GR 4 - 9 MATHEMATICS

TRAINING HANDOUT

TERM 1 & 2 2019

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MATHEMATICS
is not about
numbers, equations,
computations, or
algorithms:
it is about
UNDERSTANDING.

William Paul Thurston

TRAINING PROGRAMME

GRADES 4 - 9 MATHEMATICS

TERM 1 & 2 2019

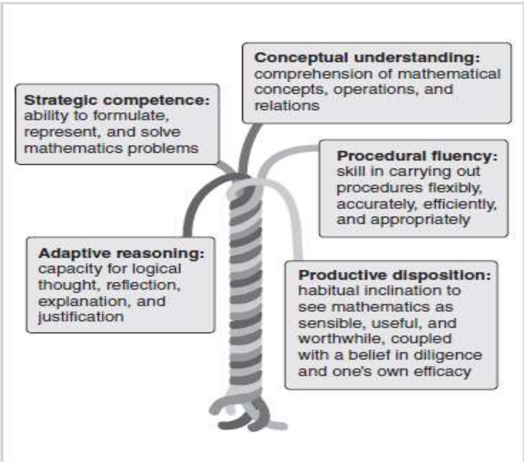
	TIME	ACTIVITY	TRAINER WORKSHOP	TEACHER WORKSHOP
1	30 minutes	Welcome, housekeeping and updates Introductions, reflections and agenda		
2	30 minutes	Pre-training activity		
3	30 minutes	Mathematics Teaching and Learning Framework		
4	1 hour 30 minutes	Conceptual understanding in a learning centred classroom – the basics and principles		
5	3 hours	Whole numbers (Grade 4 – 9)		
6	2 hours	Constructions (Grade 7 – 9)		
7	2 hours	2-D shapes and symmetry (Grade 4-6)		
8a	30 minutes	Selection of topics and preparation for participant presentations		
8b	3 hours	Presentations on conceptual understanding in a learning centred classroom		
9	1 hour 30 minutes	Orientation to trainer's guide		
10	1 hour	Training of teachers: planning session		
11	30 minutes	Post test		
10	30 minutes	Final questions and answers Closure and evaluation		

CONCEPTUAL UNDERSTANDING IN A LEARNING CENTRED CLASSROOM

INTRODUCTION

The new framework for mathematics rests on five pillars:

- Conceptual understanding
- Procedural fluency
- Strategic competence
- Mathematical reasoning
- The learning centred classroom



Source: Reprinted with permission from Kilpatrick, J., Swafford, J., & Findell, B. (Eds.), *Adding It Up: Helping Children Learn Mathematics.* Copyright 2001 by the National Academy of Sciences. Courtesy of the National Academies Press, Washington, D.C.

The ideas of the new framework for reformed mathematics in South Africa are derived and adapted from Kilpatrick's idea of "five strands in teaching and learning mathematics".
For our situation, *adaptive reasoning* has been replaced with *mathematical reasoning* (two wordings with the same meaning) and *productive disposition* has been replaced with *learning centred classroom,* which is a completely different idea in our framework.

It is suggested that conceptual understanding is created in a learning centred classroom, therefore this is our focus in Term 1 and 2, which is a shift away from the focus on procedural fluency.

CONCEPTUAL UNDERSTANDING

Your own understanding of a **concept**:

Your own understanding of **conceptual understanding**:

Your own understanding of **how concepts are formed**:

Your own understanding of **how teachers facilitate conceptual understanding**:

Notes:

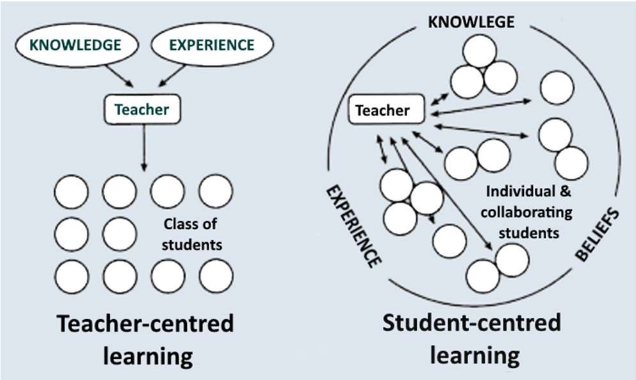
DIFFERENT ORIENTATIONS TOWARDS TEACHING

Your own understanding of **teacher centred classroom**:

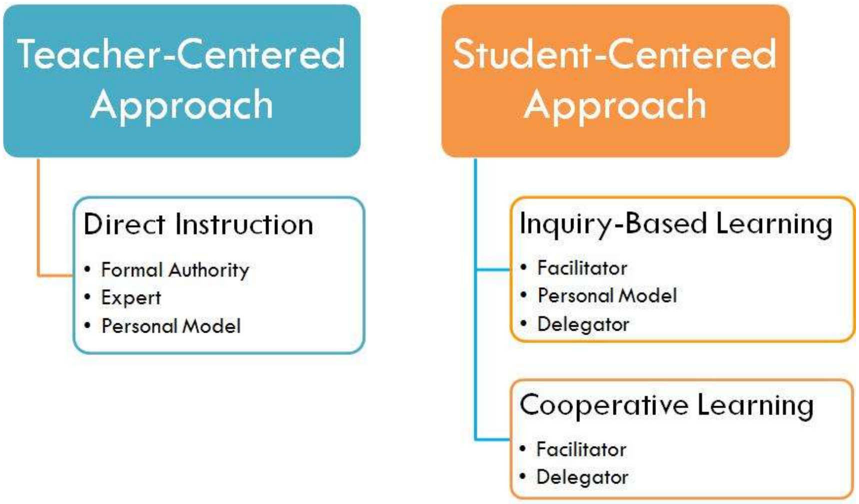
Your own understanding of **learner centred classroom**:

Your own understanding of **learning centred classroom**:

Teacher-centred vs learner-centred



https://lo.unisa.edu.au/mod/book/view.php?id=610988&chapterid=102030



https://www.plaz-tech.com/technology-in-the-classroom-making-the-shift-from-teacher-centered-to-student-centered-approach/

Notes:

Creating a learning centred classroom

- There is a specific concept in focus
- Situation is set up intentionally around the concept
- Situation is semi-structured to allow learner thinking
- Each part of the situation is meaningful for the concept
- There are many communication lines
- Realistic situations are used
- Concepts are transcended to other situations
- Learning is continuously assessed
- Learners understand the reason behind concepts
- Concepts are practiced solidifying them

Notes:

According to Daniel Willingham, successful mathematics learning requires three different abilities that must be developed and woven together. These are:

- control of facts
- control of processes
- conceptual understanding

Notes:

WHOLE NUMBERS: GRADE 4 – 9

Number sense: ***How numbers follow on each other***

Simple instructions: On their number lines, learners use one line each time to show the following:

- Counting in twos
- Marking out odd and even numbers
- Counting backwards from twenty in threes
- Counting in fives from two
- Indicating $14\frac{1}{2}$ on the second (dotted) line
- Showing the quarters of twenty

Complex instructions: On their number lines, learners use one line each time to show the following:

- Counting in multiples of sevens between 60 and 80
- Marking the prime numbers between 35 and 57
- Showing the factors of 24
- Showing the first three powers of 3
- Indicating $\frac{4}{5}$, $\frac{8}{5}$, $\frac{12}{5}$, $\frac{16}{5}$ on the second (dotted) line
- Dividing 35 by 6

Number sentences: ***To write numbers and operations in ordered ways to reach a certain result***

You have only two operation signs (+, –) and the following numbers to work with. In some items the operations are given; in some the answers only are given:

23	15	7	13	6	12	37	5	25	18	17	23
4	16	22	11	1	34	25	4	19	10	0	39

A		+		=	22	11			
B		+	1 3	=	12				
A		–		=	1				
B		–		=	1				
A				=	30				
B				=	30				
A				=	0				
B				=	0				
A		+		=	1	19	4		
B		+		=	1	18	6		
A		–		=	1				
B		–		=	1				
A	(	+	)	+	16	(16	)	+	
B	7	+	(	+	)	18	(	+	)
A		–		=					
B		–		=					
A		+		=					
B		+		=					
A		–		=					
B		–		=					

Addition and subtraction: To estimate the magnitude of change when a number is enlarged through adding or doubling and when it is reduced through subtracting or halving

Use the eight strips below, each with instruction and number line. Follow the instructions. The rule: **Nobody may calculate the answer**. The answers must be represented by a vertical line on the strip:											
1. Estimate and draw a line where you think the answer will be to: 246 + 89											
0	50	150	200	250	300	350	400	450	500	550	600
2. Estimate and draw a line where you think the answer will be to: 436 + 637											
0	100	200	300	400	500	600	700	800	900	1000	1100
3. Estimate and draw a line where you think the answer will be to: 555 + 555											
0	100	200	300	400	500	600	700	800	900	1000	1100
4. Estimate and draw a line where you think the answer will be to: 442 - 67											
0	50	150	200	250	300	350	400	450	500	550	600
5. Estimate and draw a line where you think the half of 238.											
0	20	40	60	80	100	120	140	180	200	220	240
6. Estimate and draw a line where you think the answer will be to: 38 + 83 - 42											
0	10	20	30	40	50	60	70	80	100	110	120
7. Estimate and draw a line where you think the answer will be to: 53 – 29 + 88											
0	10	20	30	40	50	60	70	80	100	110	120
8. Estimate and draw a line where you think the answer will be to: 155 + 236 + 341											
0	100	200	300	400	500	600	700	800	900	1000	1100

Multiplication and Division: To see multiplication and division are inversely related and interchangeable

My name is __________. I once had a task in school. We were writing Departmental tests and I had to make up packets of 6 sheets of paper for each learner in the Phase. There were 13 examination pads in the store room. Each examination pad had 80 pages. There were 195 learners in the Phase. In the end I had to ask Teacher Ntombi to buy more examination pads because there were not enough sheets for 195 learners. Please help me to check how many more examination pads she had to buy.
.

GRADES 7 - 9

Multiplication always results in a bigger number and division always results in a smaller number

- State two cases where this is true
- State two cases where this is not true
- Why do you think learners think this?
- What could the teacher do to alleviate the problem? Discuss with the participant next to you.

Examples where this is true:

Examples where this is not true:

Why do many learners believe that multiplication always results in a larger number?

Why do many learners believe that division always results in a smaller number?

What could the teacher do?

A number with three digits is always bigger than one with two

- State two cases where this is true
- State two cases where this is not true
- Why do you think learners think this?
- What could the teacher do to alleviate the problem? Discuss with the participant next to you.

Examples where this is true:

Examples where this is not true:

Why do learners believe that a number with three digits will always be larger than one with only two digits?

What could the teacher do?

To multiply by powers of 10, just add as many zeroes as there are 10's

- State two cases where this is true
- State two cases where this is not true
- Why do you think learners think this?
- What could the teacher do to alleviate the problem? Discuss with the participant next to you.

Examples where this is true:

Examples where this is not true:

Why do learners believe that adding zeroes when multiplying by powers of 10 is the rule?

What could the teacher do?

Why can’t we divide by zero?

Why does ‘tip and times’ work when dividing fractions?

CONSTRUCTIONS

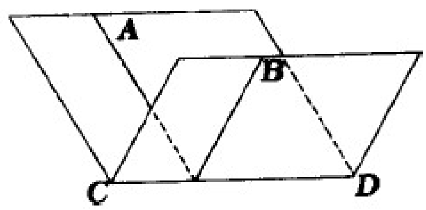
Why do we need to teach constructions?

Perpendicular lines

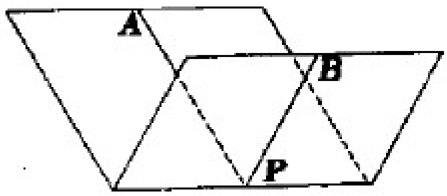
1. A line perpendicular to a given straight line
2. The perpendicular to a line at a point on the line
3. A line perpendicular to a given line passing through a given point not on the line
4. The perpendicular bisector of a given line segment

Instructions:

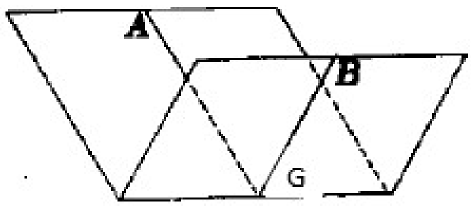
1. Draw a line segment AB.
2. Fold the page ensuring that a portion of the line segment AB folds onto itself. Make the crease.
3. Draw a line on the crease and call it CD.
4. $AB \perp CD$



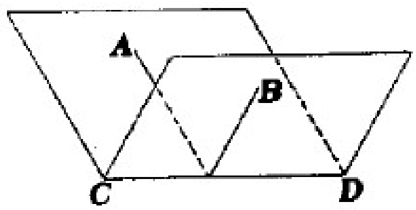
5. Using the same paper, mark a point, P, on the line AB. Fold the paper at P, ensuring that a portion of line AB folds onto itself before making the crease.
6. Draw a line on the crease. The new line is perpendicular to line AB and passes through the point P which is a point on line AB.



7. Using the same paper, turn it if necessary, so that AB is horizontal. Mark a point G anywhere above or below line AB.
8. Turn the page and fold it over so that the crease falls on point G. Mark the crease clearly, open the page and draw a line on the crease. The new line passing through G is perpendicular to Line AB.



9. Using the same paper, fold the paper so that the points A and B are on top of each other. Make a crease.
10. Draw a line on the crease and mark it CD. CD is the perpendicular bisector of AB.

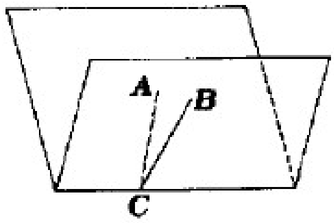


Angles

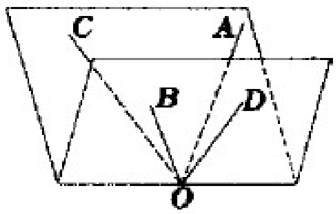
1. The bisector of a given angle
2. The formation of a right angle
3. Vertically opposite angles

Instructions

1. Using a ruler, draw line segment AB. Draw another line segment to form an angle at B. Name the other point C.
2. Fold the page so that AB and BC lie on top of each other. Mark the crease.
3. Draw a line on the crease. Name the line segment BD.
4. BD bisects $A\hat{B}C$



5. Forming a right angle was done in all 4 of the perpendicular constructions.
6. On the same piece of paper as the bisected angle, draw two line segments AB and CD. The lines must intersect. Mark the point of intersection O. (For convenience, avoid them intersecting at right angles)
7. Fold and crease the paper through vertex O, placing OB on OC. Do OA and OD also coincide?
8. Unfold the paper and repeat but this time, crease through the vertex O, placing OA on OC. Do OD and OB also coincide?



9. Repeat the instructions with a two different line segments, time permitting.

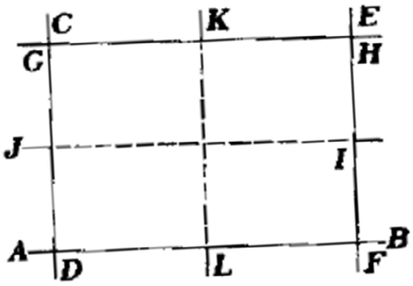
Triangles

1. Isosceles triangle
2. The base angles of an isosceles triangle
3. The sum of the angles of a triangle

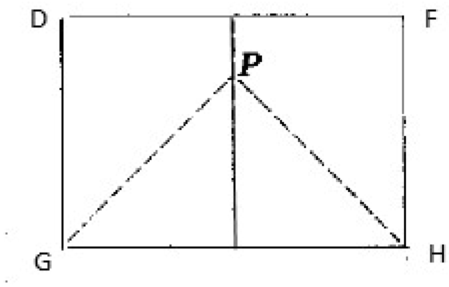
Instructions

1. To form an isosceles triangle a rectangle is required. This requires the skills used in the first four constructions.
2. Fold a line on a sheet of paper and call it AB. Mark points D and F on Line AB (D very close to A and F very close to B).
3. Fold and crease a line perpendicular to AB at D and a line perpendicular to AB at F. Label the new lines CD and EF.
4. Mark a point G on line CD (very close to C). At G, fold a perpendicular line to CD. Mark the point of intersection of this line and EF, H.

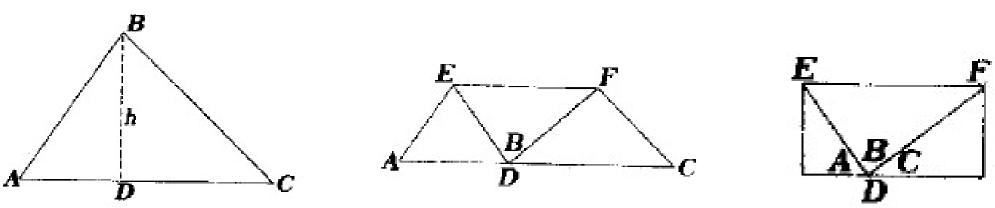
5. Confirm, by folding that GH is perpendicular to EF. Rectangle DGHF will now be used to construct an isosceles triangle.



6. Fold the perpendicular bisector of DF (named KL in the diagram above). Note that this has cut the rectangle into two equal halves.
7. Choose any point on the perpendicular bisector and label it P.
8. Fold lines from P to vertices G and H.
9. Confirm by folding that 2 triangles of equal size have been formed.
10. Triangle GPH is isosceles. The folding in the previous step confirmed that the base angles of an isosceles triangle are equal.



11. Draw any triangle. Label it ABC.
12. Use previous knowledge to draw in the perpendicular height of the triangle from vertex B. (A line perpendicular to a given line passing through a given point not on the line). Label the height BD.
13. Fold the vertex B onto the base (AC) of the triangle. Fold the base angles A and C to the base to the base of the perpendicular height (D).
14. What is known about the three angles at D? (adjacent angles on a straight line and they add up to 180^o). These are the same angles in the triangle. Hence, the angles of a triangle are also equal to 180^o.
15. This construction can also show why the area of a triangle is half the area of a rectangle.



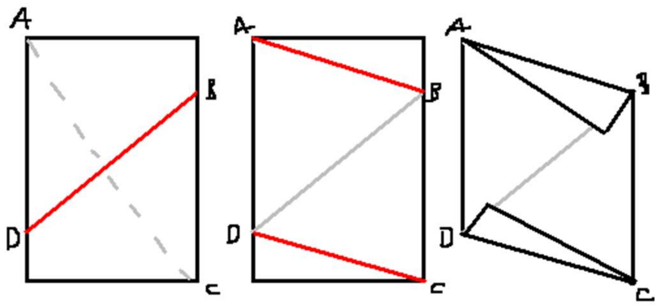
Properties of quadrilaterals

Due to time constraints we will take a brief look at how to construct a rhombus from a rectangular piece of paper to use for the investigation of some properties.

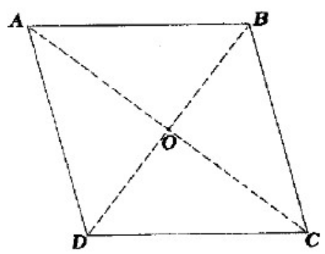
A similar exercise could be done with the rectangle, the square and the parallelogram.

Instructions:

1. Fold the paper so that the two opposite corners (call them A and C) meet. This creates a fold going from one long side to the other; call the points where this fold meets the long edges B and D so that A is on the same side as D and B is on the same side as C. Note the image for assistance.
2. Unfold.
3. Fold along AB and CD. Image below:



4. Interestingly, this is the largest possible rhombus that can be cut/folded from any given rectangle.
5. Learners could now use this rhombus to investigate the properties. Fold and then draw in diagonals AB and CD. Mark the point of intersection O.



6. Some ideas:
   - Compare AO, BO, CO and DO by folding (diagonals bisect each other)
   - Fold along the diagonal BD. What angle is the reflection of $A\hat{B}D$ ($C\hat{B}D$ – diagonals bisect the angles. This could be repeated with the other three vertices as the focus)
   - Is ΔABD congruent to ΔCBD?

LESSON DEMONSTRATIONS

The presentations must include:

Introduction (in which 'learners' are reminded of where we finished off yesterday and what they already know)

Direct Instruction (in which the next concept in the topic will be taught)

The lesson must be taught <u>exactly</u> as it would be in the classroom. Do not tell your colleagues HOW you would teach the lesson – the lesson must be taught as if you are in the classroom and your colleagues are the learners.

<u>Each</u> member of the group must be in front and assisting in some way.

A division of labour is recommended. For example, someone can prepare the 'board work' on flipchart paper, someone can present the introduction, and someone can do the actual direct instruction. If there are many worked examples to be done, these can also be split amongst participants (i.e.: the 'teacher' can change during the presentation).