Grade 5

CONTENT BOOKLET: TARGETED SUPPORT MATHEMATICS

Term 2

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). The FSS helped the DBE trial the NECT Maths, Science and language learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Provincialisation Programme. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

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CONTENTS

TOPIC 1: ADDITION AND SUBTRACTION

INTRODUCTION

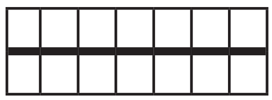
- This unit runs for 6 hours.
- It is part of the Content Area ‘Numbers, Operations and Relationships,’ which is allocated half of the total weight shared by the five content areas at Grade 5.
- This unit covers number concepts, addition and subtraction strategies within specified ranges.
- The purpose of this unit is to strengthen and expand learners’ existing number concepts and operations as a basis to master more complex ideas and calculations in the future.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Count forwards and backwards in 2s, 3s, 5s, 10s, 25s, 50s, 100s to at least 10 000 • Order, compare and represent numbers to at least 4 digits • Round off to the nearest multiple of 10, 100 or 1 000 • Represent odd and even numbers to 1 000 • Recognize place value of digits in 4 digit numbers • Add and subtract whole numbers of at least 4 digits • Use the following strategies: • estimating • building up/breaking down • using number lines • rounding/compensating • doubling and halving • addition/subtraction as inverse operations	• Count forwards and backwards in whole number intervals up to at least 10 000 • Order, compare, represent numbers to at least 6 digits • Round off to the nearest multiple of 5, 10, 100 or 1 000 • Represent odd and even numbers to 1 000 • Recognize place value of digits in 6 digit numbers • Add and subtract whole numbers of at least 5 digits • Use the following strategies: • estimating • building up/breaking down • using number lines • rounding/compensating • doubling and halving • addition/subtraction as inverse operations • calculating in columns	• Order, compare and represent numbers to at least 9 digits • Round off to the nearest multiple of 5, 10, 100, 1 000, 10 000, 100 000 and 1 000 000 • Represent prime numbers to 1 000 • Recognize place value of digits in 9 digit numbers • Add and subtract whole numbers of at least 6 digits • Use the following strategies: • estimating • building up/breaking down • rounding/compensating • addition/subtraction as inverse operations • calculating in columns • using a calculator

GLOSSARY OF TERMS

Term	Explanation / Diagram
Whole numbers	Whole numbers are the numbers you use to count with, including zero: 0, 1, 2, 3, 4.... (A fraction is not a whole number – it is a part of a whole number)
Ordering	Putting numbers in their order of size or quantity, in ascending order from smaller to bigger (fewer to more), in descending order from bigger to smaller.
Comparing numbers	When comparing the size or quantity in numbers, you may find one is bigger, smaller or the same as another; or you may find out by how much they differ.
Digit	A digit is a symbol that represents a quantity. The ten digits are: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. We use them in different positions to build up numbers. 36 is a two-digit number, of which the 3 and the 6 are both digits, 3 in the tens position which makes it worth 30 in the number 36 and 6 in the ones position which makes its value six in the number 36.
Place value and number value	Place value is the value a digit has because of its position in a number. In 3 234, the position of the first 3 gives it a place value of thousands and a number value of 3 000; the position of the second 3 gives it a place value of tens and a number value of thirty; the position of the 4 gives it a place value of ones and a number value of four.
Rounding off	Rounding is writing a number as an approximate, understood to be "about", "almost" or "closest to" a given number. We can round numbers to the nearest multiple of five or ten for example, or to the nearest multiple of hundred or thousand. We can round up to the next multiple or round down to the previous multiple. We indicate that we have rounded a number by using the symbol $\approx$, eg $38 \approx 40$
Building up and breaking down	We can write whole numbers larger than one (>1) in the parts that were added to form them (the terms of a number), or we can break down/ decompose/expand numbers into their terms. The terms of 153 are 100, 50 and 3. From the smaller numbers we can build up the bigger number by composing/contracting the terms into a whole, therefore we can write 100, 50 and 3 as the single number, 153.
Expanded Notation	Expanded notation is the form of writing a number to show its breakdown: 153 written in expanded notation is 100 + 50 + 3
Inverse operations	An operation's inverse reverses the operation, or two inverse operations undo each other. If you add 8 to 15, the sum is 23; if you subtract 8 from 23, the difference is 15. Addition and subtraction are each other's inverse operations.
Commutative property	The commutative property of numbers means that we can change the order of numbers when we add (or multiply) and the answer will not change, therefore 2 + 3 + 4 = 4 + 2 + 3.
Associative property	The associative property of numbers means that you can change the grouping of the numbers when adding (or multiplying) them and the answer will not change: 6 + (4 + 5) = (6 + 4) + 5
Multiples	Multiples of a certain number (e.g. 5) are the products when we multiply that number by any whole number: 15 is a multiple of 5, because 5 x 3 = 15

Term	Explanation / Diagram
Even and odd numbers	Even numbers can be divided into two equal groups (halved), like 18 which can be exactly divided (halved) into two groups of 9. All even numbers end with the digits 0, 2, 4, 6 or 8. Odd numbers cannot be divided into two equal groups, like 17 which cannot be exactly halved into two equal groups of whole numbers – one remains when trying to halve it. Odd numbers end with the digits 1, 3, 5, 7 or 9.
Halving and doubling	Halving is to divide a number into two equal parts, which is the same as dividing the number by two: when we halve 14, we have two equal parts of 7 each. An even number can be halved, but an odd number cannot.  Doubling is to multiply a number by 2, or to add the same number to it, so that the answer is twice as many as the number: When we double 7, we have 14. A doubled number is always even.

SUMMARY OF KEY CONCEPTS

Counting

1. In Grade 5, learners should count forwards and backwards in intervals from any number:

Example:

Count on in 40s from 5 432 (5 432; 5 472; 5 512; 5 552;...)

Example:

Count down in 15s from 973 (973; 958; 943; 928;...).

Writing numbers in their place value and expanding numbers

1. Learners should write down and understand 6 digit numbers. It helps to use a place value table:

Example:

Write down four-hundred-and-seven thousand and twenty-nine

Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Units
4	0	7	0	2	9

This table also helps to expand numbers.

Example:

Write 407 029 in expanded notation.

407 029 = 400 000 + 7 000 + 20 + 9

2. Learners' place value concepts can be strengthened further, using Flard cards (number builders) to build up and break down numbers. Ideally, each learner should have a set. (Resource 1)

Comparing and Ordering Whole Numbers

1. For learners to compare whole numbers, they must know the following facts:
   a. Place value and number value of digits in a number.
   b. The symbols < (less than), > (greater than), ≈ (approximately) and = (equal to or the same as).
   c. The meaning of the words 'ascending' and 'descending'.

2. For learners to order whole numbers, they must understand that:
   - The larger the number of digits, the larger the number.

     Example:

     98 765 < 123 456

   - The higher the value of the leftmost digit in the same position as another digit in that position, the higher the value of the number.

     Example:

     123 456 > 123 378 (400, the first instance of difference from the left, is larger than 300).

   - When we have two expressions to compare, we do the calculation before we compare the answers.

     Example:

     a. 'Which one is bigger, 88 – 33 or 77 – 33?'
        88 – 33 = 55 and 77 – 33 = 44,
        therefore 88 – 33 > 77 – 33

     b. 'Which one is smaller: 432 – 234 or 765 – 567?'
        432 – 234 = 198 and 765 – 567 = 198,
        therefore 432 – 234 = 765 – 567.

Rounding off Numbers

1. Rounding is writing a number as an approximate, a number “about”, “almost” or “closest to” a given number. We show that we have rounded a number by using the symbol ≈, eg 38 ≈ 40 and 345 ≈ 300.

Teaching Tip: For comparing, ordering, and rounding, it is helpful for learners to know where a number is positioned on a number line in relation to another number or in relation to a specific multiple of ten.

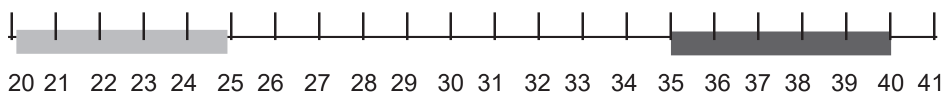
Example:
Round 23 to the nearest multiple of 5.
‘Is 23 closer to 20 or closer to 25?’

Example:

Round 22 to the nearest multiple of 5.

‘Is 22 closer to 20 or closer to 25?’

In these examples we are focusing on the spaces between 20 and 25 and between 35 and 40.



Example:

Round 36 to the nearest multiple of 5.

‘Is 36 closer to 40 or closer to 35?’

Example:

Round 39 to the nearest multiple of 5.

‘Is 39 closer to 40 or closer to 35?’

In these examples we are focusing on the space between 35 and 40.

2. A number line with intervals of 100 is marked in multiples of hundred.

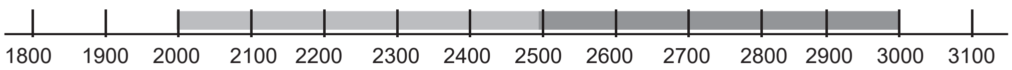
   Example:

   In this example we focus on the space between 2000 and 3000.

   a. 'Is 2735 closer to 2000 or closer to 3000?'

   b. 'Which one of 2479 and 2500 is closer to 3000?'

   c. 'Which number is in the middle between 2000 and 3000?'



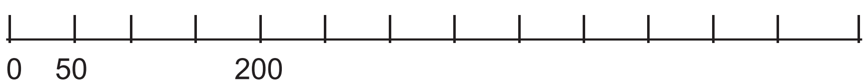
Numbers ending in 001-499 are closer to a previous multiple of thousand and are rounded down. Numbers ending in 500-999 are closer to a next multiple of thousand and are rounded up. The rule is to group 500 with the bigger numbers, so we round up a number ending in 500: 2500 ≈ 3000; 2499 ≈ 2000 which is rounding down.

Teaching Tip: A common activity at this stage is to leave out some markers on number lines with various intervals, which learners have to fill in. They need to understand the variations in intervals.

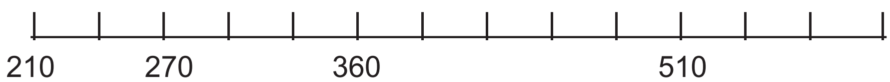
Examples:

Number lines with various intervals for completing open spaces.

a.



b.



Topic 1: Addition And Subtraction

Even and Odd Numbers

1. In Grade 5 learners investigate odd and even numbers.

Teaching Tip: Adding any three consecutive numbers is a fun way of dealing with odd and even numbers.

Example:

a. Add 5 + 6 + 7 = 18

Say odd + even + odd equals even

Show ** ** * + ** ** ** + ** ** ** *

The underlined odds combine to make an even sum.

Example:

b. Add 2 + 3 + 4 = 9

Say even + odd + even = odd

Show @@ + @@ @ + @@ @@

The underlined odd stays alone.

Addition

1. Estimating by rounding off

Example:

Calculate 35 621 + 41 198 + 22 743

35 621	rounds up to	36 000
41 198	rounds down to	41 000
22 743	rounds up to	23 000

36 000 + 41 000 + 23 000

= (30 000 + 40 000 + 20 000) + (6 000 + 1 000 + 3 000)

= 90 000 + 10 000

= 100 000

Therefore 35 621+41 198+22 743 ≈ 100 000

2. **Breaking down all numbers and adding horizontally**

Break down all numbers, all parts separated by + signs, group together the numbers that belong to the same place value and add horizontally.

Example:

Calculate 35 621 + 41 198 + 22 743

30 000 + 5 000 + 600 + 20 + 1 + 40 000 + 1 000 + 100 + 90 + 8
+ 20 000 + 2 000 + 700 + 40 + 3

= (30 000 + 40 000 + 20 000) + (5 000 + 1 000 + 2 000)
+ (600 + 100 + 700) + (20 + 90 + 40) + (1 + 8 + 3)

= 90 000 + 8 000 + 1 400 + 150 + 12

= 90 000 + 9 000 + 500 + 60 + 2

= 99 562

3. **Breaking down all numbers and adding vertically**

Break down all numbers, all parts separated by + signs, write the expanded numbers underneath each other in the place value groups that the numbers belong to.

Teaching tip: It is good to get learners in the habit of working from right to left, as this prepares them for the vertical column method.

Example:

Calculate 35 621 + 41 198 + 22 743

35 621 =	30 000 + 5 000 + 600 + 20 + 1
41 198 =	40 000 + 1 000 + 100 + 90 + 8
22 743 =	20 000 + 2 000 + 700 + 40 + 3
	90 000 + 8 000 +1 400+150 +12
	= 90 000 + 9 000 + 500 + 60 + 2
	= 99 562

4. **Adding on: Breaking down the second number only**

   Example:

   Calculate 58 291 + 32 409

   58 291 + 30 000 → 88 291 + 2 000 → 90 291 + 400 → 90 691 + 9 → 90 700

Subtraction

1. **Subtraction: Method 1: Breaking down both numbers**

   Break down both numbers. Separate the parts of the first number by + signs. All the parts of the number which you are subtracting, have – signs. Group together thousands, hundreds, tens and units to subtract.

   Example:

   84 537 – 42 213

   = 80 000 + 4000 + 500 + 30 + 7 – 40 000 – 2 000 – 200 – 10 – 3

   = (80 000 – 40 000) + (4 000 – 2 000) + (500 – 200) + (30 – 10) + (7 – 3)

   = 40 000 + 2 000 + 300 + 20 + 4

   = 42 324

2. **Subtraction: Method 2: Breaking down both numbers and compensating (“borrowing”)**

   This method may not always be as easy to do as it is for addition.

   Example:

   47 414 – 22 751

   = 40 000 + 7 000 + 400 + 10 + 4 – 20 000 – 2 000 + 700 + 50 + 1

   = (40 000 – 20 000) + (7 000 – 2 000) + (400 – 700) + (10 – 50) + (4 – 1)

   = (40 000 - 20 000) + (7 000 – 2 000)+ (300 – 700) + (110 – 50) + 3

   = (40 000 - 20 000) + (6 000 – 2 000)+ (1 300 – 700) + 60 + 3

   = 20 000 + 4 000 + 600 + 60 + 3

   = 24 663

Teaching tip: Start working from the back to make “borrowing” easier.

Note: Borrow from the first number in the brackets.

3. **Subtraction: Method 3: Breaking down the second number and compensating ("borrowing")**

   Example:

   47 414 – 22 751

   = 47 414 – 20 000 – 2 000 - 700 - 50 - 1

   = 47 414 – 20 000 → 27 414 – 2 000 →25 414 – 700 → 24 714 – 50 → 24 664 – 1 → 24 663

Solving Money Problems using Addition and Subtraction

In Grade 5 learners solve money problems using addition and subtraction skills. They are solving context free problems as well as problems in a real context, and they are working with whole numbers only.

Examples:

d. R35 432 – R13 456

e. R56 543 + R32 345 + R435

f. Manny buys chairs for R18 345 and he pays with R20 000.
How much change must he receive?

g. The school's electricity bill is R14 867, their cleaning bill is R8 576 and their food bill is R13 569. How much are these three bills together?

Resource 1 Place Value: Flard Cards

1	2	3	4	5	6	7	8	9
10	20	30	40					
50	60	70	80					
90	100	200						
300	400	500						
600	700	800						
900	1000							
2000	3000							
4000	5000							
6000	7000							
8000	9000							
10000								

TOPIC 2: COMMON FRACTIONS

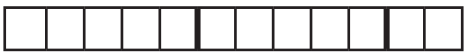
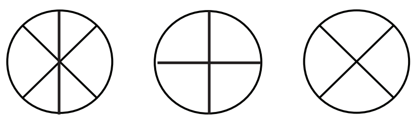
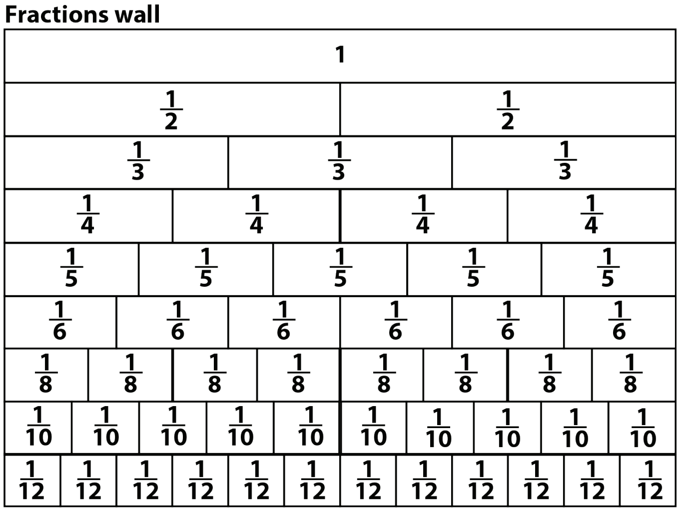
INTRODUCTION

- The unit runs for 5 hours.
- It is part of the 'Numbers, Operations and Relationships' content area, which is allocated half of the weight shared by the five content areas.
- The unit covers fraction concepts, and serves to make learners comfortable working with fractions, counting and calculating various forms of fractions.
- There is a focus on equivalent forms of fractions and solving real life problems involving fractions.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Compare and order fractions with different denominators (halves, thirds, quarter, fifths, sixths, sevenths, eighths) • Describe and compare fractions in diagram form • Recognise, describe and use the equivalence of division and fractions • Add fractions with the same denominator • Recognize and use equivalent forms of common fractions where denominators are multiples of each other • Solve problems in contexts involving fractions, including grouping and equal sharing	• Describe, compare and order common fractions to at least twelfths • Count forwards and backwards in fractions • Recognise, describe and use the equivalence of division and fractions • Add common fractions with the same denominator • Recognize and use equivalent forms of common fractions where denominators are multiples of each other • Solve problems in contexts involving fractions, including grouping and equal sharing	• Describe, compare and order common fractions including tenths and hundredths • Recognise, describe and use the equivalence between common fractions, decimal fractions and percentages • Add common fractions with the same denominator • Add and subtract mixed numbers • Recognize and use equivalent forms of common fractions where denominators are multiples of each other • Solve problems in contexts involving fractions, including grouping and equal sharing

GLOSSARY OF TERMS

Term	Explanation / Diagram
Common Fraction	A fraction is a part or parts of something or of a number of objects divided into groups. We write common fractions with one digit above and one below a fraction line, like $\frac{2}{5}$. 'Common fraction' is one type of fraction.
Denominator	The digit telling the number of equal parts into which a whole is divided, or the number of equal small groups into which a big group is divided. We write this digit under the fraction line, like in $\frac{2}{5}$.
Numerator	The digit telling how many parts or groups we are dealing with from those into which the whole is divided. That number appears above the fraction line, like $\frac{2}{5}$. The '2' shows how many parts were selected.
Mixed Number	A mixed number is a way of writing that shows all the parts, like in $\frac{12}{5}$. This is two wholes and two fifths. We see it in $2\frac{2}{5}$, which has a whole number and a fraction. 
Equivalent fractions	Fractions that have the same value: It is clear that two quarters ($\frac{2}{4}$) in the first diagram has the same value as a half ($\frac{1}{2}$) in the second and as four eighths ($\frac{4}{8}$) in the third diagram. 
Fraction wall	A diagram showing one whole in each row, divided into 2, 3, 4, 5 parts and so on. Using a ruler downwards, one can for example see on a fraction wall that two thirds has the same value as (or is equivalent to) four sixths, that two quarters is equivalent to a half and so on. **Fractions wall** 

SUMMARY OF KEY CONCEPTS

Learners in Grade 5 have to work with fractions even when they do not see them in pictures or diagrams. Most importantly, they must realize how large or small fractions are.

Linking Familiar Concepts with new Fraction Concepts

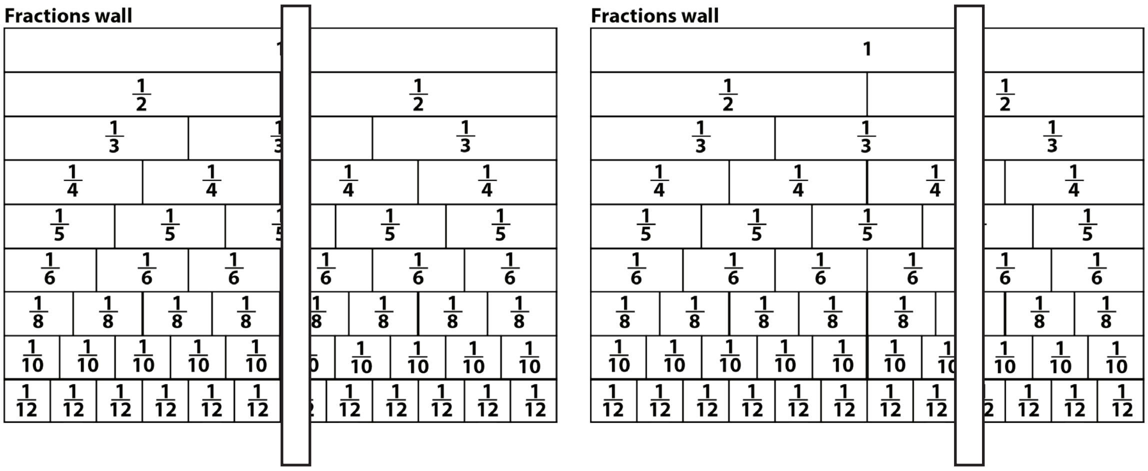
1. Learners understand the parts of fractions (numerator, denominator and fraction line) and what they stand for.
2. Now they also know that when the numerator and the denominator are the same, we actually have a whole number like in $\frac{5}{5} = 1$.
3. They learn that if the numerator is bigger than the denominator, that can be written as a whole number and a fraction, or as a mixed number, like in $\frac{12}{5}$ which is actually $2\frac{2}{5}$.

Equivalent Fractions Using the Fraction Wall

Learners discover equivalence while engaging with activities using the fraction wall. It is worthwhile spending time on talking about the fraction wall:

1. Each row of bricks in the fraction wall stands for one whole. The size of that whole is shown in the top row, which is one whole that is not divided up.
2. The second row is divided up in two, and the two halves together form one whole again. It is exactly the size of the one whole in the top row.
3. We can go on and on, for example the sixth row is divided up in six, and the six sixths together form one whole again.

4. Slide a ruler vertically across the fraction wall along the half line, as illustrated below:



Two quarters is the same size as a half, and so are three sixths, four eighths, five tenths and six twelfths.

On the second diagram, two thirds take up the same space as four sixths and as eight twelfths.

Comparing Fractions

Learners must understand that the more parts something is divided up into, the smaller the parts become. They see on the fraction wall that fractions with a larger denominator, indicates a smaller fraction.

Example:

a. Cut this 24 cm piece of wood in three equal pieces (thirds) and colour in one third.

b. Cut this 24 cm piece of wood in four equal pieces (quarters) and colour in one quarter.

c. Cut this 24 cm piece of wood in six equal pieces (sixths) and colour in one sixth.

d. Cut this 24 cm piece of wood in two equal pieces (halves) and colour in one half.

e. Which fraction of the wood is the largest? And the smallest?

f. How many centimetres is two thirds of the piece of wood?

g. Is there an equivalent piece of wood (the same length) in a. and d.?

Counting in Fractions

Learners can fill in a number chain, starting at any number, to count in fractions:

□ → + □ → + □ → + □ → + □ → + □ → + □

Example:

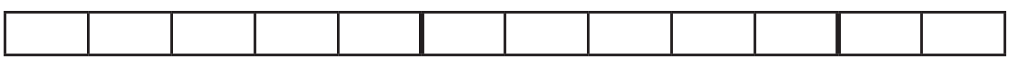
h. Start at 3 in the first block, count on in thirds, each time writing the new number in the next block.

i. Start at $2\frac{3}{4}$, count on in quarters, each time writing the new number in the next block.

j. Start at 0 in the first block, count on in fifths, each time writing the new number in the next block.

Improper Fractions and Mixed Numbers

An improper fraction is a term used to describe a fraction where the parts have formed more than a whole. The term ‘improper fraction’ is not used in the CAPS document. It should rather be shown and described using a diagram or in writing where the numerator is bigger than the denominator, like in this example:



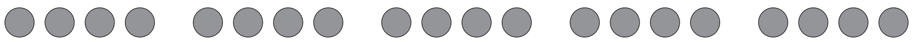
As an improper fraction we write it as $\frac{12}{5}$, and as a mixed number (whole and fraction) we write it as $2\frac{2}{5}$.

Finding a fraction of a group

We do not only get a fraction of a whole, but also a fraction of a group.

Example:

Find $\frac{1}{5}$ of twenty marbles.



Problem solving involving fractions

1. Problem solving that involves fractions, is given in the context of learners' life experience.

   Example:

   a. We have eighteen boys in this class. Two thirds of them are playing soccer. How many are playing soccer?

   b. If half of the boys were playing soccer, how many would that be?

   c. Mom uses two thirds of a loaf of bread to make us lunch for school. How much of the bread is left? How many loaves is Mom using in two days? and in three days?

Adding Common Fractions With the Same Denominator

Learners can get practice with this kind of calculation before they proceed to more complex examples.

1) $\frac{5}{8} + \frac{1}{8} = \frac{\quad}{8}$	2) $\frac{4}{10} + \frac{4}{10} = \frac{\quad}{10}$
3) $\frac{3}{7} + \frac{2}{7} = \frac{\quad}{7}$	4) $\frac{4}{3} + \frac{3}{3} = \frac{\quad}{3}$
5) $\frac{5}{9} + \frac{3}{9} = \_\_\_$	6) $\frac{3}{4} + \frac{2}{4} = \_\_\_$
7) $\frac{4}{12} + \frac{7}{12} = \_\_\_$	8) $\frac{4}{5} + \frac{3}{5} = \_\_\_$
9) $\frac{3}{6} + \frac{7}{6} = \_\_\_$	10) $\frac{4}{9} + \frac{8}{9} = \_\_\_$
11) $\frac{6}{10} + \frac{3}{10} = \_\_\_$	12) $\frac{5}{7} + \frac{6}{7} = \_\_\_$
13) $\frac{4}{5} + \frac{3}{5} = \_\_\_$	14) $\frac{4}{11} + \frac{5}{11} = \_\_\_$
15) $\frac{5}{4} + \frac{6}{4} = \_\_\_$	16) $\frac{2}{8} + \frac{5}{8} = \_\_\_$

Subtracting Common Fractions With the Same Denominator

1) $\frac{3}{6} - \frac{1}{6} = \frac{2}{6}$	2) $\frac{3}{4} - \frac{1}{4} = \frac{\quad}{4}$
3) $\frac{4}{5} - \frac{2}{5} = \frac{\quad}{5}$	4) $\frac{5}{7} - \frac{3}{7} = \frac{\quad}{7}$
5) $\frac{5}{3} - \frac{4}{3} = \frac{\quad}{3}$	6) $\frac{8}{9} - \frac{5}{9} = \frac{\quad}{9}$
7) $\frac{5}{4} - \frac{2}{4} = \frac{\quad}{4}$	8) $\frac{7}{10} - \frac{5}{10} = \frac{\quad}{10}$
9) $\frac{9}{8} - \frac{5}{8} = \frac{\quad}{8}$	10) $\frac{10}{7} - \frac{4}{7} = \frac{\quad}{7}$
11) $\frac{13}{10} - \frac{7}{10} = \frac{\quad}{10}$	12) $\frac{9}{5} - \frac{6}{5} = \frac{\quad}{5}$
13) $\frac{11}{12} - \frac{8}{12} = \frac{\quad}{12}$	14) $\frac{10}{6} - \frac{3}{6} = \frac{\quad}{6}$
15) $\frac{11}{9} - \frac{4}{9} = \frac{\quad}{9}$	16) $\frac{11}{11} - \frac{7}{11} = \frac{\quad}{11}$

TOPIC 3: LENGTH

INTRODUCTION

- This unit runs for 6 hours.
- It is part of the content area ‘Measurement’, which counts 15% of the final exam.
- The emphasis is on formal measuring of 2D shapes and 3D objects.
- The required knowledge includes measurement facts (various measurement units) and the required skills include the use of measurement instruments.
- Problem solving centres around situations in everyday contexts.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Measure 2D shapes and 3D objects formally in standard units of length • Estimate, compare, order and record formal measurements • Use and discern standard units of length: millimetre (mm), centimetre (cm), metre (m) and kilometre (km) • Use measuring instruments rulers, metre sticks, tape measures, trundle wheels • Solve problems in context involving length • Convert between any units of length including mm, cm, m and km	• Measure 2D shapes and 3D objects formally in standard units of length • Estimate, compare, order and record formal measurements • Use and discern standard units of length: millimetre (mm), centimetre (cm), metre (m) and kilometre (km) • Use measuring instruments rulers, metre sticks, tape measures, trundle wheels • Solve problems in context involving length • Convert between mm and cm; between cm and m; and m and km, including fractions of units	• Measure 2D shapes and 3D objects formally in standard units of length • Estimate, compare, order and record formal measurements • Use and discern standard units of length: millimetre (mm), centimetre (cm), metre (m) and kilometre (km) • Use measuring instruments rulers, metre sticks, tape measures, trundle wheels • Solve problems in context involving length • Convert between any units of length including mm, cm, m and km, including fractions and decimal fractions of units

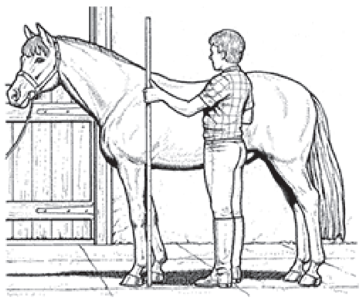
GLOSSARY OF TERMS

Term	Explanation / Diagram
Length	A one-dimensional measurement along a line which indicates the distance between two points.
Measuring Instruments	A device or a system that is used to measure a physical property, in this case, length. The instrument is usually calibrated or marked in intervals of standard units, in this case units of length.
Trundle Wheel	The trundle wheel is a measuring device for length. If the circumference of a trundle wheel is one metre, it measures one metre in one turn or rotation. If it only makes half a rotation, it has measured 50 cm. It is an easy way to find a rough distance and is often used to measure out sports fields or tracks. 
Odometer	An instrument in a car that measures the distance that the car travels.
Conversion	Changing a unit of measurement to a different but equal unit of measurement. Example: 1 cm = 10 mm 100 cm = 1 m
Estimate	Judging something (length in this case) without measuring or calculating it. Estimation is based on knowledge and experience about that which is estimated.
Standard unit of length	A single standard distance from one point to another that is used the same across the world and bears a specific name. The metric length unit is metre and this standard length is multiplied by powers of ten or divided by powers of ten to get longer and shorter standard units of length: 1000 metres = 1 kilometre $\frac{1}{100}$ metre = 1 centimetre $\frac{1}{1000}$ metre = 1 millimetre

SUMMARY OF KEY CONCEPTS

Measuring in the olden days

1. In the olden days, people measured the height of a horse with hands and that made a difference in the price of the horse. One farmer's hand was 12 cm wide and another farmer's hand was 9 cm wide. The first farmer would measure and find the horse is 12 hands high. The second farmer would find the same horse is 16 hands high. (Why is that so?) This type of problem caused people to make a rule that a hand is almost exactly 10 cm wide and they put those units on a measure stick. The standard measurement tells both farmers that the same horse is 14½ hands high.



2. People used their feet, their hands, their elbow-to-fingertip, their steps, their thumbs' width and more, to measure the length of things. This could cause great confusion and called for standard measurement units which are the same across the globe.

Measurement Facts to Know

1. Metre is our standard unit for measuring length. We use the letter 'm', meaning metre. The prefixes k (kilo-), c (centi-) and m (milli-) are used to show multiples or fractions of the standard unit.
   a. When a metre is divided up in a hundred parts, the small parts are called centimetres. We use the letters 'cm' for short when we mean centimetre. One metre is the same length as hundred centimetres
   (1 m = 100 cm).
   b. When a metre is divided up in a thousand parts, the small parts are called millimetres. We use the letters 'mm' for short when we mean millimetre. One metre is the same length as thousand millimetres
   (1 m = 1 000 mm).
   c. When a metre is multiplied by a thousand, the large length is called a kilometre. We use the letters 'km' for short when we mean kilometre. One kilometre is the same length as thousand metres
   (1 km = 1 000 m).

d. When a centimetre is divided up in ten parts, the small parts are called millimetres. We use the letters ‘mm’ for short when we mean millimetre. One centimetre is the same length as ten millimetres
(1cm = 10 mm).

2. We use a metric measuring system for length, just as we use for numbers:

Multiples of the unit ← The unit → Fractions of the unit

Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths
1 000	100	10	1			
kilometre	hectometre	decametre	metre	decimetre	centimetre	millimetre
m x 1000			m		m ÷ 100	m ÷ 1000

Comparing lengths

1. In Grade 5 learners measure in standard units of length and convert these lengths to the same unit to be able to compare them.

Example:

Measure the circumference of the wheels of four cars parked in the school yard with a measuring tape in centimetres. Round off the circumferences to the nearest 50 cm. Record the exact circumferences and the rounded numbers. Convert these lengths in centimetres to metres.

Car number	Wheel circumference in cm	Rounded to nearest 50 cm	Converted to metres

Estimate length

We can use some handy approximates in everyday life to make better judgements of length:

a. The width of one’s pinky finger is approximately one centimetre

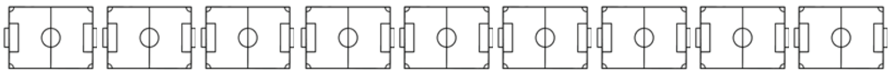
b. A pencil line is approximately one millimetre wide

c. An exercise book is approximately 20 cm wide and 30 cm long

d. The width of five exercise books next to each other is approximately one metre (about 5 cm more than a metre)

e. Learners can innovate their own approximates, compare the length of a Bic pen (14 cm) with a new pencil (18 cm)

f. A soccer field is approximately 110 meters long. Nine soccer fields next to each other make approximately a kilometre.



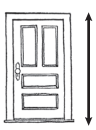
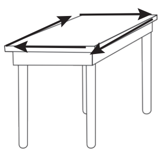
g. They can put seven Bic pens tip to end to make approximately one metre, etc.

Measurement Instruments

1. Learners must know how to choose the correct instruments for the measuring of given lengths, from a ruler, a tape measure, a metre stick, a trundle wheel and the odometer of a car. Their most available instrument is the ruler and we have to do as much as possible with a ruler.

Example:

Combined Assignment: Measuring Length in real life

Object	Estimate	Instrument	Record	Conversion
Example: Width of a chair 	50 cm	Ruler	50 cm	50cm is the same as half a metre
Height of your classroom door 	in cm			cm to m
Perimeter of (length around) a table 	in m			m to cm

Calculating and solving problems with units of length

1. Add 750 cm, 1,2 m and 2 231 cm

   Step 1: Convert all lengths to the same unit

   Step 2: Use any addition strategy to add the length

   Step 3: If the numbers are large, convert back to metre and centimetre

2. Subtract 12 500 m from 21 kilometre

   Step 1: Convert all lengths to the same unit

   Step 2: Use any addition strategy to add the length

   Step 3: If the numbers are large, convert back to kilometre and metre

3. Beauty has four pieces of red material from which she has to make bandanas for a sports day. She has 12,5 m; 3 m; 1 850 cm and 925 cm. She needs 25 metres of material to make all the bandanas. How much more material does she need?

12, 5 m 3 m 1 850 cm 925 cm

TOPIC 4: MULTIPLICATION

INTRODUCTION

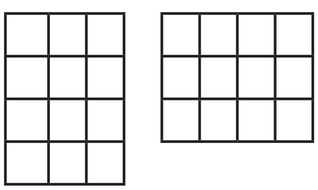
- This unit runs for 7 hours.
- It forms part of the content area ‘Numbers, Operations and Relationships’ and counts along with the other topics in this content area, and counts (along with the other topics in this content area) 50% of the marks in the final examination.
- This unit extends skills to multiplication of 3-digit whole numbers by 2-digit whole numbers.
- Learners understand and use the concepts factors, multiples and ratio.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Multiply 2-digit- by 2-digit numbers
• Estimate the answer to a multiplication calculation
• Use strategies to multiply
  • building up and breaking down numbers
  • using a number line
  • rounding off, compensating
  • doubling and halving
• Know multiples and factors of one digit numbers to 100
• Recognise, use commutative, associative and distributive property of number
• Money: Solve problems with whole numbers of rands
• Compare quantities of the same kind (ratio) and quantities of different kinds (rate)
	• Multiply 3- by 2-digit numbers
	• Estimate the answer to a multiplication calculation
	• Use strategies to multiply
	  • building up and breaking down numbers
	  • using a number line
	  • rounding off, compensating
	  • doubling and halving
	• Know multiples and factors of two digit numbers to 100
	• Know the multiplicative property of 1
	• Recognise, use commutative, associative and distributive property of number
	• Solve problems with whole numbers in financial and measurement contexts
	• Compare quantities of the same kind (ratio)
		• Multiply 4- by 3-digit numbers
		• Estimate the answer to a multiplication calculation
		• Use strategies to multiply, with or without brackets
		  • building up and breaking down numbers
		  • rounding off, compensating
		  • doubling and halving
		  • column method
		• Know multiples and factors of 2-digit and 3-digit numbers and prime factors of numbers to 100
		• Know the multiplicative property of 1
		• Recognise, use commutative, associative and distributive property of number
		• Solve problems with whole numbers involving multiplication in various contexts
		• Compare quantities of the same kind (ratio)

GLOSSARY OF TERMS

Term	Explanation / Diagram
Multiples	A number formed by multiplying two other numbers. Example: 28 is the seventh multiple of 4, since 7 x 4 = 28. 28 is also the fourth multiple of 7, since 4 x 7 = 28. The number itself is its own first multiple: 7 is the first multiple of 7 (1 x 7 = 7)
Factors	Whole numbers that divide exactly into another number, or numbers that were multiplied to make that number, like 7, 2, 14 and 4 are factors of 28.
Multiplicative Property of One	One multiplied by, or divided into a number does not change that number: one is the identity element for multiplication and division: 14 x 1 = 14; 14 ÷ 1 = 14.
Distributive Property of Multiplication over Addition	If we multiply a number by numbers that are added together, it is the same as multiplying the number by each of the other numbers. Example: Five learners each have three brothers and two sisters. To save time and space, we can write it: 5 times (+) or in numbers = 5 x (3 + 2) = 15 + 10 = 25 siblings altogether.
Terminology Used in Multiplication Equations or Calculations	9 x 4 = 36 ↓ ↓ ↓ Multiplicand x Multiplier = Product The product is the answer to a multiplication sum.
Inverse Property	Multiplication is the inverse of division. Division is the inverse of multiplication. Example: 4 x 7 = 28 ∴ 28 ÷ 7 = 4 and also 28 ÷ 4 = 7 28 ÷ 7 = 4 ∴ 4 x 7 = 28 and also 7 x 4 = 28
Commutative Law	The order of numbers in addition and multiplication may change and the answer will remain the same. Example: Rectangle a. has three blocks to the side and four down (3 x 4). Rectangle b. has four blocks to the side and three down (4 x 3). Both have 12 blocks altogether because 3 x 4 = 12 and 4 x 3 = 12 

Term	Explanation / Diagram
Halving	To divide a number into two equal parts, which is the same as dividing the number by two: when we halve 14, we have two equal parts of seven each.
Doubling	To multiply a number by two, or add the same number to it, so that the answer is twice as many as the number: when we double seven, we have fourteen. A double number is always even.
Rounding off Symbol	When one number is not exactly equal to, or the same as another number, we use the symbol $\approx$ to indicate that it is approximately, or almost the same as the other when we round off or estimate.
Financial Context	Calculating money is a calculation in a financial context. We calculate it in the currency we use, like rands and cents in South Africa.
Rate	Rate is also a ratio. It is used to compare two quantities of things that depend on each other – if one quantity changes, the other is also changing. Price and speed are instances of rate that are familiar in learners' everyday life.

SUMMARY OF KEY CONCEPTS

Multiples, factors and factorising

1. A multiple is formed when we multiply two numbers.

   Example:

   28 is the first multiple of 28, since 1 x 28 = 28

   28 is the second multiple of 14, since 2 x 14 = 28

   28 is the fourth multiple of 7, since 4 x 7 = 28.

   28 is the seventh multiple of 4, since 7 x 4 = 28.

   28 is also the fourteenth multiple of 2, since 14 x 2 = 28.

2. This means that a multiple of a number is divisible by that number.

   Example:

   28 is divisible by 1, 2, 4, 7, 14 and 28.

   This fact makes 1, 2, 4, 7, 14 and 28 factors of 28.

3. A factor is a whole number that divides into another number. Factor pairs of a number are two numbers that were multiplied to make that number. All the factors of a number are all the numbers that can be divided into that number without a remainder.

   Example:

   7 x 4 = 28, so a factor pair of 28 is 7 and 4.

   It is also true that 28 = 2 x 2 x 7 or 2 x 14,

   therefore 2 and 14 are also factors of 28.

   All numbers have 1 and themselves as factors too. All factors of 28 are:

   1; 2; 4; 7; 14; 28

   1 and 28 are a factor pair of 28 because 1 x 28 = 28

   2 and 14 are a factor pair of 28 because 2 x 14 = 28

   4 and 7 are a factor pair of 28 because 4 x 7 = 28

Multiples go in beautiful patterns, for example:

6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
(see the repeating pattern of even numbers)

8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80
(see the repeating pattern of even numbers)

7: 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112, 119, 126, 133, 140
(see that all end digits are used and repeated every tenth time)

9: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, 117,...
(see that all end digits are used and repeated every tenth time)

11: 11, 22, 33, 44, 55, 66, 77, 88, 99, 110, 121, 132, 143, 154...
(see that all end digits are used and repeated every tenth time)

4. Factorising a number is easier when we know and apply the rules of divisibility to the number:

Rules of divisibility

A number can be divided without a remainder:

- by 2, if the last digit is an even number: 324 is divisible by 2, because 4 is an even number.
- by 3, if the sum of the digits is a multiple of 3: 324 is divisible by 3, because 3 + 2 + 4 = 9.
- by 4, if the number can be divided by 2 twice, because 4 = 2 x 2.
- by 5, if the last digit is either 5 or 0: 324 is not divisible by 5, because the last digit is 4.
- by 6, if the number is divisible by 2 and by 3: 354 is divisible by 6, because it is divisible by 2 and by 3.
- by 8, if the number can be divided by 2 three times, because 8 = 2 x 2 x 2.
- by 9, if the sum of the digits is a multiple of 9: 324 is divisible by 9, because 3 + 2 + 4 = 9.
- by 10, if the last digit is 0: 324 is not divisible by 10, because the last digit is 4.

Multiplication of a 3-digit number by a 2-digit number.

1. In Grade 5, learners used various 'break-down' strategies to multiply, which we now apply to 3-digit numbers multiplied by 2-digit numbers. We break up the multiplier, or the second number. There are three options for breaking down the numbers in a multiplication sum: regarding the multiplier as the sum, the difference or the product of two numbers.

Teaching tip: Spend some time to talk to learners about the multiplier and how the multiplier was built up or formed, as follows:

Example:
In our two examples we use 12 and 35 as our 2-digit multipliers. These numbers are made up in various ways. We either add numbers to make up the number, or we multiply numbers to make up the same number:

12 = 10 + 2 (broken down into its terms) 12 = 3 x 4 (broken down into its factors)
35 = 30 + 5 (broken down into its terms) 35 = 7 x 5 (broken down into its factors)

Using the terms of the multiplier to multiply a. Sum of the terms	Using the factors of the multiplier b. Product of factors
316 x 12 = 316 x (10 + 2) = (316 x 10) + (316 x 2) Distributive property = 3160 + 632 = 3 792	316 x 12 = 316 x 3 x 4 We don't need brackets = 316 x 3 x 4 = 948 x 4 = 3 792
164 x 35 = 164 x (30 + 5) = (164 x 30) + (164 x 5) Distributive property = (100 x 30 + 60 x 30 + 4 x 30) + (100 x 5 + 60 x 5 + 4 x 5) = 3000 + 1800 + 120 + 500 + 300 +20 = 4 000 + 1700 + 40 = 5 740	164 x 35 = 164 x 7 x 5 = 164 x 7 x 5 = 1148 x 5 = 5 740

2. In the third strategy of breaking down the multiplier, we regard the multiplier as the difference between two numbers. This means we are still working with terms, but in a way where a number came about as a result of subtraction. This method is also called rounding up and compensating.

Using the terms of the multiplier to multiply: **c. Difference of the terms**	**Teaching tip:** Be extremely careful that learners understand completely what they are doing:
416 x 18 = 416 x (20 – 2) Use the distributive property = (416 x 20) – (416 x 2) = 8320 – 832 = 7 488	18 is closer to 20, so see 18 as 20 – 2. We put it in brackets to see this is our 18. We multiply 416 by 20 but it is too much. We have to subtract two times 416 from that to make sure we actually multiplied by 18.
288 x 35 =288 x (40 – 5) =(288 x 40) – (288 x 5) =(200 x 40 + 80 x 40 + 8 x 40) – (200 x 5 + 80 x 5 + 8 x 5) =(8 000 + 3 200 + 320) – (1 000 + 400 + 40) = 11 520 – 1 440 = 10 080	35 is closer to 40 so we see 35 as 40 – 5. We put it in brackets to see this is our 35. We multiply 288 by 40 but we know it is too much. We have to subtract five times 288 from that to make sure we actually multiplied by 35.

3. We can use doubling and halving in some cases to multiply, but that works well only in cases where one of the numbers is a multiple of 2, 4, 8 or 16.

Example:

288 x 35

Halving	Doubling
288	35
144	70
72	140
36	280
18	560
9	1120
9 x 1120 = 10 080	

Estimating by Rounding

Learners estimate answers to multiplication sums by rounding to check if the answers are reasonable:

Round the number that is closest to a multiple of 100:

Example:

438 x 15 ≈ 400 x 15 ≈ 6 000

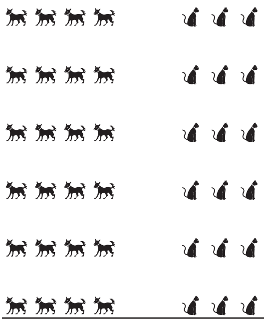
465 x 86 ≈ 465 x 100 ≈ 4 650

Ratio

Ratio is used to compare the sizes of two or more quantities, not always of the same kind. The key to understanding ratio is to understand that we are comparing the size or magnitude of sets of objects.

Example:

Where we stay, there are 24 dogs and 18 cats.



24 dogs : 18 cats

The ratio of dogs to cats is 24:18 (say twenty-four to eighteen). If we group them in as many equal groups as we can, we see that we can make six equal groups where each group has four dogs and three cats. That means that our ratio is now more simple and we can say that 24:18 is the same as 4:3.
(We actually divided both numbers by a factor that they have in common, that is 6).

24 = 6 x 4 and 18 = 6 x 3, therefore the ratio 24:18 = 4:3

We can now say that for every 4 dogs there are three cats.

TOPIC 5: PROPERTIES OF 3D OBJECTS

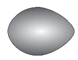
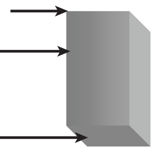
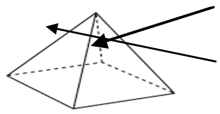
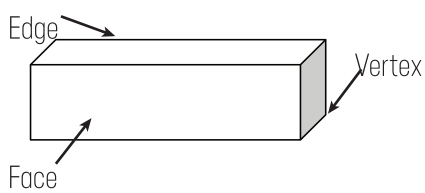
INTRODUCTION

- This unit runs for 6 hours.
- It is part of the content area 'Space and Shape' and together with the other topics in this content area, it counts for 15% in the final exam.
- The unit covers knowledge and skills pertaining to 3D objects, concepts and terminology.
- The purpose of this unit is to extend learners' knowledge and experience to include objects of the third spatial dimension and their properties.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Know and name • spheres • rectangular prisms • cylinders • cones • square-based pyramids • Distinguish, describe, sort, compare 3D objects ito • 2D shapes that make up their faces • flat or curved surfaces • Create 3D models from cut-out 2D polygons	• Know and name • cubes • rectangular prisms • other prisms • cylinders • cones • pyramids • similarities between cubes and rectangular prisms • Distinguish, describe, sort, compare 3D objectsin terms of: • 2D shapes of faces • number of faces • flat or curved surfaces • Create 3D models • make models from cut-out 2D polygons • cut open boxes to describe their nets	• Know and name • cubes • rectangular prisms • tetrahedrons • pyramids • similarities between tetrahedrons and other pyramids • Distinguish, describe, sort, compare 3D objects in terms of: • 2D shapes of faces • number of faces • number of vertices • number of edges • Create 3D models • make models from cut-out 2D polygons • use and make nets

GLOSSARY OF TERMS

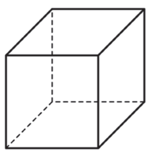
Term	Explanation / Diagram
Three-dimensional geometrical object (3D object)	Objects that occupy space and have form. We can measure such objects in three directions like a box, of which the length, breadth and height can be measured. We call it a 3D object.
Characteristics or Properties	The qualities of something, by which we recognise it, like its height, its form, etc. This is how we would describe the shape and what it looks like.
Curved surface	An object or diagram can have surfaces which are rounded and not straight, like an egg: 
Flat surface	An object or diagram with a flat suface is not curved but straight, meaning it has edges, like a box. A flat surface has a 2D shape called a face. This box has 6 rectangular faces and 12 edges. 
Prism	A solid object with a base and a top (lid) that are the same shape and all pairs of opposite sides that are rectangles of the same size lid or top side base 
Pyramid	 A solid object with a base of any shape, like a square, and sides that slope up to meet in one point on top. If the base has straight edges, the sides of a pyramid are triangle shaped.
Face **Edge** **Vertex**	A face is a flat side of a solid shape. The edge of an object is where two faces meet or where it is folded. A vertex is a point where three or more faces meet (corner) 

SUMMARY OF KEY CONCEPTS

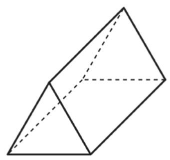
Recognition of 3D Shapes

1. 3D objects with straight surfaces

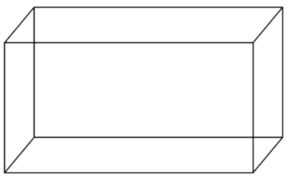
   a. Prisms
   A prism has two identical opposite faces of exactly the same size and shape. All other sides are rectangles.



Cube: all faces are equal squares

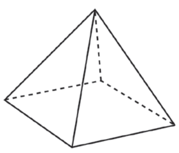


Triangular prism: two identical triangular faces parallel to each other



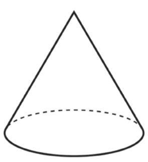
Rectangular prism: two identical parallel faces are rectangles

   b. Pyramids
   A pyramid has one polygon as its base and the sides are all triangles, meeting in a single point at the top.

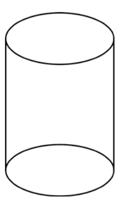


A square based pyramid has its base in the shape of a square and has four triangle faces.

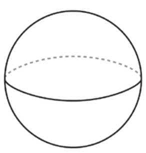
2. 3D objects with curved surfaces
   Some 3D objects do not have polygons as bases, and they have curved surfaces:



Cone has a circular base



Cylinder has a circular base and top

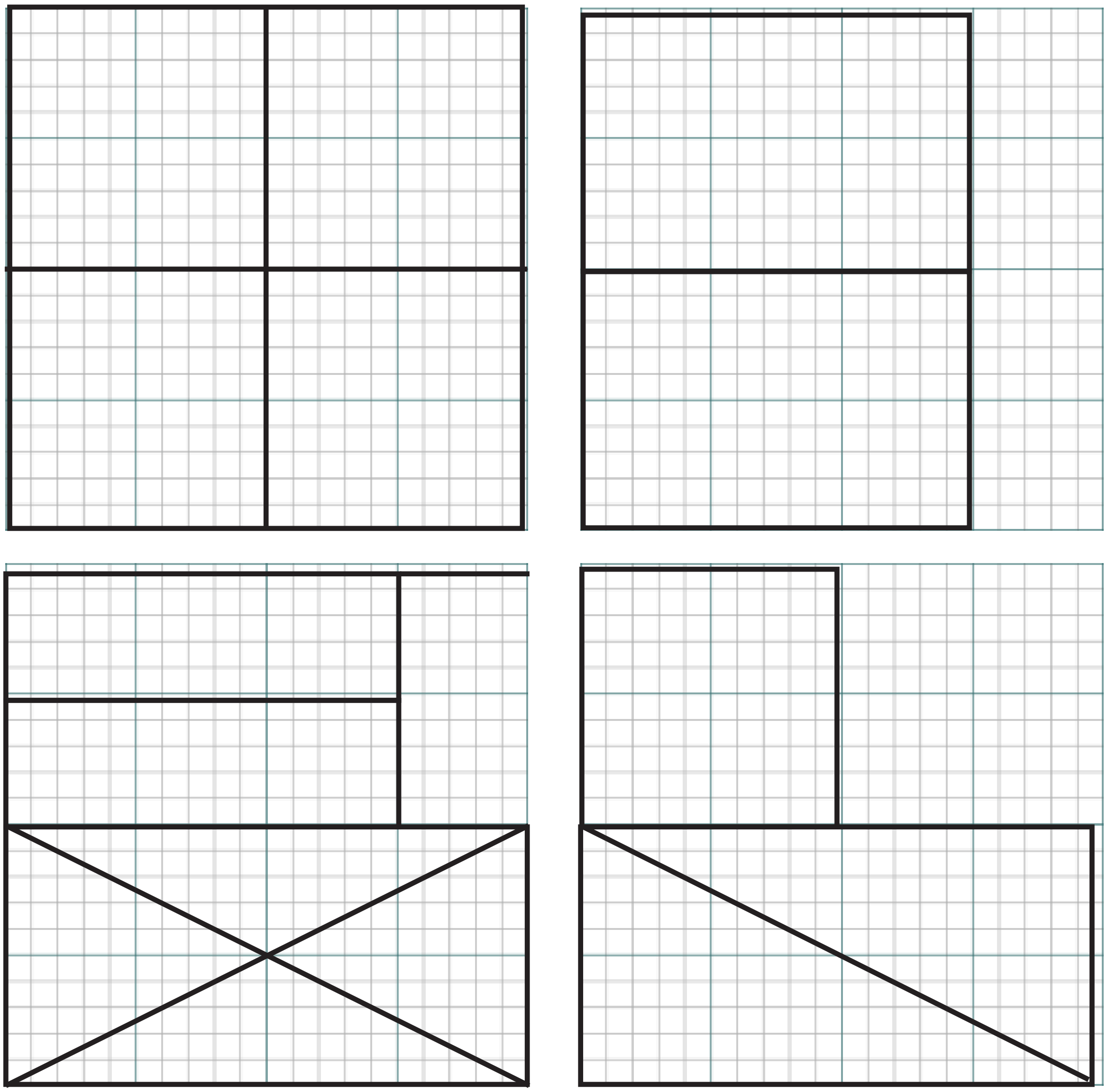


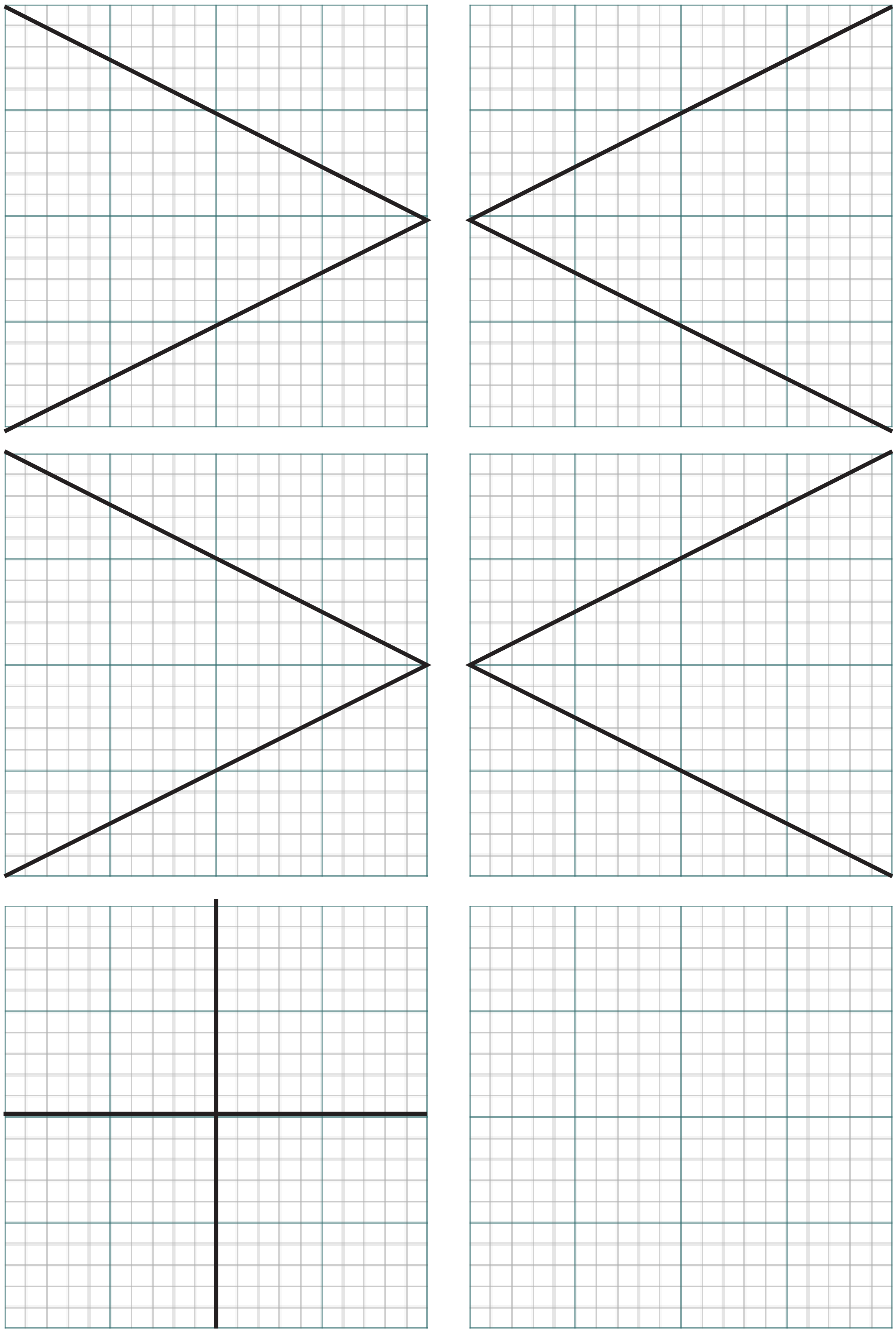
Sphere does not have a base

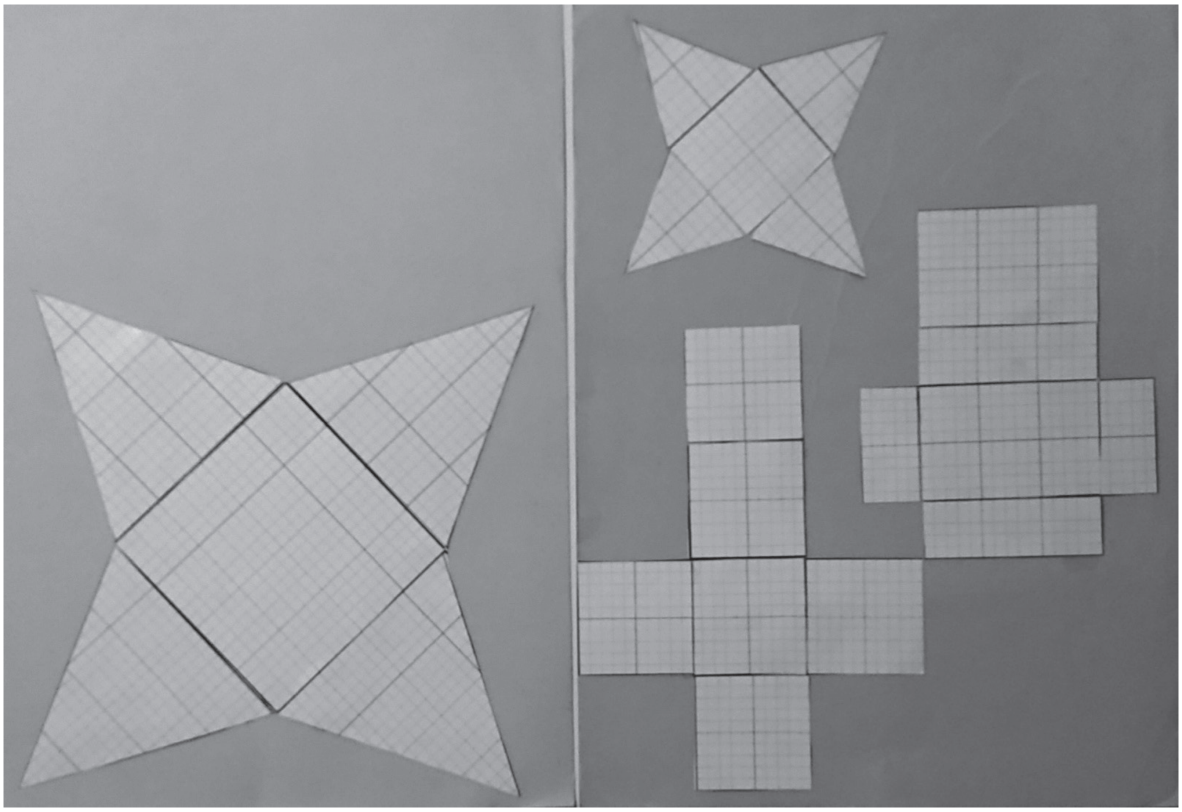
Which 3D object above reminds learners of a prism, and which object reminds learners of a pyramid?

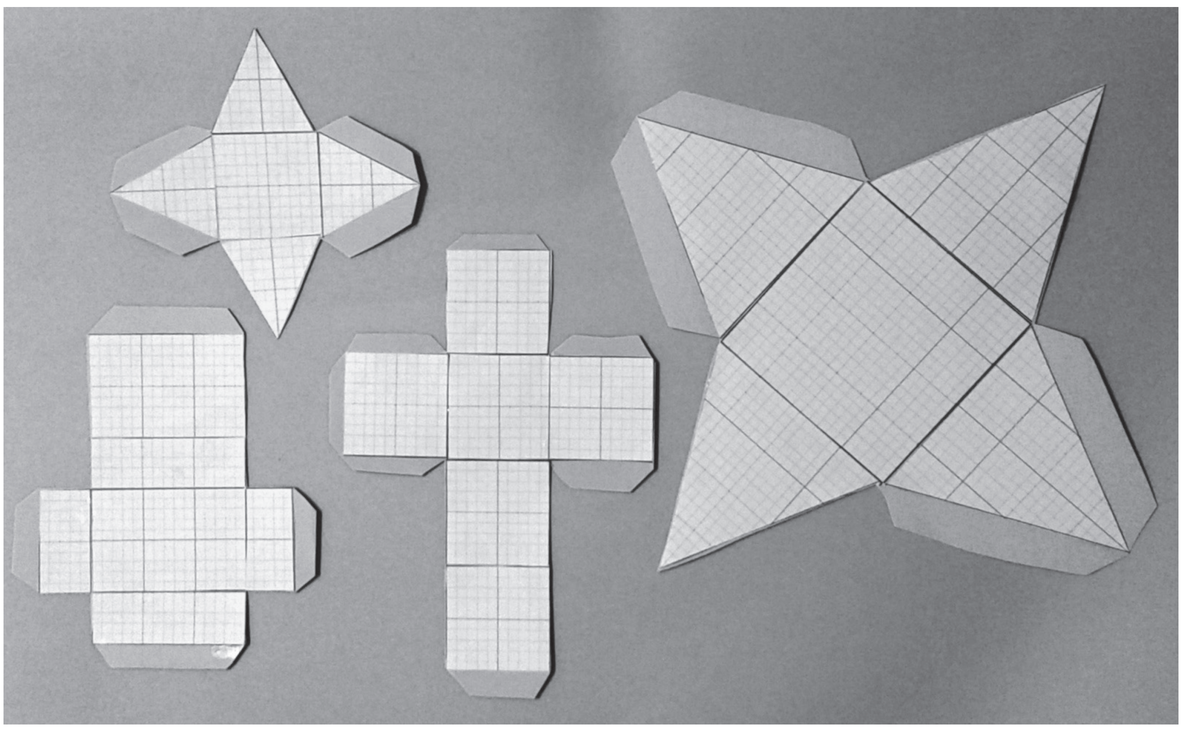
Constructing 3D objects

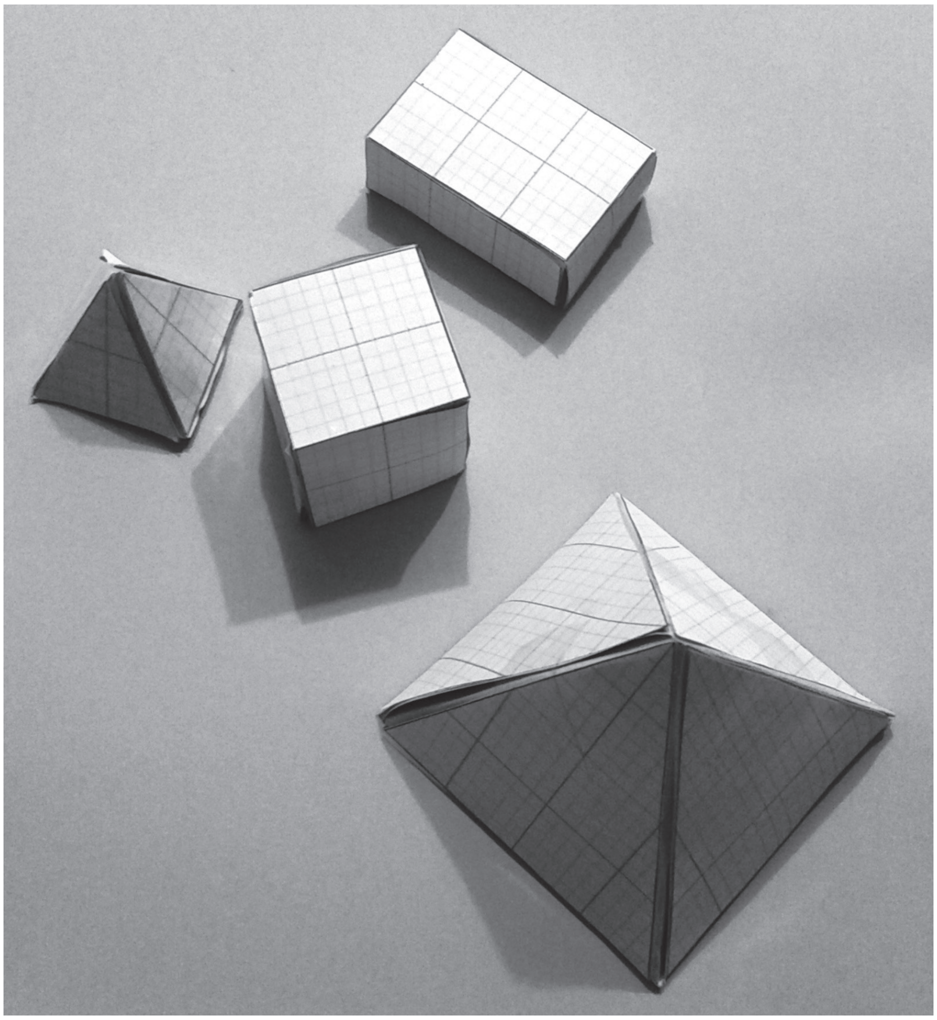
We are going to use the familiar 2D shapes to build our own 3D objects. Firstly, learners cut out the shapes on these pages, then follow the pictures to build nets and to build their own 3D objects from the basis of their familiar 2D shapes.











Hold the objects that you have built in your hand, inspect them and then complete the following table:

Name of the object	Number of rectangular faces	Number of square faces	Number of triangular faces

TOPIC 6: SYMMETRY

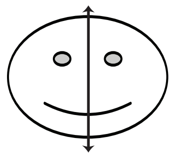
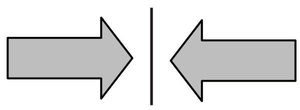
INTRODUCTION

- This unit runs for 2 hours.
- It is part of the Content Area ‘Space and Shape’ an area which is allocated 15% of the total weight shared by the five content areas at Grade 5.
- This unit covers the symmetry between and within shapes including lines of symmetry.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Recognise, draw and describe lines of symmetry in 2D shapes	• Recognise, draw and describe lines of symmetry in 2D shapes	• Recognise, draw and describe lines of symmetry in 2D shapes

GLOSSARY OF TERMS

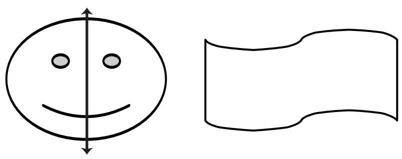
Term	Explanation / Diagram
Symmetry	Symmetry in a 2D shape means that it is made up of exactly similar parts facing each other around an axis or a line of symmetry. Example: 
Line of symmetry	The line that separates two parts of a 2D shape into exactly similar parts or that separates two shapes that are an exact reflection of each other.
Reflection	When an original image is repeated, as if in a mirror. We reflect the image along a horizontal axis or a vertical axis, or a diagonal axis. Reflections are symmetrical. Example:  arrows reflecting along a vertical axis

SUMMARY OF KEY CONCEPTS

Recognise lines of symmetry

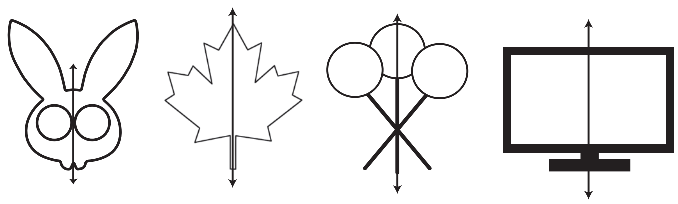
1. Many shapes have two halves that match exactly similar parts, which we call symmetry, if one half looks exactly like the other half, but they are facing each other.

Example:



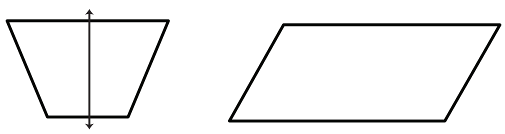
The second shape has no symmetry, although the parts appear to be similar.

Example:



All of these shapes have symmetry.

Example:

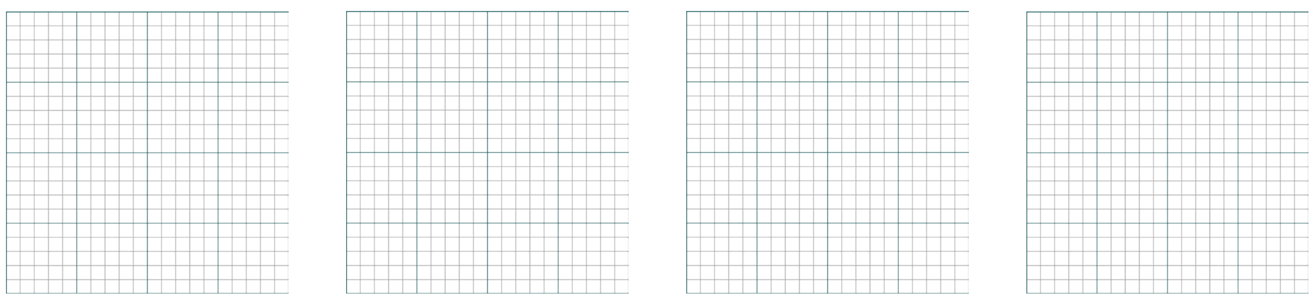


This shape has no symmetry, although the parts appear similar.

Some shapes have more than one line of symmetry.

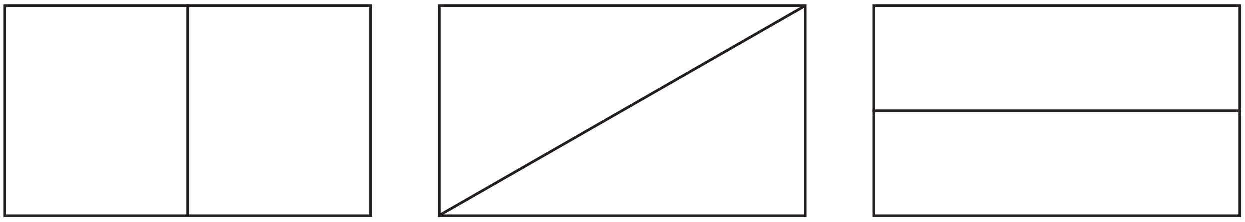
Example:

See if you can fold each of these squares in a different way to form two exactly similar halves. The fold lines are the lines of symmetry.



Example:

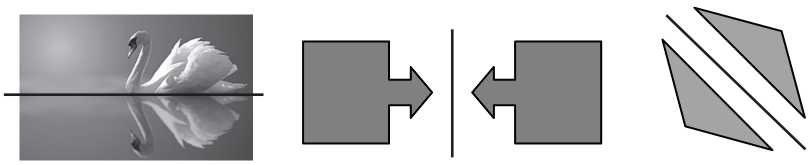
Say which of the lines in these rectangles are lines of symmetry and which are not. Explain why you say so.



Reflection

1. Reflection happens when the original image is repeated, as if in a mirror. We can reflect the image along a horizontal axis, along a vertical axis, or along a diagonal axis. Reflections are symmetrical. The images are facing each other and the axes are the same as the lines of symmetry.

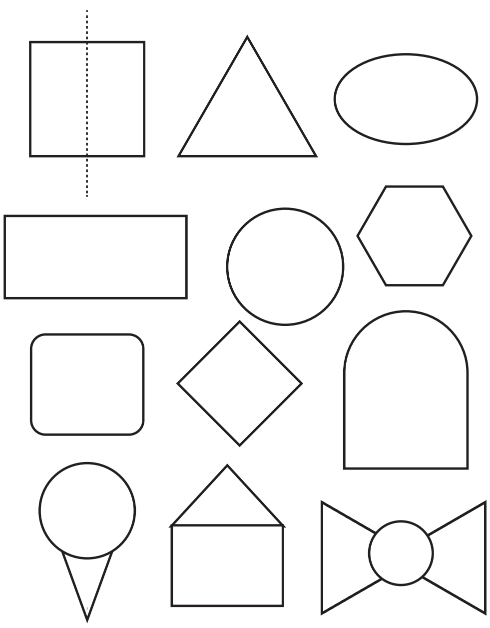
Examples:

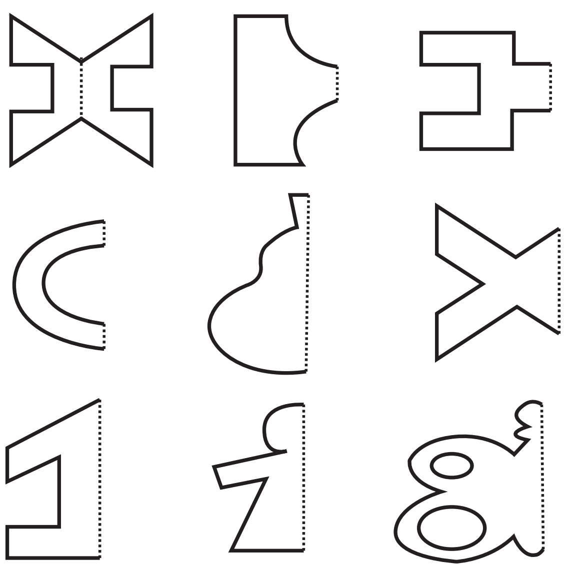


Along a horizontal axis   Along a vertical axis   Along a diagonal axis

Drawing reflections and finding lines of symmetry

1. Into the group of 2D shapes on the left, draw as many lines of symmetry as you can.
2. Onto the group of 2D shapes on the right, add a reflection to the part that is shown in the picture.





TOPIC 7: GEOMETRIC PATTERNS

INTRODUCTION

- This unit runs for 4 hours.
- It forms part of the content area ‘Patterns, functions and algebra’ and together with other similar topics, counts for 10% in the final exam.
- This unit deals with geometric (visual) patterns. Learners advance to represent these visual patterns in number form and in a diagrammatic form. They also have to find or understand the rule according to which the pattern is built.
- The purpose of this unit is to develop a sense of function, or rule-bound patterns.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Investigate and extend patterns looking for relationships and rules in • physical or diagram form • sequences with a constant difference • learners' own created patterns • Describe rules and relationships in own words • Determine input- and output values and rules for patterns and relationships using flow diagrams • Determine equivalence of different descriptions of the same relationship or pattern • verbally • in a flow diagram • by a number sentence	• Investigate and extend patterns looking for relationships and rules in • physical or diagram form • sequences with a constant difference or ratio • learners' own created patterns • Describe rules and relationships in own words • Determine input- and output values and rules for patterns and relationships using flow diagrams • Determine equivalence of different descriptions of the same relationship or pattern • verbally • in a flow diagram • by a number sentence	• Investigate and extend patterns looking for relationships and rules in • physical or diagram form • sequences with a constant difference or ratio • learners' own created patterns • represented in tables • Describe rules and relationships mathematically • Determine input- and output values and rules for patterns and relationships using flow diagrams and tables • Determine equivalence of different descriptions of the same relationship or pattern • verbally • in a flow diagram • by a number sentence • in a table

GLOSSARY OF TERMS

Term	Explanation / Diagram
Pattern	A pattern is a repeated sequence of shapes, pictures or numbers that are arranged according to a rule.
Numerical Pattern	4; 7; 10; 13 ...is a numerical pattern. Each term of the pattern has its own position and the pattern is a pattern, because all terms adhere to a general rule. The first term is important, because that is where the pattern starts. Each term's position is important and also the rule of the pattern.
Geometric Pattern	An ordered repetition of geometric shapes. Some shapes form a pattern because of their arrangement, and other form a pattern because there is a number value that we can attach to each term of the pattern.
Input Value	The input value for geometric- or number patterns is the number of the position in which the term appears in the pattern.
Output Value	The output value for geometric- or number patterns is the number value that a term of the pattern has, after we have applied the rule to the input number.
Flow Diagram	A diagram is a display of an operation or a series of operations that are performed on a number or a set of numbers. We find linear flow diagrams and the so-called 'spider diagrams'.
Relationships	In a number pattern each term has a specific relationship with the previous- and also with the next term in the pattern. This relationship is determined by the rule for the pattern.
Flow Chart	An alternative for a flow diagram, is a flow chart. This is a table that organises the number pattern and requires that any of the input values, output values or the rule has to be found.

SUMMARY OF KEY CONCEPTS

Number Patterns

In a number pattern, each term of the pattern has its own position. The basic idea behind a pattern is that all terms adhere to a general rule. The first number, the position of a term of the pattern, and the rule are the three most important elements of the number pattern.

Geometrical Patterns

1. Look at the pattern below:

   Example 1:

   ▲◊ ▲▲◊ ▲▲▲◊ ▲▲▲▲◊

   We can describe this geometric pattern in words:

   - the first term has one black triangle and one white diamond;
   - the second term has two black triangles and one white diamond;
     and so on.

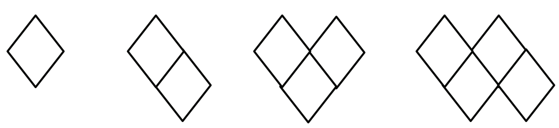
   We can also write it in numbers:

   T1: 1 (1) T2: 2 (1) T3: 3 (1) T4: 4 (1)

   In this way it becomes easy to predict any term, for example T17: 17 (1)

2. A more complex example is shown below:

   Example 2:



   We count a property of a geometrical shape. In this case we count how many lines or sides we have.

   T1: 4 T2: 7; T3: 10; T4: 13

   When we attach number values to each pattern, we are working with a numeric pattern in the end.

   The number of sides on the outside increases with three constantly:

   4; 7; 10; 13...

   The pattern starts at 4, and increases by a constant difference of 3.

Example 3:

▲◊ ▲▲◊ ▲▲▲◊ ▲▲▲▲◊

Counting the sides of all the shapes in each term, we find the following:

T1: 7; T2: 10; T3: 13; T4: 16

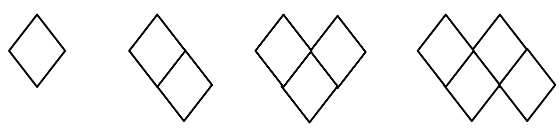
The number pattern or sequence is: 7; 10; 13; 16...

The pattern starts with 7, and increases with a constant difference of 3.

3. In Example 2 and Example 3, we see the same (constant) difference between the terms of the pattern. There is even similarity between the number values of the two patterns. However, they start at two different points. The first term in both patterns are different, therefore the value 7 is the number value of term 2 of the pattern in Example 2, where it is the number value of term 1 of the pattern in Example 3. This is an important difference!

4. Although the constant difference between the terms of both patterns is the same, they are two different patterns because they start at different points and the reason why they both go up in threes, also differs.

   In Example 2, the pattern starts with four, because of the four sides of the first diamond. A new diamond is clicked onto the previous group each time, but it shares one of its four sides with the block it clicks on to, causing an increase of three only to the number.

   In Example 3, the pattern starts with seven, because the triangle has three sides and the diamond has four sides. A new triangle is added each time, adding three sides to the next term.

   ▲◊ ▲▲◊ ▲▲▲◊ ▲▲▲▲◊

Teaching Tip: When we learn how to make a rule for a geometric pattern, let us get learners into a habit of saying: 'The rule for this pattern is that we are adding ... each time, starting at ...' For Example 2 we say: 'The rule for this pattern is that we are adding three each time, starting at four.' For Example 3 we say: 'The rule for this pattern is that we are adding three each time, starting at seven.'

Input Value

The input value for geometric- or number patterns like those in our examples, is the number of the position in which the term of the pattern appears.

Output Value

The output value for geometric- or number patterns like those in our examples, is the number value that a term of the pattern has, after we have applied the rule to the input number.

Flow Diagrams

1. A diagram is a display of an operation or a series of operations that are performed on numbers.
2. In a flow diagram, learners either find the rule that regulates the pattern, or they calculate the change that happens as the number pattern progresses, based on a given rule. We have various forms of flow-diagrams: a linear one such as in Examples 1 and 2; and the so-called ‘spider diagram’.

   a. In the linear flow-diagram (diagram flowing in a line) the rule is repeated every time and the terms follow each other in consecutive sequence.

Example 1:

The **start number** and the <u>**rule**</u> are provided. The following terms of the pattern must be found.

$4 + 5 \rightarrow \boxed{} + 5 \rightarrow \boxed{} + 5 \rightarrow \boxed{} + 5 \rightarrow \boxed{}$

Note that the rule is not only ‘5’, but ‘+5’.

Example 2:

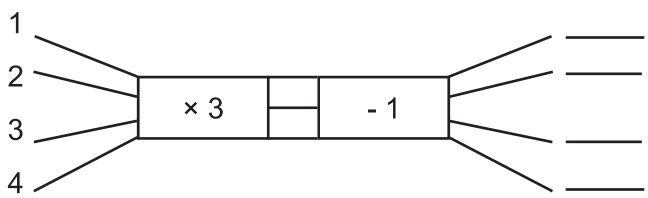
The **start number** and the <u>**next terms**</u> of the pattern are provided.
The rule must be found.

5 ____ → 9 ____ → 17 ____

b. In the spider-diagram, the rule appears once only, in a central position, the input values are to the left, the output values to the right. The input-values are following in a sequence like the term numbers.

c. EITHER the **input value** and the <u>**rule**</u> are provided and the <u>output values</u> must be found; OR the **input value** must be found while the <u>**rule**</u> and the <u>output values</u> are provided. All of this happens in the same diagram. (Spider diagrams prepare the way for functions).

Example 3:
Find the output values for a pattern following the rule x 3; – 1.

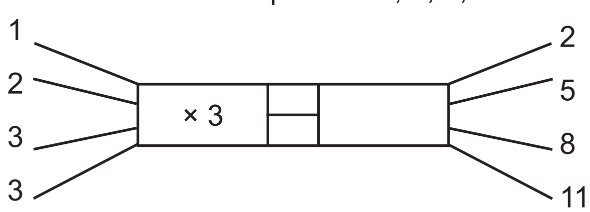


More challenging at this stage is this form of spider diagrams:

The <u>input values</u> and the <u>output values</u> are provided and the **rule** must be found.

Example 4:

Find the rule for the pattern: 2; 5; 8; 11...

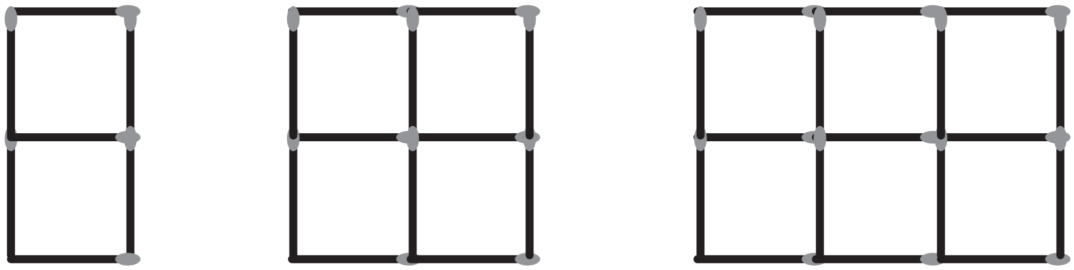


Teaching Tip: Before the above example can be given, start this type of spider diagram with either a single rule, or one of the two blocks for the rule containing a part of the rule.

3. An alternative for a flow diagram, is a flow chart. This is a table that organises the number pattern and requires that any of input values, output values or the rule has to be found.

Example:

Complete the flow chart for the pattern below and write down the rule. How many matchsticks will the eighth term of this pattern have? Which term will have 37 matchsticks?



Input	1	2	3	8		Rule:
Output					37	

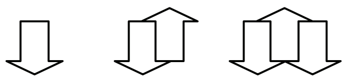
Various pattern structures

1. Patterns may develop in ascending order (with a positive difference) or in descending order (with a negative difference). Some classical geometric pattern structures are:

   a. Constant difference: Ascending order

Example:

Difference: + one arrow in the different direction

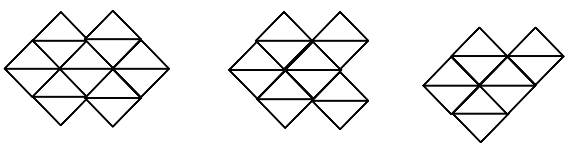


T1 T2 T3

   b. Constant difference: Descending order

Example:

Constant difference – two triangles



T1 T2 T3

TOPIC 8: DIVISION

INTRODUCTION

- This unit runs for 8 hours.
- It forms part of the content area: ‘Numbers, Operations and Relationships’ and counts a part of 50% allocated to this content area in the final exam.
- It covers division of whole 4-digit- by 2-digit numbers through various calculation strategies.
- The purpose of this unit is to deepen the understanding of division and refine calculation skills.

SEQUENTIAL TEACHING TABLE

<table>
<tr><th>GRADE 4
INTERMEDIATE PHASE</th><th>GRADE 5
INTERMEDIATE PHASE</th><th>GRADE 5
INTERMEDIATE PHASE</th></tr>
<tr><th>LOOKING BACK</th><th>CURRENT</th><th>LOOKING FORWARD</th></tr>
<tr>
<td>• Compare two or more quantities of the same kind (ratio)
• Compare two quantities of different kinds (rate)
• Divide at least whole 3-digit by 1-digit numbers
• Use the following strategies:
  • estimation
  • clue board
  • building up, breaking down
  • rounding off and compensating
  • doubling and halving
  • multiplication and division as inverse operations
• Know multiples of 1-digit numbers to at least 100
• Use properties of whole numbers
• Solve problems in financial and measurement contexts</td>
<td>• Compare two or more quantities of the same kind (ratio)
• Compare two quantities of different kinds (rate)
• Divide at least whole 3-digit by 2-digit numbers
• Use the following strategies:
  • estimation
  • building up, breaking down
  • rounding off and compensating
  • doubling and halving
  • multiplication and division as inverse operations
• Know multiples and factors of 2-digit numbers to at least 100
• Use properties of whole numbers
• Know multiplicative property of 1
• Know multiplication facts of multiples of 10 and 100</td>
<td>• Compare two or more quantities of the same kind (ratio)
• Compare two quantities different of kinds (rate)
• Divide at least whole 4-digit by 3-digit numbers
• Use the following strategies:
  • estimation
  • building up, breaking down
  • rounding off and compensating
  • doubling and halving
  • multiplication and division as inverse operations
  • long division
• Understand multiples and factors of numbers
• Use properties of whole numbers
• Know multiplicative property of 1
• Know multiplication facts of multiples of 10 and 100</td>
</tr>
</table>

GLOSSARY OF TERMS

Term	Explanation / Diagram
Division	Sharing out of a quantity into a number of equal portions or groups. Equal sharing, equal groups, rate and ratio all extensions of the same idea. Examples: a. Equal sharing: Share 35 sweets among 7 children ($35 \div 7 = 5$) b. Equal groups: Pack 35 sweets in packets of 5 ($35 \div 5 = 7$) c. Rate: Five packets of sweets cost R35, therefore the price per packet is R35 ÷ 5 = R7 (R7/packet) d. Ratio: There are 45 girls and 54 boys in Grade 5. This is a ratio of 45:54 or 5:6 if we divide each part by their highest common factor, which is 9. The girls form $\frac{5}{11}$ of the grade and the boys form $\frac{6}{11}$ of the grade.
Terms Used in a Division Equation	72 ÷ 6 = 12 ↓ ↓ ↓ dividend divisor quotient
Multiples	Multiples of a certain number (eg. 5) are the products when we multiply that number by any whole number: 15 is a multiple of 5, because $5 \times 3 = 15$
Factors	A whole number that divides exactly into another number. Factor pairs are those numbers that were multiplied to make a number. Factor the numbers 2, 14, 7 and 4 are factors of 28; 2 and 14 are a pair, 4 and 7 are a pair.
One – Multiplicative Property	One multiplied by, or divided into a number does not change that number: one is the identity element for multiplication and division.

SUMMARY OF KEY CONCEPTS

Dividing by 1, 10 and 100

Teaching Tip: We used the multiplication grid to discover what happens to a number when it is multiplied by 10: Observe the pattern in the multiples of one, ten and twenty. Now we use the multiplication grid to discover what happens to a number when it is divided by 10:

Example:

Because 7 x 10 = 70, therefore 70 ÷ 10 = 7

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	57	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	227	239	247	260
14	28	42	65	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	57	68	85	102	119	136	153	170	187	204	227	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	239	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	367	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

1. **Dividing by 0:** We cannot do that, the number will be too large to define. The answer is undefined.

   Example:

   I have 7 marbles. I give it to no children.

   How many marbles does each child get?

   7 ÷ 0 is undefined, because I could not give it, there were no children.

2. **Dividing by 1:** When we divide any number by 1, the number stays the same.

   Example:

   I have 7 marbles. I give it to one child. How many marbles does the child get?

   7 ÷ 1 = 7 The child gets all 7 marbles, because he was the only one.

3. **Dividing by 10:** When we divide a number that ends in a zero by 10, the answer looks like we have removed one zero.

   Example:

   I have 70 marbles. I share the marbles with 10 children. How many marbles does each child get?

   70 ÷ 10 = 7 Each of the ten children gets 7 marbles, because 7 x 10 = 70

4. **Dividing by 100:** When we divide a number ending in zeros by 100, the answer looks like we have removed two zeros.

   Example:

   I have 700 marbles. I share the marbles with one hundred children. How many marbles does each get?

   700 ÷ 100 = 7 Each of the hundred children gets 7 marbles, because 7 x 100 = 700

5. **Dividing 0**:There is nothing to divide, so the answer is 0.

   Example:

   I have no marbles. I want to give the marbles to seven children. How many marbles does each child get?

   0 ÷ 7 = 0 Each child gets no marbles, because there were no marbles.

Division Strategies

1. **Repeated subtraction**

   Below we are illustrating that repeated subtraction becomes a lengthy process when we are dealing with larger numbers, and it is likely that one would make mistakes.

   Example:

   874 ÷ 27

   874 – 27 → 847 – 27 → 820 – 27 → 793 – 27 → 766 – 27 → 739 – 27 → 712 – 27
   → 685 – 27 →658 – 27 →631 – 27 →604 – 27 → 577 – 27 → 550 – 27 → 523 – 27
   → 496 – 27 → 469 – 27 → 442 – 27 → 415 – 27 → 388 – 27 → 361 – 27 →334 – 27
   →307 – 27 →280 – 27 → 253 – 27 → 226 – 27 → 199 – 27
   → 172 – 27 → 145 – 27 → 118 – 27 → 91 – 27 → 64 – 27
   → 37 – 27 →10

   We subtracted 65 thirty-two times and was left with a remainder of 10

2. **Clue board**

   For a clue board, use the divisor to write down a few multiples of that number. It is usually enough to write the multiples of the number for 2, 3, 5, 10, 20, 30 and 50, depending on the dividend. Use the multiplication facts that multiplying by multiples of 10, to complete the board.

   Example 1:

   874 ÷ 27:

20 x 27 = 540	874 – 540 = 334
+ 10 x 27 = 270	334 – 270 = 64
+ 2 x 27 = 54	64 – 54 = 10
32	

   874 ÷ 27 = 32 remainder 10 or 32 $\frac{10}{27}$

2 x 27 = 54
3 x 27 = 81
5 x 27 = 135
10 x 27 = 270
20 x 27 = 540

   Teaching tip: Note that if the clue board is set up like this,

   it becomes easy to see that

   2 x 27 = 54 and 20 x 27 = 540

3. **Estimate by rounding**

   Learners estimate a division answer by rounding. Working with large numbers, we round both numbers. The estimate is not always very close,
   but it helps to check the reasonableness of our calculated answer.

Example:

874 ÷ 27

Round 874 to 900 and 27 to 30:

900 ÷ 30 = 30

The calculated answer was $32\frac{10}{27}$.

4. **Checking the answer by multiplying**

Because multiplication is the inverse of division, learners can check their answers by multiplying and adding the remainder.

Example:

Check the answer above: 874 ÷ 27 = 32 remainder 10

32 x 27

= (30 x 27) + (2 x 27)

= 810 + 54

= 864

864 + 10 = 874

5. **Solving problems in financial and measurement contexts**

Example 1:

Mom wants to buy a special of beef mince which is now marked down to R42 per kilogram. She has R765 in her purse. How many kilograms can she buy from that, and how much will she have left in her purse?

Example 2:

Thami has to cut 9.5 metres of string in strips of 33 cm each. How many strips will she be able to cut from the string that she has?

Example 3:

Challenge: If Thami has to cut a piece of string from the same length of 9.5 metres for each of 33 learners in the class, how long would that piece of string have to be?

Notes

Notes