, math anxiety etc
- Should result in interventions if necessary
- Results should not be used for promotion purposes.

Formative assessment

- Should aid the teaching process. Assessment FOR learning.
- Can be verbal or a quick assessment at the end of a lesson
- Should provide constant feedback to learners
- Results should not be used for promotion purposes

Summative assessment

- Carried after a topic or cluster of topics is complete
- Assessment OF learning
- Results are recorded and used for promotion purposes
- Examples: assignments; investigations; tests; examinations

5. Show participants the following video clips. Ask participants to turn to page 28 in the handout. While the clips are played; participants should make a note or two while watching each clip. The note can be a point they didn't know before or a point that was of particular interest to them.

- Assessment for Learning vs. Assessment of Learning
- Formative vs. Summative vs. Diagnostic Assessment
- Assessment for Learning

6. Once the clips have been played, ask participants which clip was most beneficial to them. Have a 10-minute discussion on what participants liked or learned from the information provided in the video clips.
7. After the discussion, discuss the three educational principles that should guide how we assess:

Principles for assessing mathematics learning

The content principle

- Assessment should reflect topics and applications that are critical to a conceptual understanding of mathematics
- Assessments should also reflect the processes required; for example, reasoning, problem solving, communication and connecting ideas.

The learning principle

- Assessment should enhance mathematics learning
- Students use the assessments that are given to determine what others consider to be significant

The equity principle

- Every learner should have a fair chance to demonstrate his or her best work

8. Ask participants to read the following statement carefully and to think about it.

All educational actions must support students' learning of more and better mathematics; assessment is no exception.

9. Tell participants that we have come to the end of our brief look at Assessment and in the Term 3 & 4 training, there will be a component on how to set an assessment.
10. Thank them for their attention and ask if anyone has any questions.

11	1 hour	ORIENTATION TO THE TRAINER'S GUIDE	Facilitator:	What you will need: • Trainer's guide • Training handout

Note: If you have any extra time, spend it on this activity, particularly points 4 and 6.

1. Settle participants with all their materials.
2. Give each participant a copy of the **Trainers Guide** and **Training Handout**.
3. Explain to participants that the **Trainers Guide** and **Training Handout** contains all the activities for the Term 1 & 2 training.

4. **Planning the training session:**
   a. Tell participants to look carefully at the programme at the front of the trainer's guide.
   b. Go through this programme and tell participants which activities to complete when training teachers. (This will depend on the numbers of hours for this training)

5. **Orientation to the guide and handout:**
   a. Go through each activity in the trainer's guide and look at the corresponding resources or section in the training handout.
   b. Work with participants to summarise the key steps and points of each activity.
   c. After you have done this for each activity, revise the order of activities, and the main points for each activity. For example:
      - Start with the **Guidelines for facilitators and participants.**
      - You have 30 minutes for this.
      - You must: tell participants to think about when real learning takes place; get them to discuss this with a partner; write a list of key points; discuss what is the same and different between a classroom and an adult training event; create a list of guidelines for facilitators and participants; ask participants to follow guidelines and commit to following facilitator guidelines.

6. **The point of doing this is try and ensure that trainers clearly understand each activity and internalise as much of the workshop as possible.**
7. **If time allows, allocate different activities to volunteers, and ask them to present a 'dry-run' presentation of the activity. After each presentation, ask the other participants to give feedback based on the following:**

a. Was the activity presented correctly?
b. Did the main points of the activity come across clearly?
c. Did the presenter give clear instructions?
d. Was the presenter audible?
e. Did the presenter interact effectively with participants?
f. Did the presenter manage time effectively?

8. **Finally, thank participants for their presentations, and hold a closing discussion:**
   a. Ask: Which activities are you worried about presenting or facilitating? Why?
   b. Try to address any concerns that participants may have.
   c. Wish participants well for their training.

12	1 hour	TRAINING OF TEACHERS: PLANNING SESSION	Facilitator:	What you will need:

1. Explain that this is an opportunity for Coaches and Subject Advisors to work together to talk about the logistics of the teacher training sessions in their district.
2. Allow participants to sit together in groups and discuss relevant issues.
3. If all the logistics are sorted, then participants should talk about co-facilitation, and who will present which activities.
4. They should also speak about resources in their district, like data-projectors and speakers.

13	30 minutes	POST TEST	Facilitator:	What you will need: • Copies of post test

1. Work together to hand out copies of the post-test to participants.
2. Remind participants that the purpose of these tests is to measure the success of the training, not to measure the scores of individuals.
3. Remind participants of the test conditions and available time.
4. As participants complete the test, walk around and offer practical assistance if needed.
5. Once time is up, help to collect and collate tests in an orderly fashion.

14	30 minutes	FINAL QUESTIONS AND ANSWERS CLOSURE AND EVALUATION	Facilitator:	What you will need:

1. Settle participants so that you have their attention.
2. Remind participants that we want them to IMPLEMENT THIS TRAINING IN A MEANINGFUL WAY.
3. Ask participants to think through all the materials, content, skills and information they have engaged with in this workshop. Give them time to look through materials as they do this.
4. Next, ask participants if they have any final questions.
5. Answer each question as clearly as possible. Where appropriate, involve participants in answering.

CLOSURE AND EVALUATION

1. Settle participants so that you have their attention.
2. Show participants the video: 'Teaching is tiring – but worth it'
   https://www.youtube.com/watch?v=ZSjIzi0SkjU
3. Give participants an evaluation form, briefly take them through the form, and then ask them to please complete it thoughtfully and carefully.
4. Collect the completed evaluation forms.
5. Call participants to attention and ask them to share some of the positives that they take away from this training. This can be absolutely anything: new content that they have learned or clarified; a new skill; a better understanding of the curriculum; new enthusiasm for their job; a closer working relationship with a colleague; etc.
6. Document what participants say for your report.
7. Thank the participants for their ongoing commitment to education, and to the development of South Africa and wish participants well for their own training.

Thank you for your ongoing dedication and commitment to this cause.