

Grade 6

**CONTENT BOOKLET:
TARGETED SUPPORT
MATHEMATICS**

Term 2

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education.

The NECT has successfully brought together groups of people interested in education so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). The FSS helped the DBE trial the NECT Maths, Science and language learning programmes so that they could be improved and used by many more teachers. NECT has already begun this scale-up process in its Provincialisation Programme. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers.

Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

CONTENTS

TOPIC 1: WHOLE NUMBERS - COUNTING, ORDERING, COMPARING, REPRESENTING AND PLACE VALUE	6
TOPIC 2: WHOLE NUMBERS - MULTIPLICATION	15
TOPIC 3: PROPERTIES OF 3D SHAPES	21
TOPIC 4: GEOMETRIC PATTERNS	29
TOPIC 5: SYMMETRY	35
TOPIC 6: WHOLE NUMBERS - DIVISION	40
TOPIC 7: DECIMAL FRACTIONS	49
TOPIC 8: CAPACITY AND VOLUME	60
EDUCATOR RESOURCES	69

TOPIC 1: WHOLE NUMBERS - COUNTING, ORDERING, COMPARING, REPRESENTING AND PLACE VALUE

INTRODUCTION

- This unit runs for 1 hour.
- This unit is part of the content area 'Numbers, Operations and Relationships'. This content area counts for 50% of the final exam.
- This unit covers whole numbers up to 9 digits.
- The purpose of this section is to develop skills which will assist in preparing learners for algebra. New concepts and vocabulary are introduced and built on. Rounding is a vital skill that helps learners estimate answers with more accuracy and helps them to avoid unnecessary errors.
- Mental mathematics must be done daily as the basic skills that were taught in earlier grades and skills that were taught in the first term must be tested and built on in this new term. Most CAPS aligned textbooks have examples of mental calculations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> Order, compare and represent numbers to at least 6-digit numbers Represent odd and even numbers to at least 1 000. Recognize the place value of digits in whole numbers to at least 6 digit numbers. Round off to the nearest 5, 10, 100 and 1 000 	<ul style="list-style-type: none"> Order, compare and represent numbers to at least 9-digit numbers Represent prime numbers to at least 100 Recognizing the place value of digits in whole numbers to at least 9-digit numbers Round off to the nearest 5, 10, 100 and 1000. Using a range of techniques to perform and check written and mental calculations of whole numbers including: <ul style="list-style-type: none"> estimation building up and breaking down numbers rounding off and compensating using addition and subtraction as inverse operations adding, subtracting and multiplying in columns long division using multiplication and division as inverse operations 	<p>Revise the following done in Grade 6:</p> <ul style="list-style-type: none"> order, compare and represent numbers to at least 9-digit numbers recognize and represent prime numbers to at least 100 round off numbers to the nearest 5, 10, 100 or 1 000 recognize and represent prime numbers to at least 100 order, compare and represent numbers to at least 9-digit numbers Revise prime numbers to at least 100 <p>In the FET Phase:</p> <ul style="list-style-type: none"> Learners will apply their knowledge of numbers and operations across all contexts. The FET phase requires an increasing depth of the knowledge gained in all the previous phases so that application of knowledge can be used to solve problems in a large variety of contexts. This is also extended into the algebraic concepts covered during the remainder of the senior phase and the FET phase. Number sense is the key to Mathematical Literacy in learners.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Whole Numbers	The numbers in the set $\{0, 1, 2, 3, \dots\}$ are called whole numbers. Whole numbers are counting numbers including zero.
Place Value	The value of the digit depends on its order in the number. In 65 314 728, the 2 is in the 'tens' position, so it shows a value of 20.
Rounding Off	Numbers are either rounded up or down to the nearest 10, 100 or 1 000.
Digit	A single character used in a numbering system. In the decimal system, digits are 0 to 9.
Inverse Operations	Addition and subtraction are inverse operations. Multiplication and division are inverse operations.
Order of Operation	BODMAS When there are multiple operations to be performed, there is a particular order that the operations should be performed in. <ol style="list-style-type: none"> 1. 'Brackets' [] 2. 'Order': work out exponents and roots 3. 'Division' and 'Multiplication': work left to right 4. 'Addition' and 'Subtraction': work left to right.
Estimation	The best guess arrived at after considering all the information given in a problem. To estimate is to find an answer that is as close as possible to the exact answer.
Million	A million is 1 000 000 or 1000 thousands.

SUMMARY OF KEY CONCEPTS

Counting in Millions, Ten Millions and Hundred Millions

There are many ways to practice counting, and to have learners count in the given interval such as millions, without always starting with the first multiple or with zero.

Counting can be done orally at the start of a lesson, or grids can be used. Learners can write number sets where they work with counting sets in various forms.



EXAMPLES:

Counting in millions:

1 000 000, 2 000 000, 3 000 000...

1 340 000, 2 340 000, 3 340 000...

Counting in ten millions:

10 000 000, 20 000 000, 30 000 000...

1 000 000, 11 000 000, 21 000 000 ...

12 300 000, 22 300 000, 32 300 000...

Counting in hundred millions:

100 000 000, 200 000 000, 300 000 000...

102 000 000, 202 000 000, 203 000 000...

Learners must be able to count forwards and backwards. If learners are finding this difficult, use place value tables and charts to help them see how to order the numbers correctly.

Numbers up to One Hundred Million in Words and in Numeric Form

1. In Table Form:

	HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
Digits	5	6	5	3	1	4	7	2	8
What the digit means in terms of its position	This represents 5 hundred millions	This represents 6 ten millions	This represents 5 millions	This represents 3 hundred thousands	This represents 1 ten thousand	This represents 4 thousands	This represents 7 hundreds	This represents 2 tens	This represents 8 ones or units
Numeric	500 000 000	60 000 000	5 000 000	300 000	10 000	4000	700	20	8
How you would say it	Five hundred million	Sixty million	Five million	Three hundred thousand	Ten thousand	Four thousand	Seven hundred	Twenty	Eight
What the value of each digit is in the number	The digit 5 has a value of 500 000 000	The digit 6 has a value of 60 000 000	The digit 5 has a value of 5 000 000	The digit 3 has a value of 300 000	The digit 1 has a value of 10 000	The digit 4 has a value of 4000	The digit 7 has a value of 700	The digit 2 has a value of 20	The digit 8 has a value of 8

2. In Numerals: *565 314 728*

3. In Words:

Five hundred and sixty five million, three hundred and fourteen thousand, seven hundred and twenty-eight.

4. Expanded Form:

$$500\,000\,000 + 60\,000\,000 + 5\,000\,000 + 300\,000 + 10\,000 + 4000 + 700 + 20 + 8$$

5. Identify the place and value of each digit in a number up to hundred millions (9 digits):

565 314 728

- The digit 5 is in the **hundred million** place.
- The digit 6 is in the **ten million** place.
- The digit 5 is in the **million** place.
- The digit 3 is in the **hundred thousand** place.
- The digit 1 is in the **ten thousand** place.
- The digit 4 is in the **thousand** place.
- The digit 7 is in the **hundred** place.
- The digit 2 is in the **tens** place.
- The digit 8 is in the **unit /ones** place.



Teaching Tip: Learners often misread or write numbers incorrectly when they have zeros. This can be prevented by working with place value tables, and if educators emphasise the importance of zeros as place holders in the number. Learners must know that if hundred millions are mentioned, there will be at least 9 digits in the number. Learners can practice reading numbers in sets of three and making a point of stating that each space represents a word. Example: 234 567 123 is two hundred and thirty four million five hundred and sixty seven thousand one hundred and twenty three.

Compare Numbers within Hundred Millions (9 digits):

Learners compare and order numbers according to their value. Working with place value tables and writing numbers directly below each other is a good strategy that will help learners make accurate comparisons.

1. Smaller than: <

Which number is smaller, 467 237 981 or 467 230 600?

When comparing numbers, look at the value of each digit starting from the left and gradually moving to the right of the number.

HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
4	6	7	2	3	7	9	8	1
4	6	7	2	3	0	6	0	0

0 thousands is **smaller** than **7** thousands. So, 467 230 600 is smaller than 467 237 981.

This is written as $467\ 230\ 600 < 467\ 237\ 981$.



Teaching Tip: = is a sign used for equality, where the left-hand side and the right-hand side of an expression are equal in value. There are signs that indicate inequality, where the left-hand side of an expression is not equal to the right-hand side:

$234 \neq 324$ (not equal to...)

$234 \approx 230$ (approximately equal to...)

$324 > 234$ (larger than...)

$230 < 234$ (smaller than...)

2. Greater than: >

Which number is greater, 300 712 935 or 300 712 846?

HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
3	0	0	7	1	2	9	3	5
3	0	0	7	1	2	8	4	6

A table makes it easier to work out which number is bigger or smaller than.

Work from left to right.

If they are the same, continue to compare until the values of the digits are not the same.

The values of the digits in the hundreds place are not the same.

9 hundred is **greater** than **8** hundred. So 300 712 **935** is **greater** than 300 712 **846**.

This must be written as $300\ 712\ 935 > 300\ 712\ 846$.

3. Arrange the numbers from smallest to biggest (ascending order).

324 688, 32 468, 3 246 880, 324 560 004

Look at which number has the **least** digits and this will become the first number. It will help if learners use a place value table and write numbers underneath each other.

Answer: 32 468, 324 688, 3 246 880, 324 560 004

4. Arrange the numbers from biggest to smallest (descending order)

324 688, 32 468, 3 246 880, 324 560 004

Look at which number has the **most** digits this becomes the first number in the sequence and is then followed by the next biggest number and so on.

Answer: 324 560 004, 3 246 880, 324 688, 32 468



Teaching Tip: If learners are still getting confused, then revise the rules for ordering numbers by starting with smaller numbers, and working back to longer and bigger numbers, as the learners start to grasp the concepts of comparing and ordering.

Rounding Off

This section is often difficult for learners to grasp, but remember the rules below and do a lot of oral work and mental maths with this concept.

1. Decide which digit is the last one you need to keep. You will know this because it is the digit in the place you are asked to round off to.
2. Leave it the same if the next digit is less than 5.
3. Round it up (increase by 1) if the next digit is greater than or equal to 5.



Example: Round 74 to the nearest 10

We focus on the 7 in the 10s place

The next digit is 4, which is less than 5, so no change is needed to 7

= 70

74 gets rounded down.



Example: Round 86 to the nearest 10

We focus on the 8 in the 10s place

The next digit is 6 which is 5 or more, so increase the 8 by 1 to 9

= 90

86 gets rounded up.

Learners must be able to round to the nearest multiples of 5, 10, 100 or 1000.

Prime Numbers to at least 100

Learners must be able to identify all prime numbers between 0 and 100. Prime numbers have one factor pair, in other words two factors only, that is 1 and themselves, for example $17 = 1 \times 17$. (RESOURCE 1 AT THE END OF THIS SECTION)

Learners must also know that numbers that are not prime are considered composite numbers. Composite numbers are numbers that have more than one factor pair, that is three or more factors, for example $15 = 1 \times 15$ and also $15 = 3 \times 5$.

1 is neither prime nor composite. It has one factor only.

(RESOURCE 2 AT THE END OF THIS SECTION)

RESOURCES

RESOURCE 1

PRIME NUMBERS

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

RESOURCE 2

COMPOSITE NUMBERS

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

TOPIC 2: WHOLE NUMBERS – MULTIPLICATION

INTRODUCTION

- This unit runs for 5 hours.
- This unit is part of the content area 'Numbers, Operations and Relationships'. This content area counts for 50% of the final exam.
- The unit covers multiplication of 4 digit and 3 digit numbers using a variety of techniques and skills.
- The purpose of this section is to ensure procedural fluency in the multiplication operation. Procedural fluency includes an ever improving speed:accuracy ratio.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Multiplication of 3-digit whole numbers by 2-digit whole numbers 	<ul style="list-style-type: none"> • Multiplication of 4-digit whole numbers by 3-digit whole numbers • Multiple operations on whole numbers with or without brackets 	<ul style="list-style-type: none"> • Perform calculations using all four operations on whole numbers, estimating and using calculators where appropriate

GLOSSARY OF TERMS

Term	Explanation / Diagram
Factor	Factors are numbers we can multiply together to get another number: For example: 2 and 3 are factors of 6, because $2 \times 3 = 6$.
Multiple	The result of multiplying a number by an integer [not by a fraction]. For example: 12 is a multiple of 3, as $3 \times 4 = 12$
Multiplication	The basic idea of multiplication is repeated addition. For example: $5 \times 3 = 5 + 5 + 5 = 15$
Product	The result when two or more numbers are multiplied together.
Distributive Property	The Distributive property states that multiplying a number by a group of numbers added or subtracted is the same as doing each multiplication separately For example: $3 \times [2 + 4] = 3 \times 2 + 3 \times 4$

SUMMARY OF KEY CONCEPTS

Factors and Multiples:

Learners need to know the multiples and factors of 2 digit and 3 digit numbers. An understanding of prime factors is also required.



For example:

The factors of 24 are {1, 2, 3, 4, 6, 8, 12, 24}

The prime factors of 24 are 2 and 3.



For example:

The multiples of 24 are {24, 48, 72, 96...}. Multiples continue and are infinite -meaning they do not finish- with multiples being an open and continuing set.

Partitioning (breaking up) Numbers to Multiply Them

1. Using the distributive property to multiply.



For example:

How many bottles of water are there in 237 boxes, if each box contains 24 bottles?

The number 237 can be broken up into $200 + 30 + 7$; and now the distributive property can be used to complete the problem.

$$\begin{aligned} 24 \times (200 + 30 + 7) \\ = (24 \times 200) + (24 \times 30) + (24 \times 7) \\ = 4800 + 720 + 168 \\ = 5688 \end{aligned}$$

2. The standard algorithm (long multiplication)

$$\begin{array}{r} 352 \\ \times 27 \\ \hline 2464 \\ 7040 \\ \hline 9504 \end{array}$$

This final step is the addition of two products.

Topic 2 Whole Numbers Multiplication



For example:

There are 256 people at a soccer game. If each person spends R24, how much money was made?

The factors of 24 can be 6 and 4, and these can each be factored further where 4 is 2×2 and 6 is 2×3 .

This means multiplication can be done as follows:

$$\begin{aligned}(256 \times 2) \times 2 \times 2 \times 3 \\ &= (512 \times 2) \times 2 \times 3 \\ &= (1024 \times 2) \times 3 \\ &= 2048 \times 3 \\ &= 6144\end{aligned}$$

These are not new skills as learners have practiced them in previous grades. If learners can decompose numbers into prime factors, then the final multiplication is much easier.

Estimating an Answer to Help with Multiplication

The skill of rounding off must be revised before estimation is attempted. Some learners may want to round everything off, however, it is important to stress that too much rounding has the potential to make the answer less accurate.



For example:

$$5325 \times 108 \approx 575\,100$$

The purpose with rounding is to make the estimation easier and still have a reasonable approximate answer. Rounding 5 325 to 5 000 may give a reasonable estimate, but it does not make the estimation easy enough. Rounding 108 to 100 makes the estimation easy and gives a reasonable approximate. Rounding both numbers to 5 000 and to 100 makes the estimation super easy, but the answer is far off.

The Column Method for Multiplication (standard algorithm)

This method of multiplication is suitable for the large numbers that learners should be able to multiply by the end of Grade 6.

(RESOURCE 1 AT THE END OF THIS SECTION)

M	Hth	Tth	Th	H	T	U	
			5	4	6	9	
				1	8	7	
		3	8	2	8	3	$\times 7$
	4	3	7	5	2	0	$\times 80$
	5	4	6	9	0	0	$\times 100$
1	0	2	2	7	0	3	Add all of the products to give the final answer.

Learners must be comfortable with tables, factors and multiples. This method is best taught to learners using a board, as the numbers are far too large to teach this section practically. Learners can work on multiplying two digit by two digit numbers, and then gradually increase the size of each number, until learners are comfortable and can meet the requirement of multiplying four digit numbers by three digit numbers.



Teaching Tip: Learners often forget to place zeros in the units columns when multiplying by the tens digit of a number, and two zeros when multiplying by the hundreds digit. Using the grid provided in the resource section on the next page will help learners remember the zeros.

Topic 2 Whole Numbers Multiplication

RESOURCES



RESOURCE 1

M	Hth	Tth	Th	H	T	U

TOPIC 3: PROPERTIES OF 3D SHAPES

INTRODUCTION

- This unit runs for 5 hours.
- This unit is part of the content area 'Space and Shape'. This counts for 15% of the final exam.
- Learners must be able to identify and describe various 3D shapes. This becomes the foundation of geometric reasoning that is a requirement later in the Senior and FET phases.
- The purpose of this unit is to develop an ability to work with 3D problems and understand these when working with real-life situations.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> Recognize, visualize and name 3-D objects in the environment and geometric settings, focusing on: <ul style="list-style-type: none"> rectangular prisms and other prisms cubes cylinders cones pyramids Similarities and differences between cubes and rectangular prisms Describe, sort and compare 3-D objects in terms of <ul style="list-style-type: none"> shape of faces number of faces flat and curved surfaces Make 3-D models using cut out polygons Cut open boxes to trace and describe their nets 	<ul style="list-style-type: none"> Recognize, visualize and name 3-D objects in the environment and geometric settings, focusing on <ul style="list-style-type: none"> rectangular prisms cubes tetrahedrons pyramids Similarities and differences between tetrahedrons and other pyramids Describe, sort and compare 3-D objects in terms of <ul style="list-style-type: none"> number and shape of faces number of vertices number of edges Make 3-D models using: drinking straws, toothpicks and other materials Nets of shapes 	<ul style="list-style-type: none"> Describe, name and compare the 5 Platonic solids in terms of the shape and number of faces, the number of vertices and the number of edges Recognize and describe the properties of: <ul style="list-style-type: none"> spheres cylinders Use nets to create models of geometric solids, including: <ul style="list-style-type: none"> cubes prisms pyramids cylinders

GLOSSARY OF TERMS

Term	Explanation / Diagram
Three-dimensional (3D) Object	An object with three dimensions (such as height, width and depth) like any object in the real world.
Polygon	A polygon is a plane (or flat) shape completely enclosed by three or more straight sides. Polygons are named by the number of sides they have.
Prism	A prism is a polyhedron with two identical faces parallel to each other (called the ends or the bases). All faces other than the ends are rectangles. A match box is an example of a prism in real life.
Polyhedron	A polyhedron is a three dimensional shape of which all the faces are polygons. Its name is based on the number of faces it has.
Pyramid	A special type of polyhedron. It has a solid base that is a polygon, and the other faces are triangles that meet at an apex.*
Apex	The highest point of a pyramid or cone.
Tetrahedron	A solid made up of four triangular faces. It can also be considered a pyramid with a triangular base. They are the tetrahedron [4 faces], cube [6 faces], octahedron [eight faces], dodecahedron [12 faces] and icosahedron [20 faces].
Platonic Solid	A 3D shape where each face is the same regular polygon and the same number of polygons meet at each vertex (corner). There are 5 platonic solids.
Face	The flat surface or side of a solid shape.
Edge	The intersection of the faces of a 3D shape.
Vertex	The corner of a shape or solid. The place where the edges of a shape meet.
Net	A 2D pattern of a 3D shape that can be folded to result in the 3D object.
Sphere	A perfectly round ball. This shape is made only of curved edges. Every point on the surface of the sphere is the same distance away from the centre of the shape.
Cone	A solid [3-dimensional] object with a circular flat base joined to a curved side that ends in an apex point.
Cylinder	A closed solid that has two parallel circular bases connected by a curved surface.

SUMMARY OF KEY CONCEPTS

Identifying 3D Objects

Learners must be able to recognise and name 3D objects in their natural and cultural forms, and in geometric settings. Learners should be able to identify 3D shapes in everyday objects like food tins (cylinders), match boxes (rectangular prisms) and a number of objects found around the home.

Ways in which to identify and group 3D objects:

1. Objects with a curved surface only – Spheres.

It is important to have both real-life and geometric examples for learners to use to familiarise themselves with the shape.



Sphere

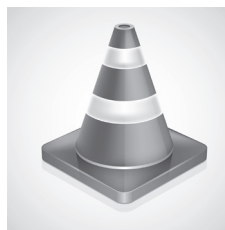
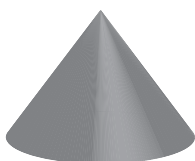
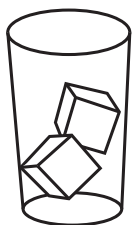


Remember that every point on the surface of a sphere is an equal distance from the centre.

This means that a rugby ball is not an example of a sphere.

2. Objects with flat and curved surfaces – Cones and Cylinders.

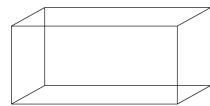
A variety of examples will help learners identify the shape when necessary.



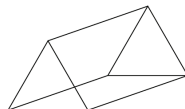
3. Objects with only straight edges – Prisms and Pyramids.

While the concepts of prisms and pyramids are being developed, learners can do some discovering on their own, to distinguish clearly between the two categories and to compare their similarities and differences.

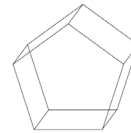
Prisms:



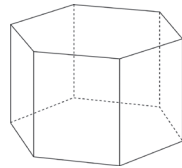
rectangular prism



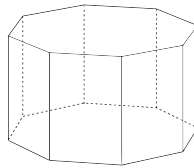
triangular prism



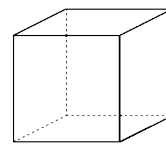
pentagonal prism



hexagonal prism

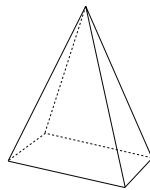


octagonal prism

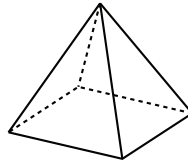


cube

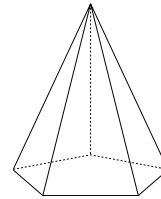
Pyramids:



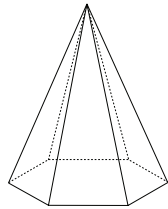
rectangular pyramid



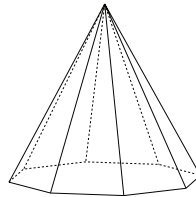
square pyramid



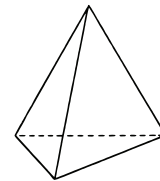
pentagonal pyramid



hexagonal pyramid



octagonal pyramid



Tetrahedron



Teaching Tip: Learners may find it difficult to tell the difference between pyramids and prisms. It is vital to explain the definitions of these shapes, and to have models for the learners to see and handle, as it will assist them in getting a better conceptual understanding.

Topic 3 Properties of 3D Shapes

Constructing 3D Shapes

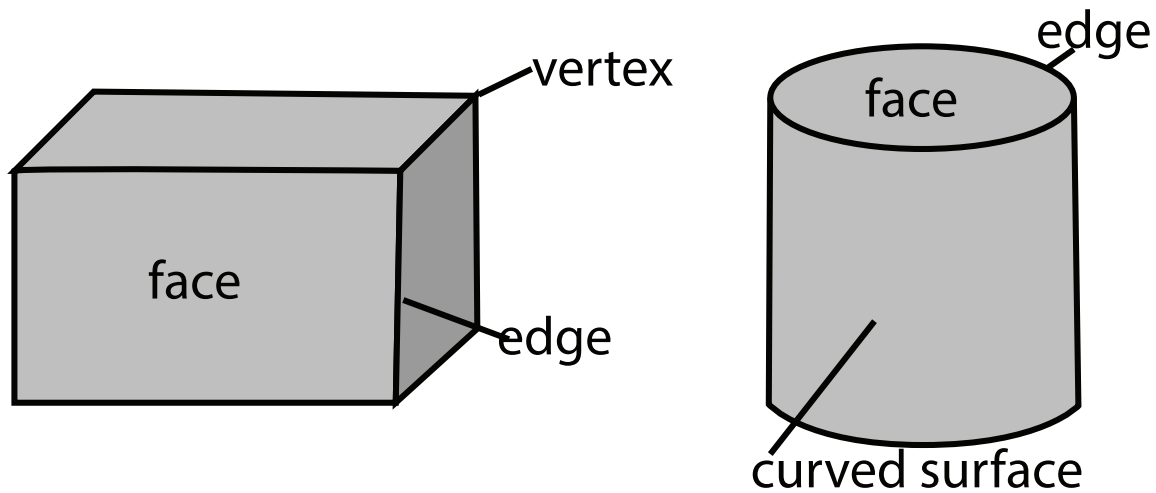
Learners can truly experience 3D shapes by constructing them by combining plane 2D shapes as the faces or sides of the 3D object. This will aid the learners in tasks that require classifying and sorting shapes. It gives the learners concrete ideas of the specific properties of the various shapes.

Included in the EDUCATORS RESOURCES SECTION are nets that can be used to construct a variety of shapes.

Opening boxes will help learners investigate the nets of items that have real-life connections, and will help to form a clearer contextual picture of the properties of the various shapes.

Describe, Sort and Compare 3D Shapes

In order to sort and compare 3D shapes accurately, learners should have a clear understanding of the parts labelled on the drawings below. These should be the criteria that are used to sort and compare shapes.



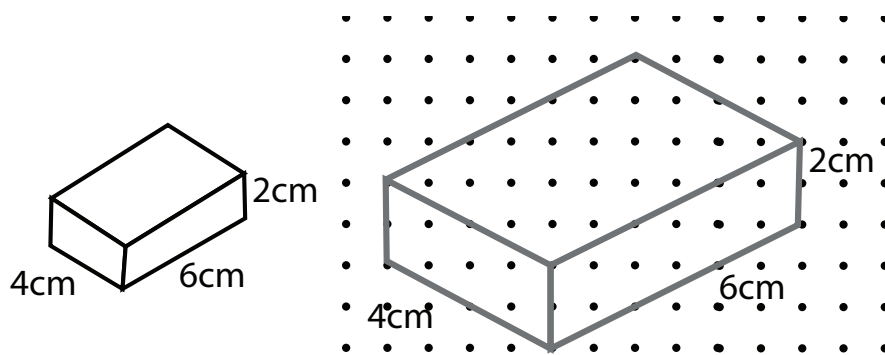
Learners can sort shapes in various ways:

1. Using the number of faces, edges and vertices (plural of vertex).
2. By sorting shapes through grouping all shapes with flat edges together, those with flat and curved edges together; and then those with curved edges only together.
3. Learners could also group shapes according to the shape of the faces.

Draw 3D Shapes

Drawing 3D shapes helps learners to develop skills that can be used in the problem solving sections later in this phase, and the phases that follow. Learners should also be capable of determining if a drawn net will result in the desired shape (RESOURCE 1).

Learners must be given the correct resources in order to be able to draw accurate 3D sketches of a variety of 3D shapes.



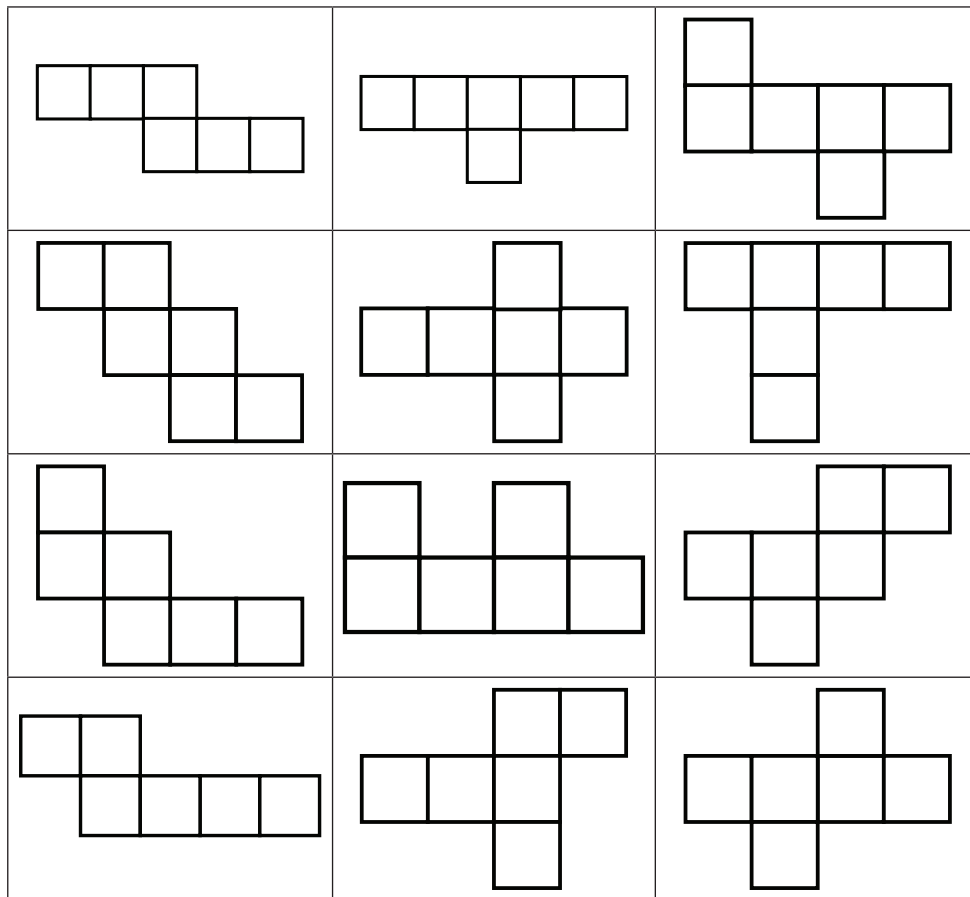
RESOURCES

RESOURCE 1

Cube Nets

Which of these nets fold up to make a cube?

To find out, copy the nets onto squared paper. Cut them out and fold them along the lines.



This exercise is also a basic lesson in constructing the cube as a regular polyhedron or platonic solid, which should have six congruent square faces.

TOPIC 4: GEOMETRIC PATTERNS

INTRODUCTION

- This unit runs for 6 hours.
- This unit is part of the 'Patterns, Functions and Algebra' content area. This counts for 10% in the final exam.
- The unit covers the concept of Geometric Patterns, which is extended as Number Patterns, and forms the foundation of the Algebra components in the senior and FET phases.
- The purpose of this section is to introduce learners to the concept of determining a general rule that can be applied in situations where a pattern develops. This is a vital building block in the development of algebraic skills.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Investigate and extend geometric patterns looking for relationships or rules represented in diagram form and not limited to constant differences or ratios • Describe observed relationships or rules in own words • Determine input values, output values and rules for patterns and relations using flow diagrams • Determine equivalence of different descriptions of the same relationship or rule presented verbally, in a flow diagram or by a number sentence. 	<ul style="list-style-type: none"> • Investigate and extend geometric patterns looking for relationships or rules represented in diagram form, in tables and not limited to constant differences or ratios • Describe observed relationships or rules in own words • Determine input values, output values and rules for patterns and relations using flow diagrams and tables • Determine equivalence of different descriptions of the same relationship or rule presented verbally, in a flow diagram, by a number sentence and in a table 	<ul style="list-style-type: none"> • Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns: represented in physical or diagram form not limited to sequences involving a constant difference or ratio of learner's own creation represented in tables represented algebraically • Describe and justify the general rules for observed relationships between numbers in own words or in algebraic language • The FET phase expands this knowledge even further looking at series of sequences.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Sequence	A sequence is a set of numbers or objects made and written according to some mathematical rule.
Inspection	Inspection is looking at something intentionally, to see patterns that show up in differences and similarity. To inspect a geometric pattern is to look at it to see if it becomes bigger or smaller, and in what way does the following term differ from the previous.
Flow Diagram	A diagram which flows horizontally from the input number[s] through the rule [operations] to the output number[s].
Geometric Pattern	A sequence of numbers or patterns based on multiplication or division is known as geometric pattern.
Rule	A given or determined method that results in the desired outputs when applied to the inputs of a pattern.
Equation	An equation is a statement that two expressions have the same value. It is a balanced calculation where the Left Hand Side [LHS] = the Right Hand Side [RHS].
Diagram	A visually ordered set of information like a geometric pattern in picture form, a table with information or a graph.
Pattern	A special arrangement of numbers or shapes. Patterns repeat or change in a constant manner.
General Term	The rule that can be used to determine any term in a given pattern.

SUMMARY OF KEY CONCEPTS

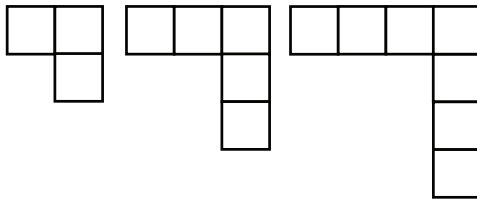
Extend Patterns and Determine Rules

The development of the concept of patterns goes through various stages:

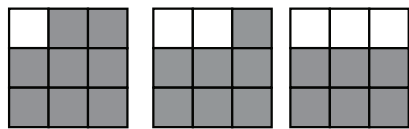
1. Packing shapes:
In the Foundation Phase, learners investigated and expanded patterns by using beads, matches, or any other small items. In this way they discovered the “rule” of the pattern they had created.
2. Drawing a pattern:
Later on learners made drawings of the patterns that they had created. Drawing the patterns allowed for further investigation and comparison.
3. In the Intermediate Phase the focus is on expanding and completing learner’s own patterns:
Activities that require completing a pattern or expanding up to 5 terms.



EXAMPLE:



Draw the next three figures in the pattern above.



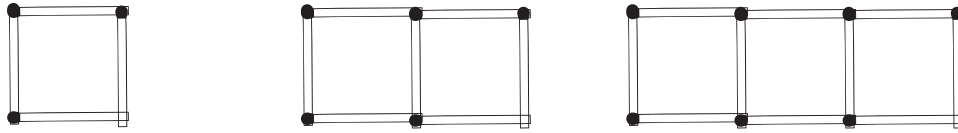
Draw the next three figures in the pattern above.

Topic 4 Geometric Patterns

Learners can be asked what would be required to produce the next figure in the geometric pattern.



EXAMPLE:



How many matches are needed to produce the 4th figure in the pattern?

(13 matches)

4. Describing patterns:

Describing a pattern in words is the basis of for finding the general term (the rule of the pattern). This is the foundation to writing rules that can be used to determine the desired outputs in a pattern.



EXAMPLE:



Learners must be able to describe in their own words how a pattern is formed. This pattern is formed by starting at 4 matches and adding three (3) more matches each time.

5. Designing own patterns

Learners can plan and design their own geometric patterns. They need to start at some point, and extend the pattern in a constant manner. Some ideas are triangles where other triangles are added on, tables where people are seated and more tables are added on, the triangular numbers, etc.



Teaching Tip: A good task for learners in order to allow them to be creative would be to ask them to design their own wrapping paper or a pattern for a table cloth or even a design to be used for the school's soccer team shirt that uses repetitive patterns of squares, circles and other shapes.

6. Developing tables from a rule:

Tables can be drawn up from a geometric pattern so that a rule can be determined.



EXAMPLE:



No. of squares	1	2	3	4	5	10	20
No of matches needed.	4	7	10	?	?	?	?

The table can help to find the rule that will be used to find terms in the pattern. It is easy to find consecutive terms, but learners should be able to determine any term by using a simple rule. This will help learners understand that patterns are used to find general solutions, thus making it easier to find larger term numbers.

An easy way to help learners find a pattern is to use the constant difference, and the first number in the pattern to make a rule.



EXAMPLE:

No. of squares	1	2	3	4	5	10	20
No of matches needed.	4	7	10	?	?	?	?
Constant difference is	3 [because 3 is added each time]						
Difference between the constant difference and the first number in the pattern.	3×1 [1st pattern which is ONE square] = 3. To get to 4 [number of matches] we need to add 1. 3×2 [2nd pattern/term] = 6. To get to 7 we need to add 1. Therefore the rule is: multiply by 3 and add 1.						

Rule = Constant difference (3) multiply by 'number of squares' and add 1.

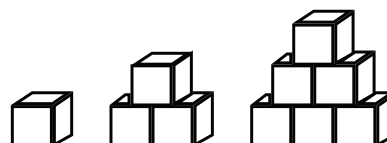
3D Geometric Patterns

At this stage, learners must start to develop a 3D pattern. Learners should know how to determine the actual pattern, and how to find the rule relating to the pattern. Learners should also be capable of completing a pattern comprised of 3D shapes.



EXAMPLE:

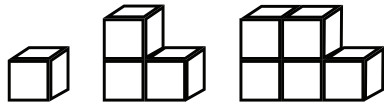
Describe the next two figures in the given pattern:



The next pattern (4th) will have 4 cubes in the bottom row, 3 in the next row, 2 in the 3rd row then one at the top. The cubes are always centred on top of the previous row. In total there will be 10 cubes.

The 5th pattern will have 5 cubes on the bottom row with one less cube on each row so there will be 15 ($5 + 4 + 3 + 2 + 1$) cubes in total.

Topic 4 Geometric Patterns



The 4th pattern will have 7 cubes. 4 on the bottom row and 3 on the next row. The three on the top row must be directly on top of the first 3 cubes in the bottom row.

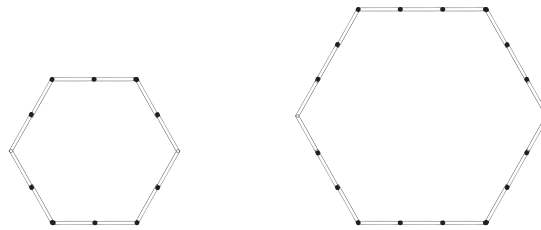
The 5th pattern will have 9 cubes. 5 on the bottom row and 4 on the top row.

Extending Patterns with a Constant Ratio

These patterns will usually represent shapes that are increasing or decreasing in size. Enlarged shapes will have a constant ratio that is greater than 1, which means that you will multiply by the same number to increase size. If the shape is getting smaller, the constant ratio is less than 1, meaning multiplication by a common fraction each time.



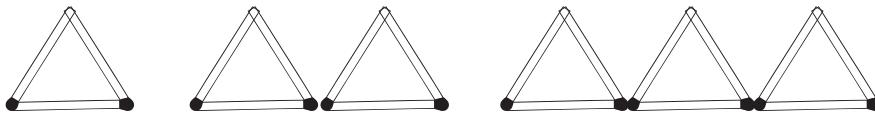
EXAMPLE:



This shape has increased to 1.5 (one and a half) times the original size. Use the length of the sides to help work this out. The larger hexagon has sides of 3 units long and the smaller hexagon has sides of 2 units long.

$$3 \text{ divided by } 2 = 1.5$$

Complete the table below for the number of matches used in this geometric pattern:



No. of Triangles	1	2	3	4	5	6
No. of Matches	3	6	9	12	15	18



For example:

For every triangle needed there are 3 matches required. Therefore, if there are 10 triangles, 30 (10 x 3) matches will be required.

TOPIC 5: SYMMETRY

INTRODUCTION

- This unit runs for 2 hours.
- This unit is part of the content area 'Space and Shape'. This counts for 10% of the final exam.
- The unit covers the concept of symmetry in shapes. The focus is on line and rotational symmetry.
- The purpose of this section is to develop skills where learners can see multiple lines of symmetry.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Recognize, draw and describe line[s] of symmetry in 2-D shapes 	<ul style="list-style-type: none"> • Recognize, draw and describe line[s] of symmetry in 2-D shapes 	<ul style="list-style-type: none"> • Symmetry is not a stand alone topic in the Senior or FET phase. However, the understanding of symmetry will be used in topics such as Functions and Transformation geometry.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Line symmetry or Reflection symmetry	Line symmetry is the symmetry of a plane [flat] shape which can be folded on that line in two parts that fit exactly on each other.
Line Of Symmetry	A line of symmetry divides a shape into two congruent parts. Some shapes have one, and some have more lines of symmetry. A square for example has four lines of symmetry, one vertical, one horizontal and two diagonal lines.
Infinite	This means that there is no end to the set, item or pattern.
Rotational Symmetry	Rotational symmetry is the symmetry of a shape that may be turned and fitted onto itself somewhere other than in its original position.
Order Of Rational Symmetry	This is the number of times a figure fits onto itself in one complete rotation [full turn].

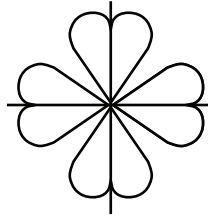
SUMMARY OF KEY CONCEPTS

Line Symmetry

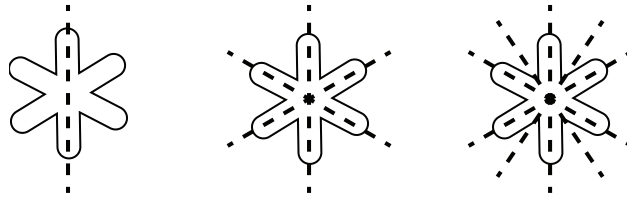
Learners must be able to indicate lines of symmetry in the presence of multiple lines of symmetry.



EXAMPLE:



This shape has both vertical and horizontal lines of symmetry

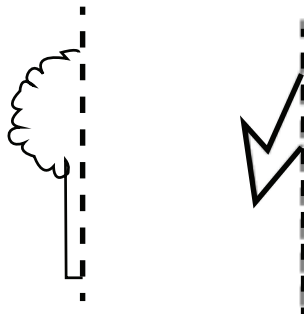


This shape has multiple lines of symmetry, and learners should be capable of determining all the lines of symmetry even along diagonal lines.

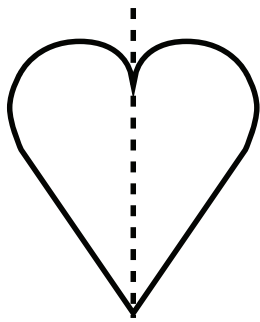
Learners should also be able to complete shapes when given a line of symmetry.



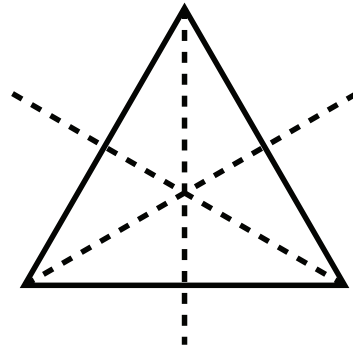
EXAMPLE:



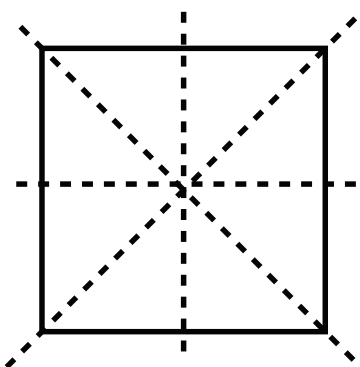
Topic 5 Symmetry



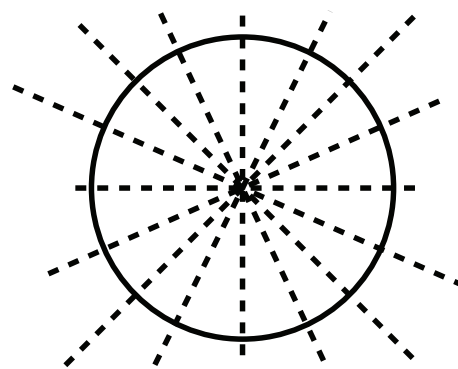
1 line of symmetry



3 lines of symmetry



4 lines of symmetry



infinite number of
lines of symmetry

Shapes like circles will have an infinite number of lines of symmetry.



Teaching Tip: Learners often struggle to complete shapes when given a line of symmetry. Using small mirrors could make it easier for learners to understand the reflective component of line symmetry.

Rotational Symmetry

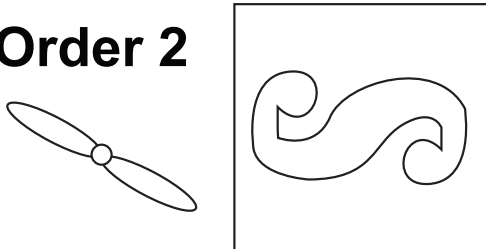
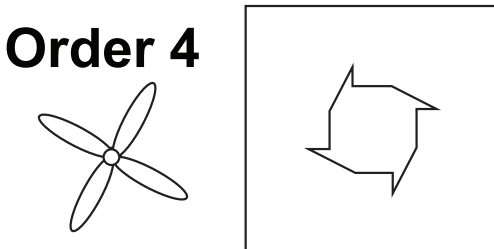
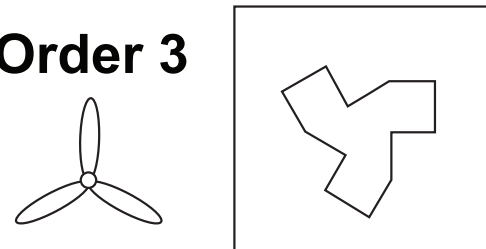
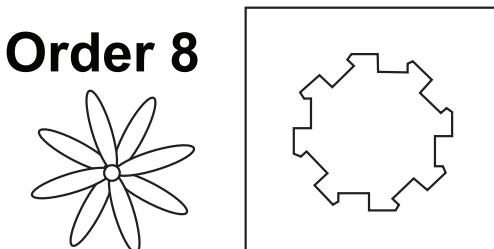
A shape has Rotational Symmetry when it still looks the same after a rotation.

How many times it matches back onto its original shape when we go around is called order. Think of propeller blades as shown below to make it a little easier.

Learners may struggle with the concept of rotational symmetry as it is not always easy for them to visualise the shape being turned and also keep track of how many times it has turned.

Cutting out shapes for the learners to hold and turn themselves would help them to grasp the idea better.

Rotation occurs about the centre of the shape.

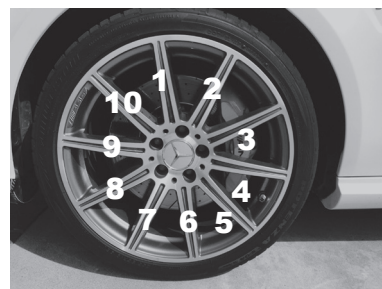
Order	Example shape	Order	Example shape
Order 2		Order 4	
Order 3		Order 8	



The example below is a good real-life contextualization of rotational symmetry.



This wheel has rotational symmetry of order 5



This wheel has rotational symmetry of order 10

TOPIC 6: WHOLE NUMBERS – DIVISION

INTRODUCTION

- This unit runs for 8 hours.
- This unit is part of the content area 'Numbers, Operations and Relationships'. This counts for 50% of the final exam.
- The unit continues the concept of division. Learners have learned strategies suitable for division in previous grades, and will now apply these skills to the division of 4 digit numbers by 3 digit numbers. Learners are introduced to the concept of long division, prime factors and divisibility rules.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Division of at least 3-digit and 2-digit whole numbers • Using multiplication and division as inverse operations 	<ul style="list-style-type: none"> • Division of at least whole 4-digit by 3-digit numbers • Using multiplication and division as inverse operations • Long division • Using a calculator 	<ul style="list-style-type: none"> • Division of at least whole 4-digit by 2-digit numbers in shorter methods. • The FET phase is focused on problem solving in various contexts thus this section draws on the skills taught in all the previous phases and combines with new calculation skills and new sections of mathematics such as Trigonometry that require extensive problem solving within the related contexts. • Shorter division techniques are used later in the senior phase and even extend to algebraic contexts.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Share	To portion or divide something between varying numbers of groups. An example is: Share 277 oranges equally into 18 bags. When this equal sharing is done, we find that there are 15 oranges in each bag and 7 oranges are left over.
Group	A second division concept is grouping, where the number of items in the group is given, and we have to establish how many groups can be formed. The above example would then be phrased as follows: There must be 15 oranges in a bag, and there are 277 oranges. How many bags [groups] can be made up?
Prime Number	A positive integer that has only two factors, 1 and the number itself. This does not include 1, as one has only one factor. 2 is the only even number that is a prime number.
Composite Number	Any number that has more than two factors.
Long Division	Long division is the name that has been given to the standard algorithm for division, where each step in the process is written vertically underneath the previous and where the answer [quotient] appears right at the top of the calculation.
Ratio	The comparison of 2 or more values. Usually written in the format 2:3. A juice concentrate can for example be made in the ratio of 2 parts juice to three parts water. Say for example I use 2 cups of juice concentrate and add three cups of water, then I have five cups of juice. So the concentrate is two-fifths of the juice and three-fifths of the juice is water.
Remainder	What is left over when you try to share a whole out into a particular number of equal parts.
Divisor	The number we are dividing by in a division calculation.
Dividend	The number being divided.
Quotient	The answer to a division calculation.
Inverse	The opposite operation being performed. The inverse of division is multiplication.
Divisible	A number is divisible by another if there is no remainder after division.

SUMMARY OF KEY CONCEPTS



Working with Factors to Understand Division

Learners must recap the basics regarding factors that were taught earlier in Grade 6 and in previous grades.

Writing numbers in factorised form is the basis of division.



EXAMPLE:

What are the factors of 125?

{1, 5, 25 and 125} are all the factors of 125.

In order to get these answers, division had to occur. Factors pair up to be multiplied and equal 125.



For example: $1 \times 125 = 125$ and $5 \times 25 = 125$

It is important in this section to teach learners the divisibility rules. These should have been covered in earlier grades, although it is always beneficial to remind learners of these rules.

Dividing by 2

Any number ending in 0, 2, 4, 6 or 8 (in other words an even number) is divisible by 2.



For example: 24 and 398 are both divisible by 2

Dividing by 3

If the sum of the digits of a number is divisible by 3 then that number is also divisible by 3.



For example:

123 is divisible by 3 because $1 + 2 + 3 = 6$ and 6 is divisible by 3.

42 105 is divisible by 3 because $4 + 2 + 1 + 0 = 7$ and 7 is not divisible by 3.

Dividing by 4

If a number can be divided by 2 twice, then the number is divisible by 4, because $2 \times 2 = 4$.



For example:

168 divided by 2 = 84

84 can be divided by 2 again, therefore 168 is divisible by 4.

Dividing by 5

Any number ending in 0 or 5 will be divisible by 5.



For example:

945 is divisible by 5

1020 is divisible by 5

Dividing by 6

If a number is divisible by 2 AND 3 then it will also be divisible by 6.



For example:

624 is divisible by 2 (it is even) and by 3 ($6 + 2 + 4 = 12$), therefore it will also be divisible by 6.

Dividing by 7 (2 Tests)

The test for divisibility by 7 is complex and learners may spend more effort trying to apply the test than just dividing the number by 7 and in that way testing for divisibility.

Dividing by 8

If a number can be divided by 2 three times, then the number is divisible by 8, because $2 \times 2 \times 2 = 8$.



For example:

168 divided by 2 = 84

84 divided by 2 = 42

42 divided by 2 = 21, therefore 168 is divisible by 8.

Topic 6 Whole Numbers - Division

Dividing by 9

If the sum of the digits of a number is divisible by 9 then that number is also divisible by 9.



For example:

4374 is divisible by 9 because $4 + 3 + 7 + 4 = 18$ and 18 is divisible by 9.

Dividing by 10

If a number ends in zero it will be divisible by 10.



For example:

13 420 is divisible by 10 because it ends in zero.

Prime Numbers in Division

Learners are introduced to prime numbers as factors, and the benefit of breaking a large number into prime factors. Learners must be aware from the onset that 1 is neither a prime nor a composite number and 2 is the only even prime number.

Learners should recognise all prime numbers between 0 and 100. It is also helpful to use the Sieve of Eratosthenes: which is explained just after the 100 chart below

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

This grid is used to determine the prime numbers between 0 and 100 using the following steps:

Step 1:

Cross out all the multiples of 2, but do not cross out 2 itself.

Step 2:

Cross out all the multiples of 3, but do not cross out 3 itself.

Step 3:

Cross out all the multiples of 5, but do not cross out 5 itself.

Step 4:

Cross out all the multiples of 7, but do not cross out 7 itself.

It may be an interesting discussion to have with the learners here as to why the multiples of 4, 6 and 8 were not part of the list of instructions. Remind them of the rules of divisibility - these numbers are all even and all of their multiples would have already been crossed out when the multiples of 2 were crossed out.

The numbers that are left are the prime numbers found between 1 and 100. The ladder method is a good tool to show learners how to write numbers as a product of their prime factors.



EXAMPLE:

Find the prime factors of 256 and 450

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
1	

2	450
3	225
3	75
5	25
5	5
	1

$$256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

and

$$450 = 2 \times 3 \times 3 \times 5 \times 5$$

This skill will help learners practice basic division with smaller numbers, and therefore recap what they have learnt in previous grades on division.

Ratio – Comparing Quantities of the Same Type

The concept of ratio is a further division concept.

Use the phrase 'for every' when introducing ratios to learners.



For example:

In a bunch of flowers there are 10 red flowers and 2 yellow flowers.

This can be written as 10 : 2 (read as 10 to 2)

10 : 2 can be simplified to 5 : 1

This can be described as: For every 5 red flowers there is one yellow flower in the bunch.

In a ratio of 5 : 1 there are 6 items in total. Therefore, the ratio actually means that 5 out of every 6 flowers are red and 1 out of every 6 flowers are yellow.



TEACHING TIP:

It would be a good idea to bring in a bottle of cool drink (it doesn't need to be full) to show learners that what is written on the label is a ratio. This tells us how many parts cool drink to mix with how many parts water. Discuss then how many parts there will be in total in your glass.



**EXAMPLE:**

Susan buys 100 small apples to feed the 2 donkeys on her plot. One donkey is much bigger than the other and she decides to share the apples in a ratio of 7: 3. How many apples does each donkey get?

Total parts is $7 + 3 = 10$

$100 \div 10$ parts = 10

Big donkey gets $7 \times 10 = 70$

Small donkey gets $3 \times 10 = 30$

Therefore, if Susan shares the apples in the ratio of 7 : 3, one donkey will get 70 apples and the other donkey will get 30 apples.

Learners must check that the numbers they get still add up to the total amount that the person is initially sharing.

TOPIC 7: DECIMAL FRACTIONS

INTRODUCTION

- This unit runs for 9 hours.
- This unit is part of the content area 'Numbers, Operations and Relationships'. This counts for 50% of the final exam.
- The unit covers the concept of decimal fractions. Learners are being introduced to decimals for the first time, so this is an important section as foundations are being laid.
- The purpose of this section is to develop the concept of decimal fractions as fractions which have powers of 10 as denominator and where a specific writing convention applies.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 6 GRADE 5		GRADE 7 SENIOR PHASE/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Basic comprehension and understanding of division and multiplication by 10, 100 or 1000. 	<ul style="list-style-type: none"> • Count forwards and backwards in decimal fractions to at least two decimal places • Compare and order decimal fractions to at least two decimal places • Place value of digits to at least two decimal places • Addition and subtraction of decimal fractions with at least two decimal places • Multiply decimal fractions by 10 and 100 • Solve problems in context involving decimal fractions • Recognize equivalence between common fraction and decimal fraction forms of the same number • Recognize equivalence between common fraction, decimal fraction and percentage forms of the same number 	<ul style="list-style-type: none"> • All the skills are revised in the senior phase and applied to algebraic contexts and extended throughout the FET phase.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Decimal, Decimal Fraction and Decimal Number	These are all words that mean the same thing. They refer to fractions that have a power of 10 as the denominator. This means that the denominator is either 10, 100 or 1000. This explanation is at Grade 6 level, but will be made clearer from Grade 7 onwards.
Decimal Places	The position of a digit to the right of a decimal point. Each successive position to the right has a denominator of an increased power of 10.
Notation	A writing system used for recording concepts in mathematics. Symbols are regularly used.
Value	What something is worth.
Terminating Decimals	This is a decimal that ends. It has a denominator that is a power of 10 and a nominator that is a whole number.

SUMMARY OF KEY CONCEPTS

Introducing Decimal Fractions

As far as possible, work with money when introducing decimal fractions. Emphasize that the decimal refers to a whole rand as well as to a fraction of a rand (but a whole number of cents).



For example:

R1.25 is one whole rand and 25 hundredths of a rand which is 25 cents.

R2.06 is two rands and 6 hundredths of a rand which is 6 cents.

Show learners how placing zeros at the end of a decimal fraction does not change the value:



For example: $0,3 = 0,30 = 0,300$ (because $\frac{3}{10} = \frac{30}{100} = \frac{300}{1000}$)

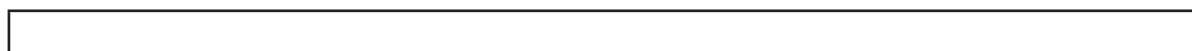
This is useful when needing to compare decimals as well as when adding or subtracting decimals and is dealt with under those sub-topics.

The following method of introducing decimal fractions requires a good understanding of common fractions (in this case particularly those with a denominator of 10).

If you think the learners need to review these skills then that would be the best way to start this section.

Once you feel learners are ready to tackle this new topic, follow these steps:

1. Draw a long rectangle on the board.



2. Split it into ten sections.



3. Ask the learners how we can label each section of the rectangle ($\frac{1}{10}$) then write this into each section

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

4. Remind learners how each of these makes a whole ($\frac{10}{10} = 1$)
5. Colour or shade some of the sections. For the sake of this example, we will shade 4 sections.

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

6. Ask the learners what fraction of the rectangle has been coloured. In this case ($\frac{4}{10}$).
7. Explain that ($\frac{4}{10}$) can be written in another way - 0,4 (no units and four tenths). Show on the rectangle why there are clearly no units (the learners need to remember that we started with one large rectangle that was split up into 10 equal parts).
8. Explain the features of the notation:
- The comma in between the 0 and the 4 is called the DECIMAL COMMA and we use it to separate the units from the tenths. Most countries use a decimal point instead of the comma, but both are correct.
 - We always write in the 0 before the decimal point because it reminds us that the whole number is less than one.
 - We say this number as “zero comma four” or “zero point 4”.

Using the resource at the end of the section you can give each learner a few of these strips and call out how many blocks to shade, then ask questions similar to above. Learners could also work in pairs and ‘test’ each other.

Counting in Decimals

This is an excellent mental maths activity for this particular unit. Learners should be able to count forwards and backwards in various decimal forms.

Start off with easy amounts to count up or down in. 0,1 will be the best place to start.

Ask learners to count from 0,1 and go up in tenths (in other words add on 0,1 each time)



For example:

0; 0,1; 0,2; 0,3; 0,4; 0,5...

Next build up to counting in decimals of 0,2 and 0,3 etc.

Once learners have mastered this, use the same decimals and ask learners to count backwards from a given number.



For example:

Count down in decimals of 0,2 from 4,6 :

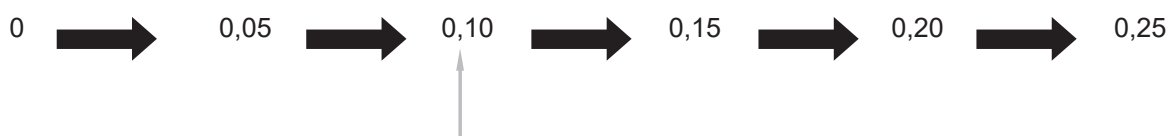
4,6 ; 4,4 ; 4,2 ; 4,0 ; 3,8 ; 3,6...

Now learners may be ready to count in hundredths (in other words decimal fractions with 2 decimal places).



For example:

Counting in decimals of 0,05.



Important teaching note: Remind learners that 0,10 can be written as 0,1 because

$$\frac{10}{100} = \frac{1}{10}$$



Teaching Tip: Learners often have trouble when counting across a number bridge if the decimal they are counting in does not fall on a whole number. This can be overcome if learners work in a table and understand that they should always proceed to the place value left of the digit once they have reached the end of that value. This is the same strategy that learners would have used for whole number counting techniques.

Topic 7 Decimal Fractions

Reading and Writing Decimal Fractions

Learners must be able to correctly read and write decimals. That is, they should know the correct name and placement according to the place value of the digit they are given.



EXAMPLE:

Numbers such as 345,23 and 34,06.

Thousands	Hundreds	Tens	Units	Decimal comma	tenths	hundredths
	3	4	5	.	2	3
		3	4	.	0	6



Teaching Tip: It is a good idea to let learners write decimals with an equivalent number of decimal places using zero to fill missing decimals. The learners must understand the significance of the digits after the comma and the portion they represent. Using value tables, such as the one above will be helpful.



EXAMPLES:

Numbers such as 324,67 (READ AS: Three hundred and twenty-four comma six seven) and 320,07 (READ AS: Three hundred and twenty comma zero seven).



Teaching Tip: Learners often try to read the numbers after the comma as normal two digit numbers, and this must be corrected. Make it a point that learners read numbers after the comma as separate digits.

Rounding off Decimal Fractions

When rounding a decimal, the interest is in the digit to the right of where the rounding requirement is – so if required to round to 2 decimal places, the digit in the 3rd place is of importance.

If that digit is 5 or higher, the digit in the correct position is rounded UP. If that digit is 4 or smaller, the digit remains as it is.



EXAMPLE:

0,547

Rounded to 2 decimal places: 0,55

(7 is in the 3rd (thousandth) position and as it is larger than 5, the 4 in the 2nd (hundredth) position will be rounded up to 5)

Rounded to 1 decimal place: 0,5

(4 is in the 2nd (hundredth) position and as it is less than 5, the 5 in the position of interest will remain the same)

Rounded to a whole number: 1

(5 is in the 1st position so round the digit in the whole number's position '0' up to 1).

Equivalent Forms

Learners must be able to change common fractions into their decimal form and vice versa. It is vital that learners understand the importance of the denominator being 10 or 100 at this stage. Remember that this is a new section, so it is a good idea to relate the section back to the division topic covered previously.



EXAMPLE:

To change $\frac{6}{25}$ into a decimal fraction:

1. Since the fraction being dealt with has a denominator that is a factor of 10, 100 or 1000, equivalent fractions can be used to change it into a fraction with a power of 10 as a denominator which can then be changed into a decimal fraction. $\frac{6}{25} = \frac{24}{100}$
2. Once the denominator is a power of 10, change into a decimal fraction ensuring that the number of decimal places matches the denominator. In other words, if the denominator is 10, there should be ONE decimal place and if the denominator is 100 there should be TWO decimal places etc.
3. In this case $\frac{24}{100} = 0,24$

Converting Decimals into Fractions



Example: Change 0,28 into a common fraction.

1. Read the decimal aloud. In this case, zero comma two eight.
2. Note the number of decimal places. In this case, two.
3. Use the rule equating number of decimal places to a denominator of 10, 100, 1000 etc. In this case, two decimal places means a denominator of 100.
4. Write the decimal as a fraction with the decimal places (28) as the numerator and the power of 10 (100) as the denominator

Solution:

$$0,28 = \frac{28}{100}$$

5. Simplify the fraction if possible by finding common factors to divide into both the numerator and denominator.

Final solution:

$$0,28 = \frac{7}{25}$$



EXAMPLE

0,35

Read as zero comma three, five or “thirty- five hundredths”.

$$0,35 = \frac{35}{100}$$

Simplified this would be: $\frac{(35 \div 5)}{(100 \div 5)} = \frac{7}{20}$

Addition and Subtraction of Decimal Fractions

The simplest way to add or subtract decimals is in columns (vertically). The decimal points or commas must be exactly underneath each other so the place values match. (Tenths underneath tenths, hundredths underneath hundredths, and so on). If need be, zeros can be placed at the end to make the decimal fractions the same 'length'.



EXAMPLES:

i. $5,1 + 1,35$ ii. $4,29 + 5,641$ (carrying over required)

$$\begin{array}{r} 5,10 \\ + 1,35 \\ \hline 6,45 \end{array}$$

$$\begin{array}{r} 4,290 \\ + 5,641 \\ \hline 9,931 \end{array}$$

iii. $5,12 - 2,452$ (borrowing is required)

$$\begin{array}{r} 5,120 \\ - 2,452 \\ \hline 2,668 \end{array}$$



Teaching Tip: Learners often do not place the decimals correctly underneath each other when they need to perform various operations. It helps to get learners to draw columns until they have mastered this technique. Learners must ensure that decimals are the same length when adding them or subtracting them as this will eliminate unnecessary errors.

Multiplication of Decimal Fractions

Multiplying by powers of 10

This is very easy to do - for every 10 being multiplied by, the digits move one place to the left.

Remember to check that the number has become larger, as it should when multiplying.



EXAMPLES:

$$54,21 \times 10 = 542,1$$

$$406,156 \times 100 = 40615,6$$



Teaching Tip: Learners must realise that the comma is a FIXED point, and that only the digits move, and not the comma, when being multiplied by multiples of 10. The digits move to the left of the comma as the number gets bigger. It is also helpful to remind learners that divide and multiply are inverse operations, and that they can check their answers by using the opposite operation to revert back to the question.

Topic 7 Decimal Fractions

Compare and Order Decimal Fractions

Use decimal fraction place value columns to compare decimal fractions.



EXAMPLE:

Compare 0,45 and 0,045.

Units	Decimal comma	tenths	hundredths	Thousandths
0	,	4	5	0
0	,	0	4	5

Looking at the Units the numbers seem similar, but looking at the tenths, hundredths and thousandths reveals that 0,450 is the bigger decimal number.

The numbers must be compared from the left to the right.



Teaching Tip: Decimals can only be compared if they contain the same number of decimal places.

Remind learners that by putting another zero at the end of a decimal fraction does not change its value ($0,4 = 0,40$ because $\frac{4}{10}$ is the same as $\frac{40}{100}$).

'Topping up' a decimal fraction by adding zeros so that the decimals match each other in length will help in all calculations without changing the value of the decimal fraction.

RESOURCES

Introduction to Decimal Fractions

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{10}$
----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------	----------------

TOPIC 8: CAPACITY AND VOLUME

INTRODUCTION

- This unit runs for 5 hours.
- This unit is part of the content area 'Measurement'. This counts for 15% of the final exam.
- The unit covers the two related concepts of volume and capacity.
- The purpose of this section is to develop a clear idea of the two concepts in three-dimensional measurement, which is later used as the context for problem solving.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 5	GRADE 6	GRADE 7 SENIOR PHASE/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
<ul style="list-style-type: none"> • Practical measuring of 3D objects by estimating, measuring, recording, comparing and ordering • Calculations and problem solving involving capacity and volume which can include conversions 	<ul style="list-style-type: none"> • Practical measuring of 3D objects by estimating, measuring, recording, comparing and ordering • Calculations and problem solving involving capacity and volume which can include conversions • Conversions should include fraction and decimal forms 	<ul style="list-style-type: none"> • Use appropriate formulae and conversions between SI units to solve problems and calculate the surface area, volume and capacity of: <ul style="list-style-type: none"> • cubes • rectangular prisms • triangular prisms • cylinders • Investigate how doubling any or all the dimensions of right prisms and cylinders affects their volume • Use and convert between appropriate SI units, including: <ul style="list-style-type: none"> • $\text{mm}^3 \leftrightarrow \text{cm}^3$ • $\text{cm}^3 \leftrightarrow \text{m}^3$ • Use equivalence between units when solving problems: <ul style="list-style-type: none"> • $1 \text{ cm}^3 \leftrightarrow 1 \text{ ml}$ • $1 \text{ m}^3 \leftrightarrow 1 \text{ kl}$

GLOSSARY OF TERMS

Term	Explanation / Diagram
Capacity	This is the amount of liquid a container can hold, on its inside. It is related to volume as the following example illustrates: A cubic container 10 cm long, 10 cm wide and 10 cm high, has a volume of 1 000 cm ³ and a holding capacity of 1000 ml or one litre.
Volume	The amount of space taken up by a 3D object. It is related to capacity, but uses a different set of SI [Standard International] units. 3D objects have 3 dimensions and therefore volume is recorded as cubic units, as in our example above. [Note: The exponent used to indicate square units in two-dimensional shapes, is two; the exponent used to indicate cubic units in three-dimensional objects is three.]
Convert	This is a concept referring to the changes from one unit of measurement to another.
Kilolitre	This is one of the SI units for capacity. 1000l = 1kl
Litre	This is another SI unit for capacity. 1000ml = 1l.
Error of Parallax	This error occurs when learners do not look at liquid inside a jug correctly and thus do not read measurements accurately. The error of parallax is avoided by looking at the level of that which is measured from exactly the same level, not looking up or down, but looking straight to it, for example looking at the top level of milk in a measuring jug holding the jug up to match with one's own eye level.
Measuring Instrument	These instruments include measuring cylinders, jugs and any other calibrated container used to determine the capacity of a liquid.
Calibration	Calibration is a way to ensure that the same amount per unit is used in measuring instruments, for example a litre jug when manufactured in China has to hold exactly the same amount of water as one manufactured in Hong Kong. For that purpose, they need to be calibrated against one standard amount to ensure both correspond with the correct standard. We say it is a calibrated measure.
Gradation lines	Numbered intervals on a measuring instrument.

SUMMARY OF KEY CONCEPTS



Comparing Volume and Capacity

Learners must have a clear understanding of the difference between volume and capacity.

1. Volume is the amount of space taken up by a 3D object, while capacity is the measure of an object's ability to hold a substance (the focus here will be on liquid)
2. Volume is measured in cubic units of length, like cm^3 , while capacity (of liquid) is measured in millilitres, litres etc
3. Volume is calculated by multiplying three dimensions of the object.
At this level, length, breadth and height are the best examples. .



Teaching Tip: Give learners a cup filled with water. Tell them the capacity of the cup is 250 ml, but have them observe that the cup cannot be filled to the brim because it may spill over. This means the volume of the water in the cup will not be 250 ml, and as they drink the water, the volume will decrease. They must notice that although the volume of the water in the cup is changing, the capacity of the cup stayed the same.

Kilolitres, Litres and Millilitres

1. Basic conversions

$$1 \text{ Kl} = 1\,000 \text{ l}$$

$$1 \text{ l} = 1\,000 \text{ ml}$$

$$\frac{1}{2} \text{ l} = 500 \text{ ml}$$

$$\frac{1}{4} \text{ l} = 250 \text{ ml}$$

$$\frac{1}{5} \text{ l} = 200 \text{ ml}$$

$$\frac{1}{10} \text{ l} = 100 \text{ ml}$$

$$1000\text{ml} \div 2 = 500\text{ml}$$

$$1000\text{ml} \div 4 = 250 \text{ ml}$$

$$1000\text{ml} \div 5 = 200 \text{ ml}$$

$$1000\text{ml} \div 10 = 100 \text{ ml}$$

It is important for learners to be familiar with everyday items such as teaspoons, spoons, cups, jugs; and bottles like milk cartons, drinks containers and any other item that has a capacity occupied by a volume of liquid or solid.

(RESOURCE 1 AT THE END OF THIS SECTION)

If possible, gather as many different kinds of bottles and containers as you can and allow the learners to work with these.



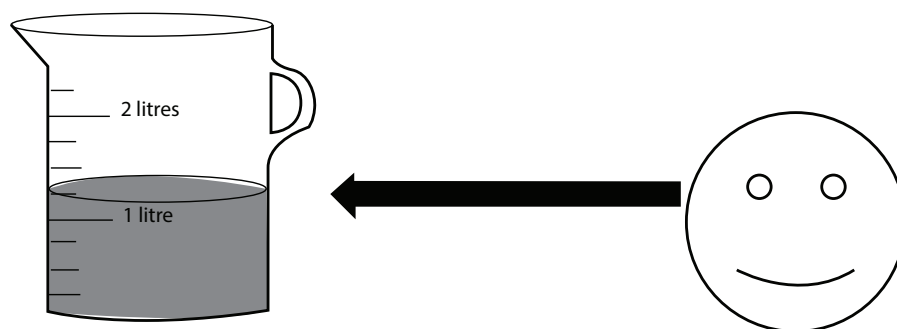
Example:

See how many medicine teaspoons you need to fill a 500ml bottle of cool drink, or how many cups of water to fill a 2 litre drinks container, or even how many tablespoons it would take to put water in a cup.

Learners must use estimation skills and actual measurement skills to determine the relationship between various containers.

Reading Scale Accurately- The Error of Parallax

1. When measuring an amount of liquid in a measuring jug or beaker, take care to read the amount correctly.
2. To do this, the eye must be at the same level as the top of the liquid.
3. Take care to understand the scale on the measuring cylinder. Sometimes not all spaces are marked. For instance, the litre point may be marked and then spaces below this may be marked too.
4. Look at how many marks there are and then divide that number by 1000ml.
5. This will give you the amount measured at each mark.



In the example above there are 4 divisions so: $1000ml \div 4 = 250ml$.

This means that each gradation line represents 250 ml and that the jug contains 1250 ml or 1 litre 250 ml or 1,250 l.



Teaching Tip: Learners often struggle to find the scale of gradation lines. It is good practice to give learners many examples of gradation lines with one or two intervals not filled in. This is an excellent mental maths activity for this section. Encourage learners to use a ruler to help them read the correct gradation interval when they are reading measurements from a picture. This is especially helpful if the gradations are quite close together, or if the value lies between two gradation lines on a beaker or jug.

Rounding Off

Learners must be able to extend their knowledge of rounding throughout different topics, as the rules never change, with learners instead starting to round numbers that are larger. Learners must be able to round to the nearest 5, 10 or 100

Conversions: *ml* and *l* and *Kl*

Use the following to convert:

- $1\ l = 1\ 000\ ml$
- $1\ 000\ ml = 1\ l$
- $1\ 000\ l = 1\ Kl$
- $1\ 000\ 000\ ml = 1\ Kl$

1. Converting litres to millilitres (x 1000):

$$9\ l\ to\ ml$$
$$9\ l = 9\ 000\ ml\ (9 \times 1000ml)$$

2. Converting litres and millilitres to millilitres:

$$2\ l\ 453\ ml$$
$$= 2\ 453\ ml$$

3. Converting fractions of a litre to millilitres:

$$\frac{1}{2}\ l = 500\ ml\ (1\ 000 \div 2 = 500)$$

$$\frac{1}{4}\ l = 250\ ml\ (1\ 000 \div 4 = 250)$$

$$\frac{1}{5}\ l = 200\ ml$$

$$\frac{1}{10}\ l = 100\ ml$$

4. Converting millilitres to litres ($\div 1000$):

$$1\ 250\ ml = 1\ l\ 250\ ml$$

5. Convert to a fraction of a litre:

$$6500ml = 6\frac{1}{2}l$$

6. Convert a fraction of a litre to millilitres:

$$3\frac{1}{2}l = 3500ml$$

7. Using conversions to compare capacities:

Capacities can be given in different forms and learners will need to convert in order to compare and insert an equal sign, less than sign or greater than sign.



For example:

$$6\frac{1}{2}l * 6500ml$$

$$6\frac{1}{2}l = 6500ml$$

$$3\frac{1}{4}l * 3300ml$$

$$3\frac{1}{4}l < 3300ml$$

8. Conversions involving kilolitres:

$$5000l = 5Kl$$

$$7Kl = 7000l$$

When converting between measurements that are two units apart, it is easier to convert to the next measurement before finally converting to the one required.

In other words, when converting from *millilitres* to *kilolitres* rather first change the *millilitres* to *litres* first and then change into *kilolitres*.



For example:

Change 4KL into *millilitres*.

$$4KL = 4000l = 4000000ml$$

Change 6 500 000ml into KL

$$6500000ml = 6500l = 6,5KL$$

Topic 8 Capacity and Volume

9. Using conversions to arrange measurements in ascending or descending order:



For example:

Arrange the following in ascending order:

$1,35l$; $1475ml$; $1\frac{1}{2}l$; $1,450l$

First write all the measurements in the same unit of measurement. In this case, *millilitres* may be the easiest.

$$1,35l = 1,350l = 1\,350ml$$

$$1475ml$$

$$1,5l = 1,500l = 1\,500ml$$

$$1,450l = 1450ml$$

Now rearrange but remember to use the original units given when writing them in either ascending or descending order.

Ascending: $1,35l$; $1,450l$; $1475ml$; $1\frac{1}{2}l$



Teaching Tip: Learners should convert units to the same unit before they can compare or order the measurement. It is useful to show them using real-life examples, like comparing who gets more if one is given 2 *litres* of water versus 10 cups of water, if each cup holds 250 *ml* of water. Learners also get confused as to when they should multiply and when to divide. A good way to overcome this is to remind them that if they are converting from a larger unit to a smaller one, they are making many smaller pieces and should therefore multiply. When learners are converting from a smaller unit to a larger unit, they are combining small units into a larger unit and should therefore divide.

Calculations with Capacity

Convert before calculating, if necessary:

1. $250\text{ ml} + 1\,457\text{ ml} = \underline{\hspace{2cm}}$

$$\begin{array}{r} 250 \\ + 1\,457 \\ \hline 1\,707 \end{array}$$

2. $4\text{ l} + 25\text{ l} = \underline{\hspace{2cm}}$

$$= 29\text{ l}$$

3. $346\text{ ml} + 1,647\text{ l}$

Convert to ml first

$$\begin{array}{r} 346 \\ + 1\,647 \\ \hline 1\,993\text{ ml} \end{array}$$

Note: There is no need to change this back into litres unless specified in the question

4. $\frac{5}{8}\text{ l} + \frac{2}{8}\text{ l}$

$$= \frac{7}{8}\text{ l}$$

5. $48\text{ ml} \times 7 = \underline{\hspace{2cm}}\text{ ml}$

$$\begin{array}{r} 48 \\ \times 7 \\ \hline = 336\text{ ml} \end{array}$$

6. $925\text{ ml} \div 5 = \underline{\hspace{2cm}}\text{ ml}$

$$\begin{array}{r} 185\text{ ml} \\ 5 \overline{)925} \end{array}$$

Topic 8 Capacity and Volume

RESOURCES

RESOURCE 1

Useful capacities used every day

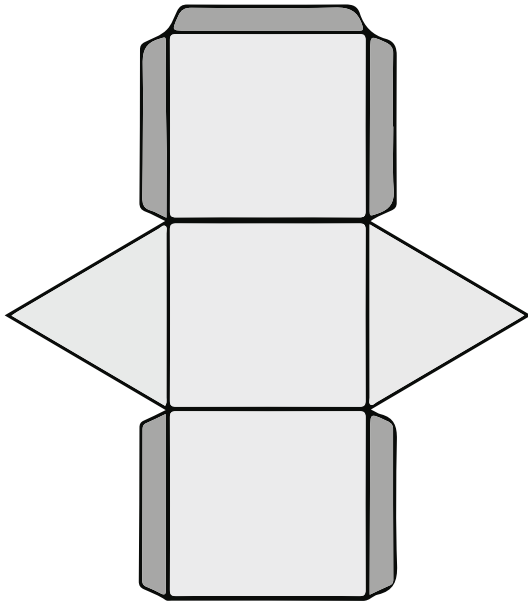
<i>Teaspoon (Tsp.)</i>	<i>5ml</i>
<i>Tablespoon (Tbsp.)</i>	<i>15ml</i>
<i>1 Cup</i>	<i>250ml</i>
$\frac{1}{4}l$	<i>1 Cup</i>
$\frac{1}{2}l$	<i>500ml</i>
$\frac{3}{4}l$	<i>750ml</i>
<i>1l</i>	<i>1000ml</i>

EDUCATOR RESOURCES

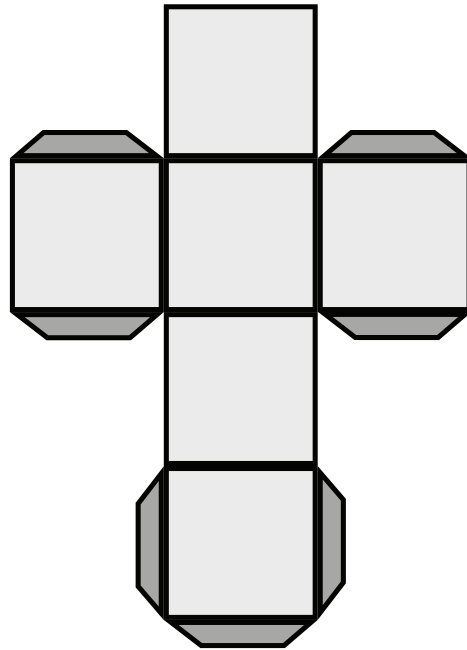


Properties of 3D Objects - nets to assist in constructions

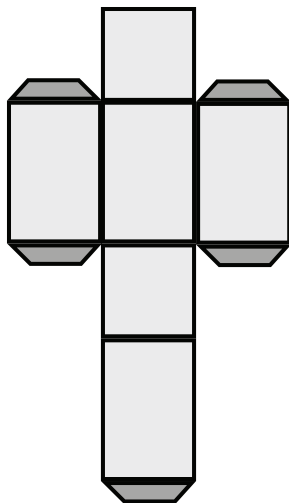
Triangular Prism



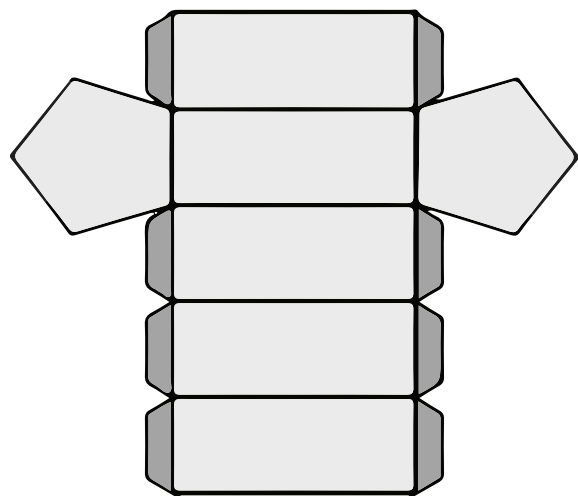
Cube



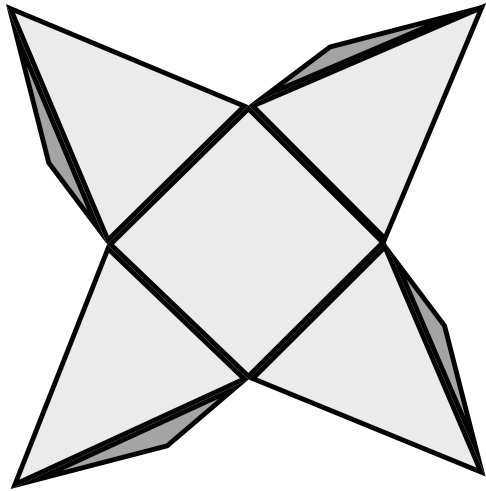
Rectangular Prism



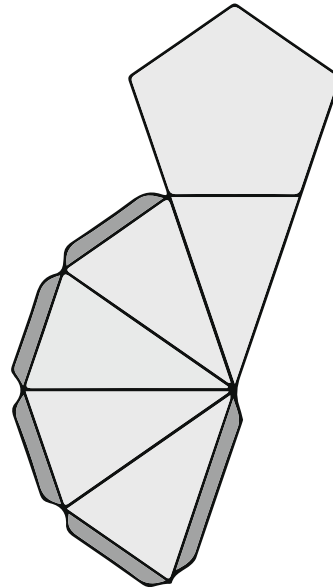
Pentagonal Prism



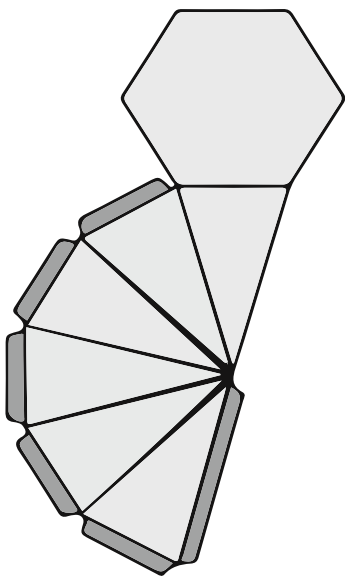
Square Base Pyramid



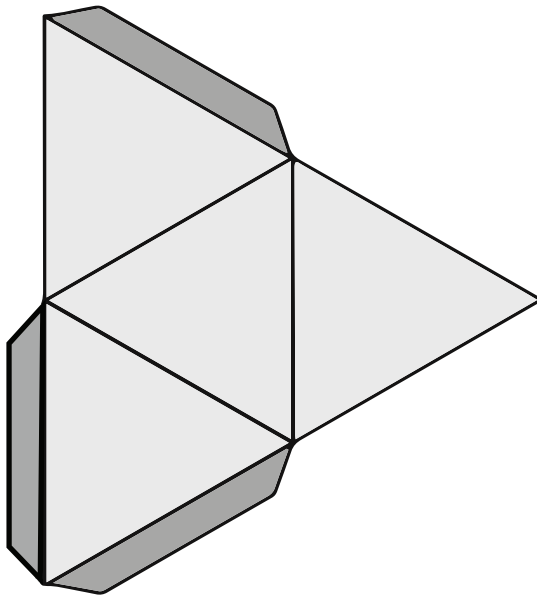
Pentagonal Base Pyramid



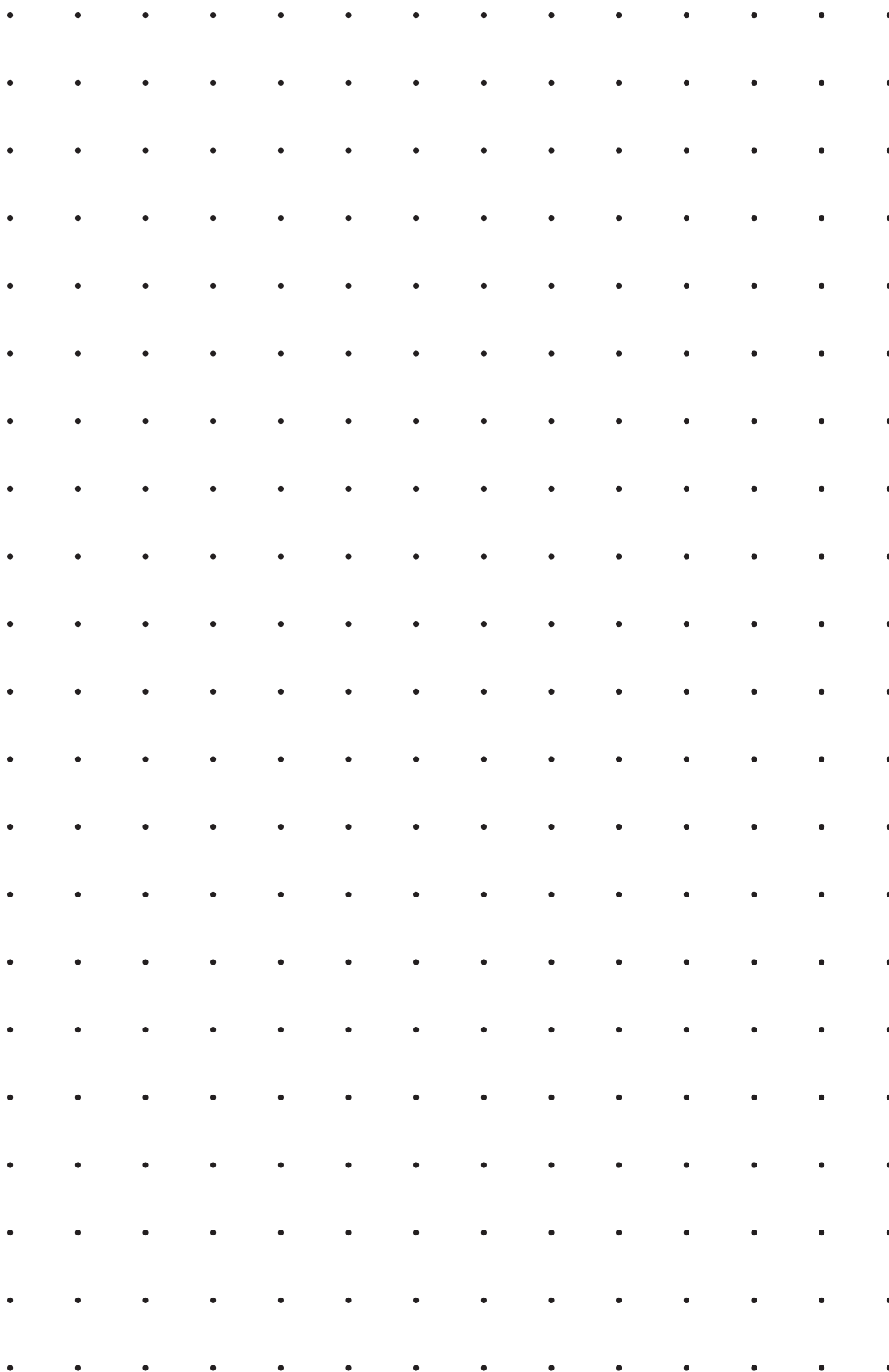
Hexagonal Base Pyramid



Tetrahedron



Dot paper to assist learners in drawing 3D Objects



Resource for Decimal Fractions

Converting decimals to fractions and fractions to decimals.

Show how you converted the following fractions and decimals.

Decimal	Showing your conversion	Fraction
		$\frac{8}{10}$
		$\frac{2}{10}$
0.25		
0.065		
		$\frac{15}{100}$
0.8		