

MATHEMATICS CONTENT BOOKLET: TARGETED SUPPORT



A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers,

This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?

In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education and to help the DBE reach the NDP goals.

The NECT has successfully brought together groups of relevant people so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?

One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers. The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this embedding process.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

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Principles of teaching Mathematics

INTRODUCTION: THREE PRINCIPLES OF TEACHING MATHEMATICS

PRINCIPLE 1: TEACHING MATHEMATICS DEVELOPMENTALLY

What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which determine whether children are ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract, comes from Piaget (1969). We adopted a refined version of that idea though, which works very well for mathematics teaching, namely a "concrete-representational-abstract" classification (Miller & Mercer, 1993).
- It is not possible in all cases or for all topics to follow the "concrete-representational-abstract" sequence exactly, but at the primary level it is possible in many topics and is especially valuable in establishing new concepts.
- This classification gives a tool in the hands of the teacher to develop children's mathematical thinking but also to fall back to a previous phase if it is clear that the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phase.

How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- Where applicable, the initial concrete way of teaching and learning the concept is suggested and educational resources provided at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- In most cases pictures (semi-concrete) and/or diagrams (semi-abstract) are provided, either at the clarification of terminology section, within the topic or lesson plan itself or at the end of the lesson plan or topic as an educational resource.
- In all cases the symbolic (abstract) way of teaching and learning the concept, is provided, since this is, developmentally speaking, where we primarily aim to be when learners master mathematics.

PRINCIPLE 2: TEACHING MATHEMATICS MULTI-MODALLY

What is multi-modal teaching and what are the benefits of such an approach?

- We suggest a rhythm of teaching any mathematical topic by way of "saying it, showing it and symbolising it". This approach can be called multi-modal teaching and links in a significant way to the developmental phases above.
- Multi-modal teaching includes speaking about a matter verbally (auditory mode), showing it in a picture or a diagram (visual mode) and writing it in words or numbers (symbolic mode).
- For multi-modal teaching, the same learning material is presented in various modalities: by an explanation using spoken words (auditory), by showing pictures or diagrams (visual) and by writing words and numbers (symbolic).
- Modal preferences amongst learners vary significantly and learning takes place more successfully when they receive, study and present their learning in the mode of their preference, either auditory, visually or symbolically. Although individual learners prefer one mode above another, the exposure to all three of these modes enhance their learning.

How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the "say it" or auditory mode.
- The pictures and diagrams provide suggestions for the "show it" mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the "symbol it" or symbolic mode of representation.

PRINCIPLE 3: SEQUENTIAL TEACHING

What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths the approach to teaching needs to be systematic. Teaching concepts out of sequence can lead to difficulties in grasping concepts.
- Teaching in a systematic way (according to CAPS) allows learners to progressively build understandings, skills and confidence.
- A learner needs to be confident in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- Ongoing review and reinforcement of previously learned skills and concepts is of utmost importance.
- Giving learners good reasons for why we learn a topic and linking it to previous knowledge can help to remove barriers that stop a child from learning.
- Similarly, making an effort to explain where anything taught may be used in the future is also beneficial to the learning process.

How can this approach be implemented practically?

If there are a few learners in your class who are not grasping a concept, as a teacher, you need to find the time to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again. This could cause difficulties when trying to keep on track and complete the curriculum in the time stipulated. Some topics have a more generous time allocation in order to incorporate investigative work by the learners themselves. Although this is an excellent way to assist learners to form a deeper understanding of a concept, it could also be an opportunity to catch up on any time missed due to remediating and re-teaching of a previous topic. With careful planning, it should be possible to finish the year's work as required.

Another way to try and save some time when preparing for a new topic, is to give out some revision work to learners prior to the start of the topic. They could be required to do this over the course of a week or two leading up to the start of the new topic. For example, in Grade 8, while you are teaching the Theorem of Pythagoras, the learners could be given a homework worksheet on Area and Perimeter at Grade 7 level. This will allow them to revise the skills that are required for the Grade 8 approach to the topic.

What does this look like in the booklet?

At the beginning of each topic, there will be a SEQUENTIAL TEACHING TABLE, that details:

- The knowledge and skills that will be covered in this grade
- The relevant knowledge and skills that were covered in the previous grade or phase (looking back)
- The future knowledge and skills that will be developed in the next grade or phase (looking forward)

THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

Concrete: It is the real thing	E REAL THING	representational: It looks l	Looks Like the real thing	ABSTRACT: IT IS A SYMBOL FOR THE REAL THING	or the real thing
Mathematical topic	Real or physical For example:	Picture	Diagram	Number (with or without unit)	Calculation or operation, general form, rule, formulae
Counting	Physical objects like apples that can be held	$\mathcal{A} \mathcal{A} \mathcal{A}$	000	6 apples	$= 6$ or $^{-1}$
	and moved	D D D	0 0 0		or $\frac{1}{2}$ of $6 = 3$ or $\frac{2}{3}$ of $6 = 4$
Length or distance	The door of the classroom that can be measured			80 cm wide 195 cm hidh	Perimeter: $2L + 2W = 390 + 160$ = 550cm
	physically				Area: $L \times W = 195 \times 80$ $= 15\ 600 cm^2$ $= 1.56m^2$
Capacity	A box with milk that can be poured into glasses	10		1 litre box 250 ml glass	$4 \times 250ml = 1 \ 000ml = 1 \ litre$
					or 1 litre $\div 4 = 0.25$ litre
Patterns	Building blocks			l; 3; 6	n (n+1)
					7
Fraction	Chacalate bar			ں 10 10	$ \begin{cases} 6 &= 1 \\ 12 &= 2 \\ or & \frac{1}{2} & of 12 &= 6 \end{cases} $
Ratio	Hens and chickens		*** * *** *** * ***	4:12	4:12 = 1:3 Of 52 fowls V_4 are hens and 3_4 are chickens. ie 13 hens. 39 chickens
Mass	A block of margarine			500g	500g = 0.5 kg or calculations like 2 ½ blocks = 1.25kg
Teaching progres	sses from concrete -> to	Teaching progresses from concrete -> to -> abstract. In case of problems, we fall back <- to diagrams, pictures, physically	blems, we fall back	 - to diagrams, pictures 	, physically.

MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

Examples	SPEAK IT: to explain the concept	SHOW IT: to embody the idea	SYMBOL IT: to enable mathematising
	 Essential for introducing terminology in 	and retrieving concepts	 Essential to assist <u>mathematical thinking</u>
	context	 Supports understanding through the visual 	about concepts
	 Supports learning through the auditory 	pathway	 Supports the <u>transition</u> from situations to
	pathway	 Important to <u>condense</u> a variety of information 	mathematics
	 Important to link mathematics to everyday realities 	into a single image	 Important to <u>expedite</u> calculation and problem solving
FP: Doubling and	"To double something, means that we make	1. Physical objects:	D
halving	it twice as much or twice as many. If you	Example: Double 5 beads	7 + 7 =
	got R50 for your birthday last year and this	Halve 12 beads	
	year you get double that amount. it means		7 + 🛄 = 14
	this year you got K100. If Mom is doubling the	S.	
	recipe for cupcakes and she used to use 21/2	Example: Double	$\sum + \bigcup = 14$, but
	cups of flour. it means she has to use twice		
	as much this time."		+ = 14
		Halve]
	"To halve something, means that we divide	******	2 times $7 = 14$
	it into two equal parts or share it equally. If I		double of 7 is 14
	have RIG and I use half of it. I use R8 and I		
	am left with R8. If we share the 22 Astro's		14 – 7 =
	in the box equally between the two of us. you	3. Diagrams:	
	get eleven. which is one half and I get eleven.		4 - [] = 7
	which is the other half."		
			14 divided by $2 = 7$
			14 halved is 7

Note how important it is to support the symbolising by saying it out: 1. 3: 6	I: 3; 6; 10 I: 3; 6;10;15	Inspecting the terms of the sequence in relation to their number values: T1: 1 = 1	The value of term 1 is 1 T2: 3 = 1+2	The value of term 2 is the sum of two consecutive numbers starting at 1	T3: 6 = 1+2+3 The value of term 3 is the sum of three consecutive numbers starting at 1	T4: 10 = 1+2+3+4 The value of term 4 is the sum of four consecutive numbers starting at 1	T5: 15 = 1+2+3+4+5 The value of term 5 is the sum of five consecutive numbers starting at 1	T9: 45 = 1+2+3+4+5+6+7+8+9 The value of term 9 is the sum of nine consecutive numbers starting at 1	We can see that the value of term n is the sum of n number of consecutive numbers. starting at 1.
0 (o oo ooo Draw the next term in this pattern.	TI T2 T3 T4 o		Describe this pattern. What is the value of the 9th term of this pattern [T9]? 0		To draw up to the ninth term of this pattern, is a safe but slow way. It is even slower to find out but drawing which term has a value of 120 for	example. One is now almost forced to deal with this problem in a symbolic way.		
"If we see one shape or a group of shapes that is growing or shrinking a number of times, every time in the same way, we can	say it is torming a geometric pattern. It we can find out how the pattern is changing every time, we can say we found the rule	of the sequence of shapes. When we start working with geometric patterns, we can describe the change in normal language.	Later we see that it becomes easier to find the rule if there is a property in the shapes that we can count. so that we can give a	number value to each , or each term of the sequence"	"You will be asked to draw the next term of the pattern, or to say how the eleventh term of the pattern would look, for example. You may also be given a number value and	you may be asked, which term of the pattern has this value?"			
IP: Geometric patterns									

$ \begin{array}{ } 4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a \\ - 2ab + 2a^2b + a^2b \\ \end{array} $	= -3a + 5a - a + 4b + 4b - 2ab - 2ab - a2 + 3a2b + 2a2b + a2b	$= a + 8b - 4ab - a^2 + 6a^2b$					
Although not in a real picture, a mind pic- ture is painted, or a mental image to clarify the principle of classification:	 Basket with green apples (a) Basket with green pears (b) 	 Basket with green apples and green pears (ab) 	 Basket with yellow apples (a²) 	 Basket with yellow apples and green pears (a²b) 	Or in diagrammatic form	a da da	a ²
"We can simplify an algebraic expres- sion by grouping like terms together. We therefore have to know how to	sport fruit in a number of baskets and explain the variables or the unknowns	in terms or muits. Try to visualise the following pictures in your mind:"	-			-	-
SP: Grouping the terms of an algebraic							

TOPIC 1: COMMON FRACTIONS

INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area 'Numbers, Operations and Relationships' an area which is allocated 50% of the total weight shared by the five content areas in Grade 5.
- For Term 3, this unit covers a range of equivalent fractions, simple calculations with fractions and problem solving contexts with fractions up to twelfths.
- The purpose of this unit is to extend the fraction concept to prepare the way for decimal fractions and include percentages as fractions with a denominator of hundred.

GRADE 4	GRADE 5	GRADE 6
INTERMEDIATE PHASE	INTERMEDIATE PHASE	INTERMEDIATE PHASE
 LOOKING BACK Compare. order fractions with different denominators [halves. thirds, quarters, fifths. sixths, sevenths, eighths] Describe and compare common fractions in diagram form Add fractions with the same denominators Recognise and describe that division and fractions are equivalent concepts Solve fraction problems in context including grouping and equal sharing Recognise and describe equivalent fraction forms where denominators are multiples of each other 	 CURRENT Count forwards and backwards in fractions Compare, order common fractions to at least twelfths Add and subtract fractions with same denominators Add fractions which result in whole numbers Recognise, describe and use the equivalence of division and fractions Solve problems with fractions including equal sharing and grouping Recognise and describe equivalent forms of fractions of which the denominator is a multiple of another 	 LOOKING FORWARD Compare, order fractions including hundredths Add and subtract fractions of which one denominator is a multiple of the other Add and subtract mixed numbers Determine fractions of whole numbers Solve problems with fractions including equal sharing and grouping Find percentages of whole numbers Recognise and describe equivalent forms of fractions with denominators [1- or 2 digit] which is a multiple of another Recognise percentage and decimal fraction forms of a common fraction

SEQUENTIAL TEACHING TABLE

GLOSSARY OF TERMS

Term	Explanation / Diagram
Common fraction	a. A fraction is a part or parts of a whole that has been shared equally into a number of pieces.b. A fraction can also be a part of a number of things that have been divided into equal groups.
Denominator	The number which tells us the number of equal parts into which one whole has been divided, or the number of equal smaller groups into which a bigger group has been divided. We write that number under the fraction line, $\frac{2}{5}$ [5 is the denominator].
Numerator	The number that tells us how many parts or groups we are dealing with and which appears above the fraction line, $\frac{2}{5}$ [2 is the numerator].
Fractions and whole	We speak of a fraction when we describe the parts into which something has been divided and where the parts have not yet formed or exceeded a whole. In the way we write a fraction, it means that the numerator [the number on top] is smaller than the denominator [the number in the bottom]. When the numerator and denominator are the same, we have a whole.
Mixed numbers	A fraction of something refers to less than a whole, but where we have more than a whole, the numerator [the number in the top part of the fraction] is bigger than the denominator [the number in the bottom part]. $\frac{17}{5}$ [seventeen fifths] works out to be three wholes and a fraction of two fifths [$3\frac{2}{5}$ a mixed number or mixed fraction]
Simplify fractions	 We can simplify fractions, or write them in their simplest form: a. If the numerator is larger than the denominator, we bring the fraction to a mixed number: 23/6 = 35/6 b. If the numerator and the denominator can both be divided by the same number, we do that: 6/8 = 3/4 because both 6 and 8 can be divided by 2
Equivalent fractions	Equivalent fractions are fractions that are equal in size but have different names. The numerator and the denominator of one equivalent fraction is always a multiple of the numerator and the denominator of the other equivalent fraction. This means that $\frac{6}{24}$ is the same as $\frac{3}{12}$ and it is the same as $\frac{1}{4}$: $\frac{3}{8}$ is the same as $\frac{9}{24}$.

SUMMARY OF KEY CONCEPTS

Introduction

The focus for this term is on the fractions in the range up to twelfths. During this term learners view fractions as division within diagrams and situations. It is most important to understand the idea behind fractions along with writing fractions in symbolic (number) notation.

Common fractions



Example:

a. Two-fifths of the bar is shaded (not all the blocks are shaded, only a part of them).



b. Eight of the twenty learners in the class play soccer (the whole class does not play soccer, only part of the class plays soccer). The part of the class that plays soccer is $\frac{8}{20}$ (eight twentieths) or $\frac{4}{10}$ (four tenths) or $\frac{2}{5}$ (two fifths) of the class.



Fractions and a whole

Example: $\frac{6}{6}$ of the first pizza is the whole pizza, but $\frac{5}{6}$ of the second pizza is not yet a whole pizza (or not any more!)







Example:

The first bag is full and has 50 potatoes in it. A second bag has 40 potatoes in it - it has $\frac{40}{50}$ of the potatoes compared to the first bag. We can also say the second bag has $\frac{4}{5}$ of the whole bag. If the bag with 50 potatoes is considered to be one full bag, then together we can say that we have 1 and we can say we have 1 and $\frac{40}{50}$ or 1 $\frac{4}{5}$ bags of potatoes



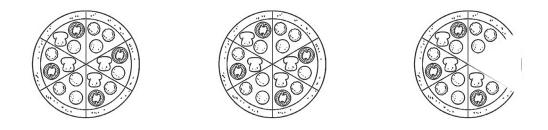


Mixed numbers

Example:

a. $\frac{17}{6}$ of pizza is an improper fraction, which we write in the form of a mixed number, $2\frac{5}{6}$. (Two wholes and five sixths)

Note below that there are 2 full pizzas and $\frac{5}{6}$ of a pizza. This is because $\frac{17}{6}$ is made up of $\frac{6}{6}$ + $\frac{6}{6}$ + $\frac{5}{6}$



Example:

b. There is a full bag and a bag with 40 potatoes, which makes $1\frac{4}{5}$ bags of potatoes.



Counting in fractions

1. Learners can do verbal counting exercises with fractions in mental maths, counting forwards and backwards in fractions, jumping a fraction and counting over whole numbers.



Teaching tip:

A simple exercise like this, helps learners in more than one way. It assists them in:

- understanding the magnitude of fractions when ordering and comparing them;
- realising and expressing equivalence of fractions;
- understanding that all the fractions of a kind add up to a whole;
- understanding when common fractions go over to mixed numbers.



Example:

"Count in ninths, each fraction from one-ninth to one-and-three-ninths"

(one-ninth, two-...)

 $\frac{1}{9}; \frac{2}{9}; \frac{3}{9}; \frac{4}{9}; \frac{5}{9}; \frac{6}{9}; \frac{7}{9}; \frac{8}{9}; \frac{9}{9}, \text{ which is 1; 1 and } \frac{1}{9}; 1 \text{ and } \frac{2}{9}; 1 \text{ and } \frac{3}{9}.$ Example:

"Count from two-ninths to one and seven-ninths by counting every second

ninth." (two-ninths, four-...)

 $\frac{2}{9}; \frac{4}{9}; \frac{6}{9}; \frac{8}{9}; 1 \text{ and } \frac{1}{9}; 1 \text{ and } \frac{3}{9}; 1 \text{ and } \frac{5}{9}; 1 \text{ and } \frac{7}{9}.$

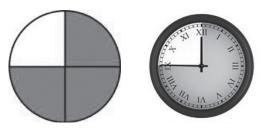
Describing fractions in various ways

 In Grade 5, learners should be making the shift from just recognising fractions or diagrams of fractions which are given, to producing diagrams of fractions and writing them in a symbolic form. This can be done by combining three modalities of their understanding of fractions:



Example:

Show quarters of a whole on a circle; show quarters on a clock; use the clock to tell what three quarters of 12 is; what three quarters of sixty is and so on. Draw a diagram of a circle, of a clock, of the hours and of the minutes which correspond with three quarters. Shade and write $\frac{3}{4}$ of a circle and $\frac{3}{4}$ of a clock with the diagram. Include $\frac{3}{4}$ of 12 hours; $\frac{3}{4}$ of 12 hours is 9 hours; $\frac{3}{4}$ of 60 minutes is 45 minutes.



 $\frac{3}{4}$ of a circle

 $\frac{3}{4}$ of a clock $\frac{3}{4}$ of 12 hours is 9 hours $\frac{3}{4}$ of 60 minutes is 45 minutes

Comparing fractions with a fraction wall

We can compare fractions where one whole is divided into multiples of a specific number. Learners can build their own fraction walls, or they can cut out the fraction walls similar to the one below.



Teaching tip:

Specific fraction walls that are built up showing the image of equivalence together with the numbers, create a better concept of equivalence than a mixed wall.

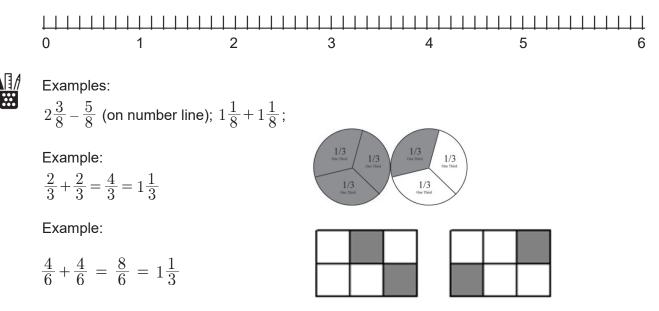
	<u>1</u>	<u>1</u> 3			4 	<u>2</u> 3				<u>}</u> }	
(<u>1</u> 6	<u>-</u> (2	<u>.</u>	<u>3</u> 3	<u>-</u>	<u>1</u> 3	<u>-</u> [5 3	<u>(</u>	<u>)</u> j
$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{10}{12}$	$\frac{\underline{12}}{\underline{12}}$

<u>1</u> 5		<u>2</u> 5		50 31	5	$\frac{4}{5}$	-	ي ب	<u>5</u>
$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$

Adding and subtracting common fractions

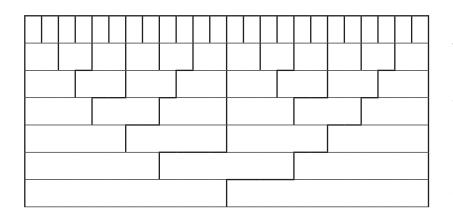
with the same denominator

 Learners add and subtract fractions with the same denominators, add fractions of which the sum is an improper fraction and convert the fraction answer to a mixed number. Diagrams are still used, the name of the fraction must be said and the number symbols must be added. A number line can be used to assist with adding and subtracting fractions.



Equivalent Fractions

1. In Term 1 we aimed for understanding of equivalence before we started writing the symbols in equations. This term, the focus is on diagrams and symbols to confirm the idea of equivalent fractions. We work on fraction equivalents up to twelfths this term.



24 is a multiple of 2, 3, 4, 6, 8 and 12. It is the 12th multiple of 2, the 8th multiple of 3, etc. We can also say 24 can be divided into 2, 3, 4, 6, 8 and 12, because these numbers are all factors of 24.



Teaching tip:

Let learners see that both the numerators and denominators of a fraction that is equivalent to another, are either factors or multiples of the other.

Example:

All fractions with denominators that can be divided by 5, have exact fifths.

Fifths: $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$ and $\frac{4}{5}$

Tenths: $\frac{2}{10} = \frac{1}{5}$, $\frac{4}{10} = \frac{2}{5}$, $\frac{6}{10} = \frac{3}{5}$ and $\frac{8}{10} = \frac{4}{5}$

	I				

Revision of Understanding Fractions as a Part of One Whole

and of a Whole Number

1. This term we revise the idea that a fraction is not only part of a whole, but that there are fractions of whole numbers of items too, where the group is seen as the whole.



Example:

Thirds of 6: This strip is 6 units long. $\frac{1}{3}$ of 6 = 2; $\frac{2}{3}$ of 6 = 4; $\frac{3}{3}$ of 6 = 6

1	2	3	4	5	6

Making a tenths and hundredths strip that is about a metre long

These strips are each about 10 cm long. If you cut ten of them and stick them with the shaded "handle" to the next strip, you have a strip of 100 cm long. This can help to start thinking of fractions of a hundred, or fractions of a metre.

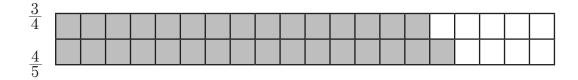


Comparing more fractions

- 1. We can compare different fractions or portions of something to find out which one is the smallest and which one is the biggest.
- Which fraction is the biggest: two and a half or one and a half? $2\frac{1}{2} > 1\frac{1}{2}$ because the wholes are more!
- Which fraction is the biggest: one fifth or two fifths? $\frac{2}{5} > \frac{1}{5}$ because $\frac{2}{5}$ has more pieces!
- Which fraction is the biggest: one sixth or one fifth? $\frac{1}{5} > \frac{1}{6}$ because fifths are bigger than sixths
- Which fraction is the biggest: three eighths or three fifths? $\frac{3}{5} > \frac{3}{8}$ because eighths are smaller than fifths
- Which fraction is the biggest: three quarters or four fifths? $\frac{3}{4}$ or $\frac{4}{5}$?
- 2. It is fairly easy to compare fractions with whole numbers, fractions with the same denominators and fractions with the same numerators. When both the numerator and the denominator of fractions differ, it is not so easy to compare. Use two rows of quad paper.

Teaching tip:

Use the number of blocks that is the product of the two denominators: to compare $\frac{3}{4}$ and $\frac{4}{5}$, use 20 blocks. Find $\frac{3}{4}$ of 20 in the first row and $\frac{4}{5}$ of 20 in the second row.



TOPIC 2: MASS

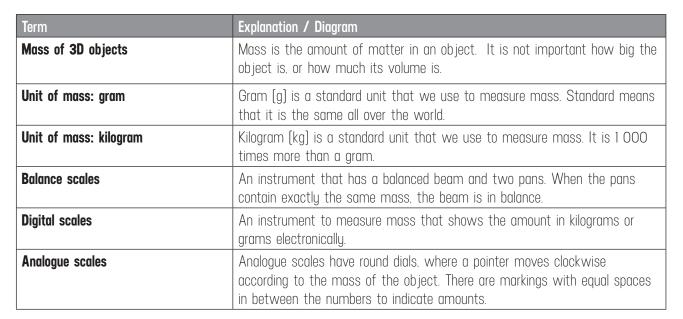
INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area 'Measurement', an area which is allocated 15% of the total weight shared by the five content areas in Grade 5.
- This unit covers practical measuring of 3D objects, the use of measuring instruments in grams and kilograms, calculations and problem solving related to mass.
- The purpose of this unit is to consolidate learners' sense of how much a kilogram and a gram is, to judge the suitability of measuring instruments and to read numbered and unnumbered gradations at various intervals.

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Practical work to estimate, measure, record, compare and order mass of 3D objects	 Practical work to estimate, measure, record, compare and order mass of 3D objects 	 Practical work to estimate. measure, record, compare and order mass of 3D objects
• Measuring instruments include bathroom scales, kitchen scales and balances	 Measuring instruments include bathroom scales, kitchen scales and balances 	• Measuring instruments include bathroom scales, kitchen scales and balances, analogue
 Units used are grams and kilograms 	 Units used are grams and kilograms 	and digitalUnits used are grams and
Calculations involve converting between units, whole units and fractions	 Calculations involve converting between units, whole units and fractions 	 kilograms Calculations involve converting between units, whole units.
Problem solving in mass contexts	 Problem solving in mass contexts 	fractions and decimals to two places
		 Problem solving in mass contexts

SEQUENTIAL TEACHING TABLE

GLOSSARY OF TERMS



SUMMARY OF KEY CONCEPTS

Understanding the Concept of Mass

 Mass is the amount of material, substance, matter or "stuff" in an object. The size or the volume of the object does not determine its mass, but how tight (dense) the substance is.



Example:

The size of 1 kg sugar, 1 kg flour, one litre of water (which has a mass of 1 kg) and a 1 kg block of polystyrene differ a lot.



2. The mass of something that grows bigger, becomes more, because the substance it is made of, becomes more. The mass of the growing pig(let) is shown below:











e. 72kg

f. 96kg

a. 3kg b. 12kg c. 24kg

d. 54kg



Standard Metric Units of Mass

1. The standard unit for measuring mass is the gram. This is used worldwide. Standard means that it is the same all over the world. A small paperclip has a mass of about 1 gram. A 10c coin has a mass of 2 grams; a 50c coin has a mass of 5 grams. A R5 coin has a mass of almost 10 grams.



 Kilogram (kg) is also a standard unit that we use to measure mass of larger items. It is 1 000 times more than a gram. A pile of 1 000 paper clips has a mass of 1 kilogram. One litre of water has a mass of one kilogram (of course without the mass of the container – just the water!).

Estimating Mass in Grams and Kilograms

It is easier to estimate mass if we know the mass of some things in our world in the standard measures that we use. In Grade 5 we only use grams and kilograms. Our markers for estimating mass, are a small paperclip (1g), a 10c coin (2g), a 50c coin (5g), a R5 coin (10g).

Our markers for 250g and 500g are blocks of margarine and for a kilogram it is a litre of water.













500 grams

1 gram

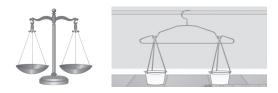
2 grams 5 grams

10 grams 250 grams

1 000 grams (1 kilogram)

Using Mass Measurement Instruments

1. Balancing scales are instruments that have a balanced beam and two pans. When the pans contain exactly the same mass, the two beams are at the same height (exactly level). To find the mass of objects, one can place them in one pan and put standard weights in the other, until the pans are at the same height. You can make a balance with a clothes hanger and margarine tubs on strings. Use coins to find the mass (in grams) of half an apple, a small packet of chips, an eraser, a pencil and a pen. Record your findings in a table.



 Digital scales are instruments that show the amount in kilograms or grams electronically. Bathroom scales usually measure in kilograms and kitchen scales measure in grams. Digital scales have no pointers like on an analogue scale, but show the reading in numbers immediately.



3. Analogue scales are marked in numbers of the measure units of mass, with spaces in between, much like a ruler. Some show kilograms only, some show grams only and some show both kilograms and grams. Most analogue scales have only some numbers marked, with lines and spaces in between for the missing numbers. Analogue scales have round dials, where a pointer moves clockwise according to the mass of the object.







Example:

0 | | | | | | 1 | | | | | 2 | | | | | 3 | | | | | 4 | | | | | 5 kg

The mass is more than ____ kg but less than ____ kg, and closest to ____ kg. It is ____ kg and ___g

Estimation, rounding and calculation

1. At this stage it is necessary to estimate within calculations by rounding up or rounding down, and to do regular operations within a mass context, now using grams and kilograms.

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Teaching tip:

Give learners a single problem like the pancake example.

Example:

At the school sports day, the following recipe is used. It makes 25 pancakes.



NO FLOP PANCAKE RECIPE

375 g self-raising flour
7½ g salt
4 eggs
45 g melted margarine
720 ml water

Use the table below to calculate how much of each of the ingredients will be needed to make 50 pancakes, 100 pancakes, 250 pancakes, and 800 pancakes. Where the mass of the ingredients is more than 1 000g, convert the mass to kilograms and grams.

Ingredients	Recipe for 25 pancakes	Recipe for 100 pancakes	Recipe for 250 pancakes
	Recipe x 1	Recipe x	Recipe x
Flour	375 g		
Salt	$7 \ \frac{1}{2}$ g		
Eggs	4		
Margarine	45 g		
Water	720 ml		

Solution:

Ingredients	Recipe for 25 pancakes	Recipe for 50 pancakes	Recipe for 100 pancakes	Recipe for 250 pancakes	Recipe for 800 pancakes
	Recipe x 1	Recipe x 2	Recipe x 4	Recipe x 10	Recipe x 32
Flour	375 g	750 g	1 500 g = 1 kg 500 g	3 750 g = 3 kg 750 g	12 000 g = 12 kg
Salt	7½ g	15 g	30 g	75 g	240 g
Eggs	4	8	16	40	128
Margarine	45 g	90 g	180 g	450 g	1 440 g = 1 kg 440 g
Water	720 ml	1 440 ml =1 litre 440 ml	2 880 ml =2 litre 880 ml	7 200 ml =7 litre 200 ml	23 040 ml =23 litre 40 ml

TOPIC 3: ADDITION AND SUBTRACTION

INTRODUCTION

- This is a combined unit which runs for 1 + 5, i.e. 6 hours.
- It is part of the Content Area 'Numbers, Operations and Relationships' a topic which is allocated 50% of the total weight shared by the five content areas in Grade 5.
- This unit covers some number concepts and skills up to 5 digit numbers and arithmetic strategies for addition and subtraction.
- The purpose of this unit is to strengthen and expand learners' existing number concepts and to start applying the vertical column method or standard algorithm for addition and subtraction.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE		GRADE 5 INTERMEDIATE PHASE		GRADE 6 INTERMEDIATE PHASE		
LO	oking back	Cl	IRRENT	LO	oking forward	
•	Describe, order, compare whole numbers up to at least 5 digit numbers	•	Describe, order, compare whole numbers up to at least 6 digit numbers	•	Describe, order, compare whole numbers up to at least 9 digit numbers	
•	Round off to 10. 100 and 1 000	•	Round off to 5, 10, 100 and 1 000	•	Round off to 5, 10, 100, 1 000, 100 000, 1 000 000	
•	Represent odd and even numbers to at least 1 000	•	Represent odd and even numbers to at least 10 000	•	Represent prime numbers to at least 100	
•	Add and subtract whole numbers of at least 4 digits	•	Add and subtract whole numbers of at least 5 digits	•	Add and subtract whole numbers of at least 6 digits	
•	For addition and subtraction use strategies of building up and breaking down: number lines: rounding off and compensating: using addition and subtraction as inverse operations	•	For addition and subtraction use strategies of building up and breaking down: number lines: rounding off and compensating: using addition and subtraction as inverse operations: adding and subtracting in columns	•	For addition and subtraction use strategies of building up and breaking down: number lines: rounding off and compensating: using addition and subtraction as inverse operations: adding and subtracting in columns: using a calculator	

GLOSSARY OF TERMS

Term	Explanation/Diagram					
Whole numbers	Whole numbers are numbers we use to count, including zero: 0,1,2,3,4					
Digit	A digit is a symbol that we use to represent a quantity. There are ten digits which we use in different positions to build up numbers. 36 is a two-digit number, of which 3 and 6 are digits. 3 is in a position that makes it worth 30 and 6 is in a position that makes its value six.					
Building up and breaking down	We break down (expand) a number into its terms $(36 = 30 + 6)$. We build up a number by writing the terms as one number $(100 + 50 + 3 = 153)$.					
Rounding up or rounding down	Rounding involves either increasing or decreasing a number by writing it as an approximate closest to a given number like 5 (16 \approx 15; 28 \approx 30).					
Compensating	Compensating is a strategy to add and subtract (mostly used in mental maths) where you change the second number to a "friendly number" like 40 by rounding it up or down and then you adjust the answer.					
Column method to add and subtract	Also called the "standard algorithm" for addition and subtraction. In this method, numbers are written vertically below each other, with all units in one column, all tens in one column and so on. Calculations are done from right to left, starting at the units.					
Inverse operations	Inverse operations are opposite operations that undo each other. Addition and subtraction are inverse operations. Example: Add 89 to 567, the sum is 656 [567 + 89 = 656] Subtract 89 from the sum of 656, the difference is 567, which is the starting number [656 - 89 = 567]					
Estimation	When we do a close guess of the actual answer, we estimate. We do some thinking and some calculation in our heads (through mental maths) but we do not actually calculate the answer. Rounding is a handy way of estimating.					

SUMMARY OF KEY CONCEPTS

Introduction

Concepts important for learners to know in Grade 5:

The zero product property: When we multiply a number by zero, we get zero $(34 \times 0 = 0)$.

Identity property of one for multiplication and division: When we multiply or divide any number by one, the number stays the same (69 x 1 = 69 and 72 \div 1 = 72)

The commutative property of number for addition and multiplication: It does not matter in which order we add or multiply numbers $(15 + 16 = 16 + 15 \text{ and } 12 \times 15 = 15 \times 12).$

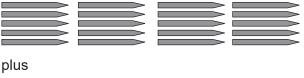
The associative property of number for addition and multiplication: It does not matter how we group numbers to add or multiply them $[(15 + 3) + 5 = 15 + (3 + 5) \text{ and } (12 \times 2) \times 5 = 12 \times (2 \times 5)]$

The distributive property of number:

Two terms which must both be multiplied by the same number, can be written like this: $7 \times (5 + 3)$ or $12 \times (20 - 8)$

Example:

Distribute five red pencils and three yellow pencils to four boys so they all get the same amount of each colour.



4 x 5 red pencils



4 x 3 yellow pencils

In numbers, we write it like this:

 $4 \times (5 + 3) = (4 \times 5) + (4 \times 3) = 20 + 12 = 32$

Rounding up or down to the nearest multiple of 5

1. In Grade 5, learners need to round to the nearest 5. We can round up to the next multiple of 5 or down to the previous multiple of 5. A number line can help us to understand this new idea:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52



Example:

- 3 is closer to 5 than to 0
- 16 is closer to 15 than to 20
- 34 is closer to 35 than to 30

Estimating

When we do a close guess of the actual answer, we estimate. We do some thinking and some calculation in our heads (through mental maths) but we do not actually calculate the answer. Rounding is a handy way of estimating.



Example:

There are three rows of bicycles at school. If you count the first row, it is 8. That is almost 10. There are three rows, so you can roughly estimate there are about 30 bicycles. If you calculate it exactly, there are 8 + 9 + 9 = 26, which is close enough to your estimation of 30.

Compensating

In compensating we change the second number to a "friendly number" like a multiple of 10, eg 28 to 30 (rounding up or down). Then we adjust the answer. It works especially well with 8 or 9.



Example: 456 + 28

Because we know 28 = 30 – 2, we go: 456 + 28 -> 458 + 30 -> 488 – 2 = 48

(We have added two more than we should have, so we subtract those 2 again)

Topic 3 Addition and Subtraction

Addition and subtraction strategies

There are two ways in which we use expanded notation in addition:

- a. both parts are expanded
- b. only the second part is expanded
- 1. Expanded notation: break-down method (both parts expanded)
 - a. Horizontal: Expanded notation: break-down method (both parts expanded)



Example:

52 713 + 28 224

- = 50 000+2 000+700+10+3+20 000+8 000+200+20+4 (expand both numbers)
- $= 50\ 000 + 20\ 000 + 2\ 000 + 8\ 000 + 700 + 200 + 10 + 20 + 3 + 4$ (group)
- = 70 000 + 10 000 + 900 + 30 + 7 (add like terms)

= <u>80 937</u>

b. Vertical: Expanded notation: break-down method (both parts expanded)



Example:					
52 713	+	28 22	24		
3	+	4	=	7	add units horizontally
10	+	20	=	30	add tens horizontally
700	+	200	=	900	
2 000	+	8000	=	10 000	
50 000	+	20 000	=	70 000	
<u>52 713</u>	+	28 224	=	80 937	add totals vertically

2. Expanded notation: break-down method (only one part expanded)



Example: 24 435 + 18 749

24 435 + 10 000-34 435 + 8 000-42 435 + 700-43 135 + 40-43175 + 9-43 184

Note: Learners need to be clear on why equal signs CANNOT be used where the arrows are.

3. Column method (standard algorithm)

In this method, we write numbers vertically below each other, all units in one column, all tens in one column, etc. We calculate from right to left, starting at the units. If the sum of digits in a column has two digits eg 5 + 9 = 14, the second digit is "carried" to the next column to add there.

HT Th H T U ¹⁸ ¹⁸ ¹⁴ ¹³ 8 + <u>6 2 7 9 4</u> <u>1 5 1 2 3 2</u>

SUBTRACTION

There are also two ways in which we use the expanded notation in subtraction:

- a. both parts are broken down or expanded
- b. only the second part (subtrahend) is expanded or broken down

a. Expanded notation (break-down method, both parts expanded)

1. Horizontal: Expanded notation (break-down method, both parts expanded)

Break down both parts, the minuend (the minuend is the first number and the subtrahend is the second number) in brackets (separated by + signs) followed by the subtrahend, (separated by – signs), then group together the Th's, H's, T's U's, each pair bracketed and separated by – signs, but brackets separated by + signs), add the totals.

NB: This method is complicated and has many opportunities for mistakes! Learners should be guided carefully to understand what they are doing.

NB: START SUBTRACTING FROM THE RIGHT, IE THE UNITS

Topic 3 Addition and Subtraction

Example: 45 232 – 18 438 = (40 000 + 5 000 + 200 + 30 + 2) –10 000 – 8 000 – 400 – 30 – 8 (Bracket and add all minuend numbers and subtract each subtrahend number)
= (40 000 − 10 000) + (5 000 − 8 000) + (200 − 400) + (30 − 30) + (2 − 8) (Group for subtraction in each bracket, but add the differences)
= $(40\ 000 - 10\ 000) + (5\ 000 - 8\ 000) + (200 - 400) + (20 - 30) + (12 - 8)$ (Start subtracting from the units: unit minuend borrows from tens minuend)
= (40 000 – 10 000) + (5 000 – 8 000) + (100 – 400) + (120 – 30) + 4 (Tens minuend borrows from hundreds minuend)
= (40 000 – 10 000) + (4 000 – 8 000) + (1 100 – 400) + 90 + 4 (Hundreds minuend borrows from thousands minuend)
= (30 000 – 10 000) + (14 000 – 8 000) + 700 + 90 + 4 (Thousands minuend borrows from ten thousands minuend)
= (30 000 - 10 000) + 6 000 + 700 + 90 + 4
= 20 000 + 6 000 + 700 + 90 + 4
= 26 794

a. Vertical: Expanded notation (break-down method, both parts expanded)

Break down both parts, write one below the other in a column, writing the Th's, H's, T's U's of each part next to each other, subtract to the sides, then add the totals downwards.

Teaching tip:

Leave lines open in between!

NB: START SUBTRACTING FROM THE UNITS

Exampl	e: 84 232 – 6	61 438	3		
	2	_	8	= (cannot)	
	12	-	8	= 4	< Open line, filled in if/when needed
+	20 30	_	30	= (cannot)	
	120	-	30	= 90	< Open line, filled in if/when needed
+ 1	00 200	_	400	= (cannot)	
	1 100	-	400	= 700	< Open line, filled in if/when needed
+ 300	0 4 000	- 1	000	= 2 000	
					< Open line, filled in if/when needed
+	80 000	- 6	000 00	= <u>20 000</u>	
				= <u>22 794</u>	

b. Expanded notation (break-down method, subtrahend expanded)

1. Horizontal

NB: START SUBTRACTING FROM THE LARGEST, IE THE THOUSANDS



Example: 84 232 - 61 438 84 232 - 60 000 → 24 232 - 1 000 → 23 232 - 400 → 22 832 - 30 --> 22 802 - 8 → <u>22 794</u>

OR

84 232 - 60 000 - 1 000 - 400 - 30 - 8 = 24 232 - 1 000 - 400 - 30 - 8 = 23 232 - 400 - 30 - 8 = 22 832 - 30 - 8 = 22 802 - 8 = 22 794

b. Standard algorithm (column method) If the first number in the column is smaller than the second, we "borrow" from the next column to make the calculation possible.



Examp	le:				
	HTh	Th	Н	Т	U
	4 5	3 4	¹¹ 2	12 3	¹ 2

	4 5	3 4	¹¹ 2	12 3	¹ 2
_	1	4	3	8	4
	3	9	8	4	8

TOPIC 4: VIEWING OBJECTS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area 'Space and Shape (Geometry)' an area which is allocated 15% of the total weight shared by the five content areas in Grade 5.
- This unit covers the viewing of side- and top views as well as plan views and maps, a skill which is supposed to be developed in Geography and practised in Mathematics.
- The purpose of this unit is to develop the mental ability to translate and interpret various views and plans of an object or an area. Since the concepts have been learned in Grade 4, the time allocated to this topic may be spent on practising the skills involved in viewing objects.

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
 Recognise and match different views of picture representations of the same everyday object 	 Link the position of the viewer relative to side- and top views of single everyday objects 	 Link the position of the viewer relative to the view of single everyday objects and collections of objects
 Identify everyday objects from different views Interpret top views of scenes 	 Link the position of the viewer relative to the view of groups of everyday objects 	 Link the position of the viewer relative to the view of composite everyday objects
and plans	 Link the position of the viewer relative to the view of everyday scenes 	 Link the position of the viewer relative to the view of everyday scenes
	 Link an object or scene to the plan or map of the object or scene 	 Link an object or scene to the plan or map of the object or scene

SEQUENTIAL TEACHING TABLE

GLOSSARY OF TERMS

Term	Explanation/diagram
View	What a person can see, or take into the eye, from where they are at that moment.
Position	 When we view an object, there are two positions: The place where a person is when looking at the object, is the person's position. The place where the object is at which the person is looking, is the object's position.
Perspective	The way we draw a 3D shape to make it look like a real object.
Scale	Scale is used for example in plans of houses and for maps to make a picture many times smaller than the real object or area.

SUMMARY OF KEY CONCEPTS

Introduction

In Grade 5, the viewer takes various positions to look at the same area or object.

View

• If you are next to something, you look straight to it and you have a side view of it.

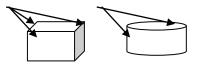


 If you are above something, you look down on it and you have a top view of it.



Perspective

The way we draw a 3D shape to make it look 3D and to give the right impression of its height, width, depth, and the position where it is standing. The way we draw the shape, gives perspective like in the pictures underneath.



Scale

We use scale in plans of houses or for area maps, to draw something in a way that it is a small image of something large.

 In house plans, we take different views of the house and make drawings of those. The pictures must be on scale, for example one metre of the house is one centimetre on the plan. This is a scale of 1:100, because 1 metre is the same as 100 cm.

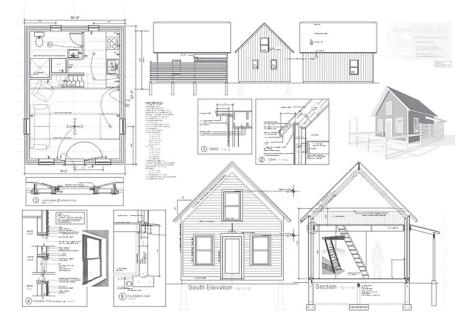
Viewing a house plan from different perspectives



•

Example:

Study this plan of a house and answer the questions following.



- a. Which picture is a front view of the house?
- b. Which pictures are side views of the house?
- c. Which picture is a back view of the house?

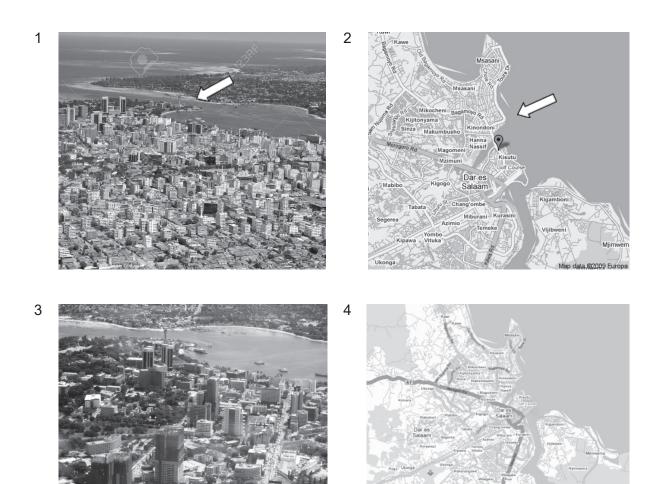
Viewing a city on a map from a top view



Learners should be able to answer questions similar to those in the example below.

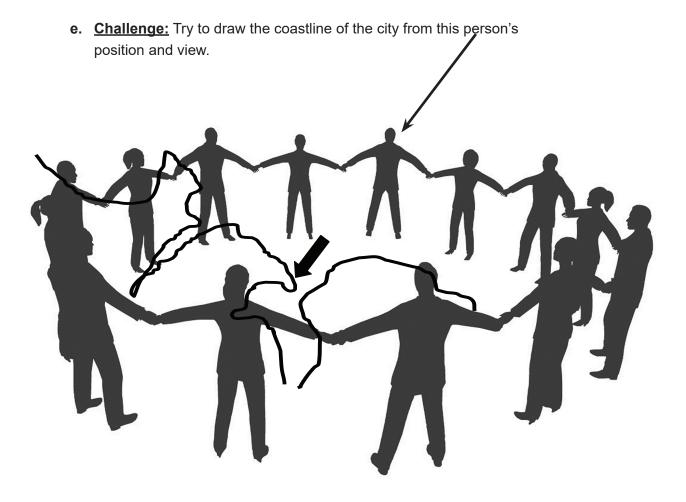
Example:

Below is an aerial photo and a map of a part of Dar Es Salaam city.



- a. Compare the two aerial photos (1 and 3) of Dar Es Salaam in Tanzania. Spot the similarities by taking some specific buildings or other markers that you identify.
- b. Compare the maps of Dar Es Salaam (2 and 4) next to the photos and find the area in the map that corresponds with the area in the photos. Use the arrows to help you.
- c. Which photo is taken from the closest height (distance) above the city?
- d. Choose from the people below who could have taken the three different photos of the city if the city is the oval in the middle of them.

Topic 4 Viewing 3D Objects



TOPIC 5: PROPERTIES OF 2D SHAPES

INTRODUCTION

- This unit runs for 4 hours.
- It is part of the Content Area 'Space and Shape (Geometry)' an area which is allocated 15% of the total weight shared by the five content areas in Grade 5.
- This unit is a continuation of the concepts learned in Term 1, now applied in exercises.
- The purpose of this unit is to consolidate the ability to construct polygons by using straight and curved lines and to judge the size of angles smaller and bigger than right angles.

SEQUENTIAL TEACHING TABLE

GRADE 4 FOUNDATION PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	
LOOKING BACK	CURRENT	LOOKING FORWARD	
Range of 2D shapes includes circles. squares. and rectangles. regular and irregular polygons. triangles. pentagons and hexagons	Range of 2D shapes includes circles, squares, rectangles, regular and irregular polygons, triangles, pentagons, hexagons and heptagons	 Range of 2D shapes expands to include parallelograms and octagons in addition to circles, squares, rectangles, regular and irregular polygons, triangles, pentagons, hexagons and heptagons 	
 Recognise, visualise and name 2D shapes in the environment and in geometric settings 	 Identify and name regular and irregular 2D shapes in the environment and in geometric settings 	 Identify similarities and differences between rectangles and parallelograms 	
• Describe, sort and compare properties of 2D shapes in terms of straight and curved sides and the number of sides	 Describe and create 2D shapes with straight and curved sides. Determine the number and length of sides, angles in shapes, (right sides, angles angles and 	 Range of angles expands to include straight- and reflex angles and a revolution in addition to right angles. acute angles and obtuse angles 	
	angles. acute angles and obtuse angles]	 Draw circles using compasses and create patterns with circles 	

GLOSSARY OF TERMS

Term	Explanation/diagram	
Polygon	A plane figure with at least three straight sides and angles.	
Heptagon	A two-dimensional shape or figure with seven straight sides and seven angles	
Right angle	When a line is placed onto another straight line so that the two angles that is forms, are the same size, the two angles are both right angles. A right angle is also the angle at which something falls to the floor.	
Acute angle	An acute angle is an angle smaller than a right angle. The two rays or arms of the angle that meet at the vertex, form a sharp point.	
Obtuse angle	An obtuse angle is larger than a right angle, but does not yet form a straight line. The two rays of the angle meet at the vertex and form a blunt point.	
Vertex of angle	A vertex (plural: vertices or vertexes) is a point where two or more curves, lines, or edges meet.	
Regular and irregular shapes	Regular shapes with straight sides are shapes of which all sides and angles are the same. All shapes with straight sides that are not the same length, are irregular shapes. A circle is a regular shape with curved sides.	
Curved side	A curve is similar to a line, but it is not straight.	

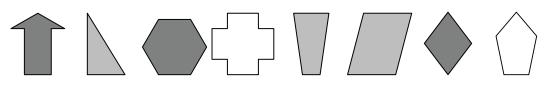
SUMMARY OF KEY CONCEPTS

Introduction

We create right angles, acute angles, obtuse angles and 2D shapes with curved sides.

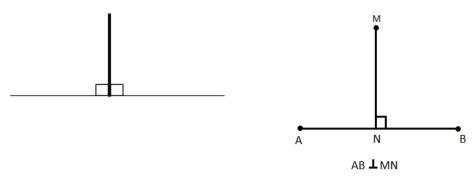
Polygons

Examples of plane figures with at least three straight sides and angles:



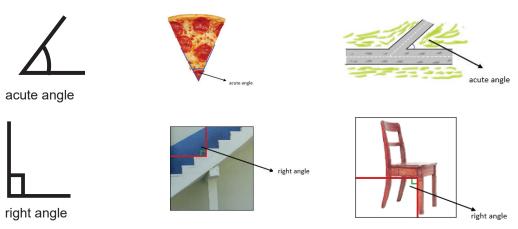
Perpendicular lines and right angles

The vertical line in these pictures form right angles with the horizontal lines.



Acute angles

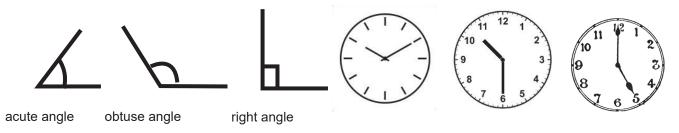
Examples of an acute angle and a right angle with some real life examples:



Encourage learners to look for their own real-life objects that have acute angles or right angles in them. Ask them to find five of each between now and tomorrow when you will discuss them again.

Obtuse angles

Acute-, obtuse- and right angles and the hands of a clock that form obtuse angles:

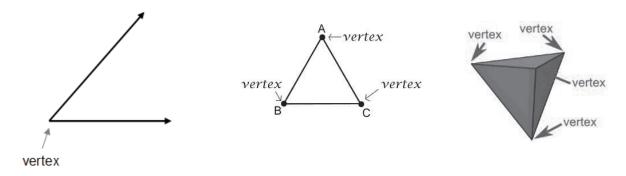


Ask learners if they can tell you some times that would form acute angles or right angles.

Vertex

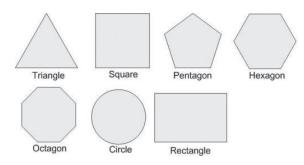
A vertex is the point where two lines meet to form an angle. It is also the point where lines meet in a two-dimensional shape as well as the point where edges meet in a threedimensional shape. More than one vertex are known as vertices.

It may assist learners if the word 'corner' is used when first explaining the concept of a vertex.



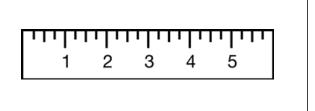
Regular shapes

The 2D shapes with straight sides below are regular shapes, because all their sides and angles are the same. A circle is a regular shape with curved sides (An oval is not a regular shape).

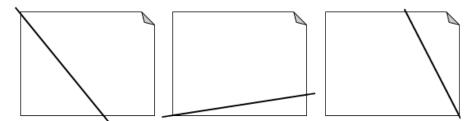


Creating angles

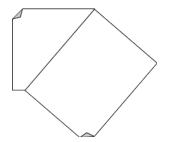
1. We can create a right angle on paper with the corner of a ruler or of a page.



2. We can create acute angles by cutting a page in any way through one of the corners that is a right angle.



3. We can create obtuse angles by adding an acute angle to a right angle.

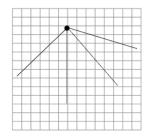


4. We can create any size of angle from a point on quad paper.

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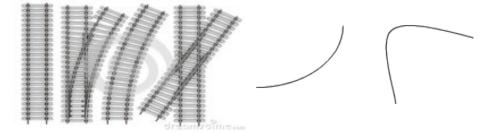
Teaching tip:

Use a ruler to help find a right angle. An angle that disappears underneath the ruler, is acute and an angle that sticks out from underneath the ruler, is obtuse.



Curve

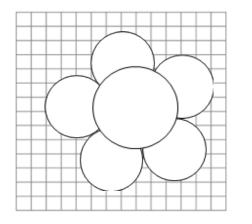
A curve is similar to a line, but it is not straight. We can say it is bent.

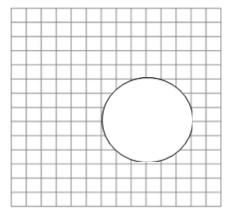


Allow learners to be creative. This topic could also be linked to Art.

Creating shapes with curved sides

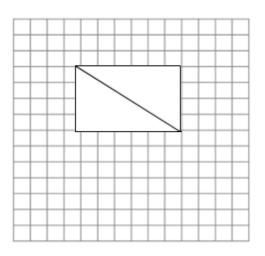
With the lid of your glue stick, create a flower on the quad paper. The first flower in the example has five petals. Draw one with four petals.

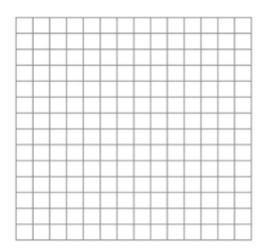




Creating polygons from triangles

We can create polygons by putting triangles together, each time adding one more angle and one more side. Look at the example below where two triangles were connected to form a rectangle. Now create a pentagon by adding a triangle to the rectangle. Create your own polygon by drawing a triangle and adding triangles to it.





TOPIC 6: TRANSFORMATIONS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area 'Space and Shape' an area which is allocated 15% of the total weight shared by the five content areas in Grade 5.
- This unit covers the creation of composite 2D shapes, including shapes with lines of symmetry.
- The purpose of this unit is to introduce the concept of manipulating a 2D shape in ways that do not affect its form, but its position, orientation and direction.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
 Recognise, draw and describe lines of symmetry in 2D shapes 	 Recognise, draw and describe lines of symmetry in 2D shapes 	 Continue the work and concepts learned in Grade 4 and 5
 Create composite 2D shapes by putting together various 2D shapes with line symmetry 	 Use transformations to build composite 2D shapes by tracing and moving 2D shapes 	• Transform 2D shapes through reflection, translation, rotation, enlargement and reduction
 Tessellate patterns with 2D shapes, some with line symmetry 	by rotation, by translation or by reflectionUse transformations to	 Use transformations to describe shapes in the world, in nature and from our
• Describe patterns in terms of the line of symmetry with an informal idea of the transformations, reflection, translation and rotation	 tessellate patterns with 2D shapes Observe and recognise symmetry and transformations in nature and in the 	 cultural heritage Describe transformations in terms of reflection, rotation, translation, enlargement and reduction
• Observe and recognise symmetry and transformations in nature and the environment	 environment Use reflection, rotation and translation Describe patterns 	

Term	Explanation / Diagram	
Symmetry	The quality of having two parts that match each other.	
Tessellation	A pattern made of one or more shapes:the shapes must fit together without any gapsthe shapes should not overlap	
Transformation	A change in a 2D shape, where its direction, position or orientation changes. Different types of transformation are reflection, translation and rotation.	
Reflection	A transformation in which a geometric figure is reflected across a line. creating a mirror image.	
Translation	A type of transformation where the original image is repeated, but has moved its position to the left or the right, and/or up or down.	
Rotation	The original image is turned around about a point, clockwise or anticlockwise.	
Composite Shapes	Shapes that are made up from a number of other shapes.	

SUMMARY OF KEY CONCEPTS

Symmetry

1. Symmetry is a quality of some 2D shapes. This means they will have at least two parts that will be exactly the same size and shape.

The line of symmetry is the line that cuts the shape into these equal parts. A shape could have one line of symmetry, no lines of symmetry (it isn't a symmetrical shape) or more than one line of symmetry.

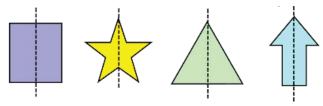


Example – no line of symmetry



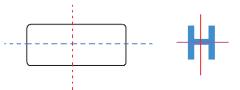


Example – one line of symmetry

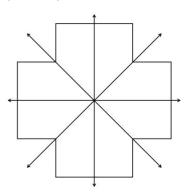




Example – two lines of symmetry



Example - four lines of symmetry



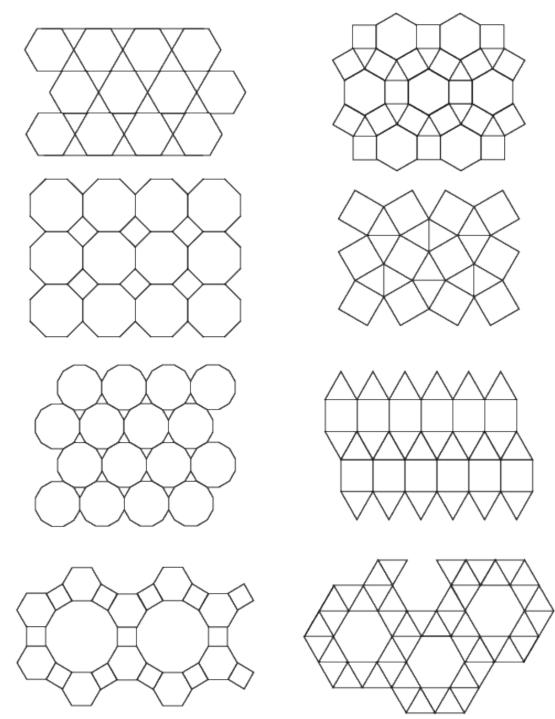
Ask learners to discuss with a partner how many lines of symmetry they think a circle might have.

Tessellation

1. Tessellation is an arrangement of 2D shapes, especially of polygons, closely fitted together in a repeated pattern without gaps or overlapping.



Examples:

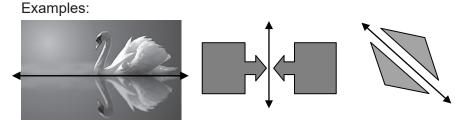


Teaching tip: Tesselations is an excellent choice of topic to link with Art. Learners enjoy creating and then colouring their own tesselations.

Transformations

1. Reflection is a type of transformation where the original image is repeated, as if in a mirror. We can reflect the image along a horizontal line, a vertical line, or a diagonal line.





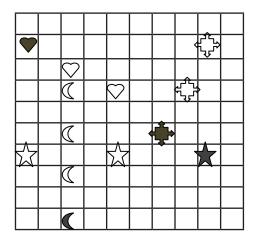
Along a horizontal line

- Along a vertical line Along a diagonal line
- 2. Translation is a type of transformation where the original image is repeated, but has moved its position to the left or the right, and/or up or down.



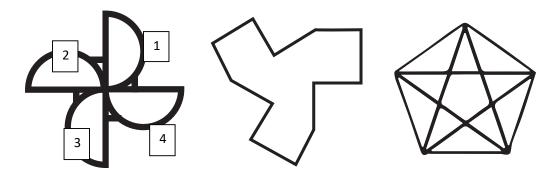
Examples:

- The heart has moved two right and one down each time;
- The shape with arrows has moved two up and one right each time;
- The moon has moved two up each time;
- The star has moved four left each time



3. Rotation is a type of transformation where the original image is turned around a point, clockwise or anticlockwise.

Note: Rotations are usually the most difficult transformation to recognise. Spend some time allowing learners to trace a shape (so that it can fit exactly on top of the original shape) then using a pencil to hold it in the centre before slowly turning it to show that it is being rotated and how it would look in different positions.



TOPIC 7: TEMPERATURE INTRODUCTION

- This unit runs for 2 hours.
- It is part of the Content Area 'Measurement' an area which is allocated 15% of the total weight shared by the five content areas in Grade 5.
- This unit covers the new concept of temperature, including a feel for hot and cold, the measurement, reading, recording and calculating of temperature.
- The purpose of this unit is to develop learners' sense of temperature and to introduce them to the instruments and the Celsius measurement scale used to measure temperature. Temperature presents the natural opportunity to make the transition to negative whole numbers or integers.

GRADE 4 GRADE 5 GRADE 6 INTERMEDIATE PHASE INTERMEDIATE PHASE INTERMEDIATE PHASE LOOKING BACK LOOKING FORWARD CURRENT The topic is not covered Understand the range within • Understand the range within at Grade 4 which temperature occurs in our which temperature occurs in our everyday lives everyday lives Estimate and compare different • Estimate and compare different temperatures temperatures • Measure and record temperature Measure and record temperature • with thermometers with analogue and digital thermometers Read temperature in degrees Celsius • Read temperature in degrees Celsius Solve problems in a temperature • context • Solve problems in a temperature

context

SEQUENTIAL TEACHING TABLE

GLOSSARY OF TERMS

Term	Explanation/diagram
Temperature	Temperature is a real or actual measure of cold or hot where a number value is attached to an object or to the environment being cold or hot.
Body temperature	Body temperature is the degree of heat required by our bodies to be healthy. The normal body temperature of a human being is 37°C.
Freezing point of water	The temperature at which a liquid turns into a solid when it cools down, is called that liquid's freezing point. Water freezes at a temperature of O°C and turns into solid ice.
Boiling point of water	The temperature at which a liquid turns into a gas when it heated up, is called that liquid's boiling point. Water boils at a measured temperature of 100°C and turns into steam.
Thermometer	An instrument for measuring temperature in numbers. It is usually a narrow, sealed glass tube marked with numbers and has a bulb at one end with mercury inside, which moves up along the tube as it expands when it heats up, and moves down as it shrinks when it cools down.
Degrees Celsius (°C)	Degrees Celsius are the units on the temperature measurement scale where a temperature of 0°C represents the melting point of ice and 100°C represents the boiling point of water.
Minimum and maximum temperature	The minimum temperature is the lowest, and the maximum temperature is the highest temperature that the environment reaches in a given period, like a day or a month or a year.

SUMMARY OF KEY CONCEPTS

Temperature

 With our bodies we feel when it is cold or hot or when an object is cold or hot. We differ in the way we feel hot and cold – what one person thinks is a hot bath, may not feel hot for another person. What one person thinks is too cold for a cup of tea, may still be warm enough for someone else.



2. Temperature is a real or actual way to tell how hot or cold it is. We measure it with an instrument which gives a count of exactly how cold or how hot something is.

The Celsius scale

- Degrees Celsius (°C) is the unit on the Celsius measurement scale where the temperature of melting ice is 0°C and 100°C is the temperature of the boiling point of water.
- The Celsius scale also goes below 0°. Just as we count above zero in 1, 2, 3, and so on, we then count below zero in minus one (-1), minus two (-2), minus three (-3) and so on.

Thermometer

- 1. A thermometer is an instrument for measuring temperature. It is a narrow, sealed glass tube marked with numbers and has a bulb at one end with mercury which moves up in the tube as it expands when it heats up, and moves down as it shrinks when it cools down.
- The thermometer tube is marked for its purpose. A thermometer for baking may go up to 400°C, and a thermometer to measure the temperature of the air, may go anything from -20 to 55°C. Below is a thermometer by which we measure body temperature. It goes up to about 43°C.



Body temperature

Body temperature is the degree of heat that a human body maintains or keeps on the inside of the body. In degrees Celsius, the normal body temperature is 37°C. When the body temperature is too high, we say the person has a fever and that happens when the body temperature goes up over 38°C. When the body temperature is too low for the body to work well (hypothermia), the body temperature drops below 36°C.



Fever and hypothermia

Freezing point

The temperature at which a liquid turns into a solid when it cools down, is called that liquid's freezing point. Different liquids freeze at different temperatures. Water freezes at a temperature of 0°C and then turns into solid ice. When the frozen (solid) liquid is heated up again, it melts back to liquid when it reaches the same temperature. Therefore, the freezing point and the melting point of a liquid are the same, and in water it is 0°C.



Freezing point and melting point

Boiling point

The temperature at which a liquid turns into a gas when it is heated up, is called that liquid's boiling point. Different liquids boil at different temperatures. Water boils at a temperature of 100°C and turns into steam. When the steam cools down again, it returns back to liquid or condensates.



Boiling point

Maximum and minimum temperature

The minimum temperature is the lowest temperature, and the maximum temperature is the highest temperature that the environment reaches in a given period, like a day or a month or a year.

The highest temperature ever recorded on earth was 58° C in libya and the lowest ever recorded was -89° C in Antarctica. In South Africa our maximum temperature does not easily go above a high of 45° C and our minimum temperature in certain regions sometimes reaches a low of -15° C.

Deserts normally have a very high maximum temperature during daytime and a very low minimum temperature during the night. It can range from a maximum of 36°C to a minimum of 6°C.

Feeling temperature

There are a few pointers that we can use to start estimating temperature:

- The temperature of an ice cube in our hands is 0°C
- The water from a fridge is about 5°C
- When the temperature is below 15°C, one would possibly wear three layers of clothes
- When the temperature is between 15°C and 25°C, one would possibly wear two layers of clothes
- The temperature of your skin on the outside is about 35°C
- Our hands feel as if they are burning if the temperature of the water is about 50°C
- Food at a temperature of 75°C will not burn your mouth
- Food at a temperature of 85°C will burn your mouth

TOPIC 8: DATA HANDLING INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area 'Data Handling' an area which is allocated 10% of the total weight shared by the five content areas in Grade 5.
- This unit covers the collection and organising of data, different forms of representing data including words, pictograms, bar graphs and pie charts, analysing, interpreting and reporting data and determining the mode of the data set.
- The purpose of this unit is to further develop learners' ability to understand how everyday events and situations are represented in a diagrammatic- or graphical form with which numerical values can be associated and which allows interpretation and prediction.

GRADE 4 INTERMEDIATE PHASE LOOKING BACK	GRADE 5 INTERMEDIATE PHASE CURRENT	GRADE 6 INTERMEDIATE PHASE LOOKING FORWARD	
Collect and record data through tally-marks	Collect and arrange data from smallest to largest	Collect and arrange data from smallest to largest	
 Represent data in words. bar graphs. pictograms (one-to-one correspondence) and pie 	 Represent data in words. pictograms (many-to-one correspondence) and bar graphs 	 Represent data in words, in a pictogram (many-to-one correspondence), single and double bar graph, pie chart 	
 charts Answer questions related to data categories 	 Answer questions related to data categories, sources and contexts 	Answer questions related to the data	
Critically read and interpret	• Read and interpret data	 Set and administer a simple questionnaire to collect data 	
data	• Summarise data verbally and	• Read and interpret data	
 Summarise data verbally and written 	written. draw conclusions and make predictions	 Summarise data, draw conclusions and make predictions 	
	• Determine the mode of the data set	• Determine the mode and median of the data set	

SEQUENTIAL TEACHING TABLE

Term	Explanation/diagram	
Data	Facts about something that happens in this world (usually large numbers of things) and which we can count and give a number value to it.	
Data set	A data set consists of many different pieces of information that are related to one another.	
Statistics	Statistics is created after one has worked through the data and you have put it into tables, charts or graphs.	
Tally marks	Tally marks are a way of counting by only using ones. They are most useful in counting or tallying while things become more. like the score in a match, or the cars passing the school. We group the ones in fives as follows: NN NN II for 12	
Scaled pictograph	A picture symbol for something real, which is the oldest form of statistics (found in Egypt from before 3 000 BCE). One picture can represent one or more objects.	
Bar graph	A bar graph (also known as a bar chart) is a visual tool that uses bars to compare data among categories. A bar graph may run horizontally or vertically. The important thing to know is that the longer the bar, the greater its value. Bar graphs consist of two axes.	
Pie chart	A Pie Chart is a special chart that uses "pie slices" to show relative sizes of data. The circle is divided into sectors, where each sector shows the relative size of each value.	
Mode	The mode is the value that occurs most often in a data set. If no number is repeated, then there is no mode for the set. There can also be more than one mode.	
Data source	A way of finding out what the source of data is, is to ask: "Where does this data come from?" A data source can be many things. It can be a survey, tallies, a business's record of sales, or a dataset.	
Prediction	A forecast or a statement about a future event of which one is not sure yet. When one looks at a data set, one can make some reasonable predictions.	

SUMMARY OF KEY CONCEPTS

Introduction

We shall be working with a data set that gives us the opportunity to practice most of the skills and knowledge needed in Grade 5.

Collecting data

1. Tally is a do-word:

Example: Sir Thabo coached the long distance runners at school from 2010 to 2015. Each year he tallied the number of runners in each grade who ran the big annual race.

- 2. Data collection: This means that Sir Thabo collected the data or the facts about the number of learners in each grade who did the 5 000 m race each year.
- 3. Tally table: The table that Sir Thabo used, looked like this.
 - A tally table has a heading that describes clearly exactly what data has been collected.
 - The table has columns going down and rows going to the sides.
 - We calculate the number of rows by adding one row for the column heading, the number of categories (in this case the four classes), and a row for the column total.
 - A tally table has three columns. The first column would be for the grades, the second column for the tallies and the third column for the number.

Example: The first tally table is completed already. Now complete the other tables in the same way by using the following information:

•	2010:	Gr. 4: 2 ;	Gr. 5: 5 ;	Gr. 6: 8 ;	Gr. 7: 12
•	2011:	Gr. 4: 3 ;	Gr. 5: 7 ;	Gr. 6: 9 ;	Gr. 7: 13
•	2012:	Gr. 4: 4 ;	Gr. 5: 8 ;	Gr. 6: 11 ;	Gr. 7: 16
•	2013:	Gr. 4: 4 ;	Gr. 5: 8 ;	Gr. 6: 12 ;	Gr. 7: 16
•	2014:	Gr. 4: 5 ;	Gr. 5: 10 ;	Gr. 6: 15 ;	Gr. 7: 18
•	2015:	Gr. 4: 6 ;	Gr. 5: 10;	Gr. 6: 17 ;	Gr. 7: 21

Runners of 5 000 m in 2010

Grade	Tally	Number
Grade 4		2
Grade 5	#	5
Grade 6	₩	8
Grade 7	₩ ₩	12
Total		27

Runners of 5 000 m in 2013

Grade	Tally	Total
Grade 4		
Grade 5		
Grade 6		
Grade 7		
Total		

- 4. Data source: In data and statistics, it is always important to ask where data comes from, or what the source of the data is. Some sources of data cannot be trusted as much as other sources. Rate the following sources of data for the data set that we are working on, by saying which data source you trust most and which data source you trust least:
 - a. There was a camera up at the finish of the race for the past three years and all learners who finished, can be seen on the video recordings.
 - b. Sir Thabo did not keep notes, but he was sure that he could remember how many learners ran the race every year. This is how they set up the numbers on the list.
 - c. Each year from 2010 to 2015, Sir Thabo used the same type of table to tally the number of runners for each grade that ran the big annual race.

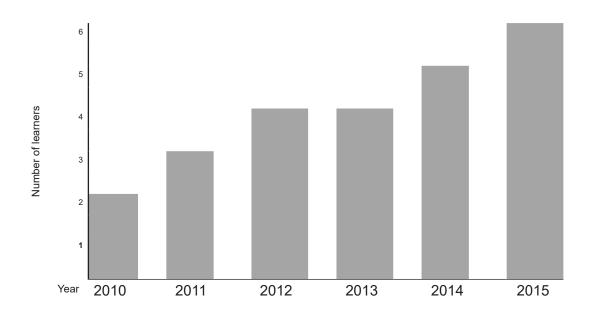
Representing data

- 1. We are now going to look at the data that Sir Thabo has collected and present it in different ways in graphic forms.
- 2. Pictograph or pictogram:
 - A pictogram has a heading in the first row, saying exactly what data is presented.
 - It has two columns with as many rows as the categories of data.
 - The first column has the names of the categories.
 - The second column has the pictures.
 - It has a last row for the key, to explain what the picture means.
 - One can use one picture for each runner, or one picture for more than one runner, as long as you explain that clearly in your key.

- If the number of learners cannot be divided exactly by the number that the picture represents, the picture must be divided to show the exact number. In the example of 27 learners below, each smiley face represents four learners. There are six full faces and one where one $\left(\frac{1}{4}\right)$ part is cut out, meaning that face stands for three learners.
- Now complete rows in the the pictogram for 2011 to 2015 too.

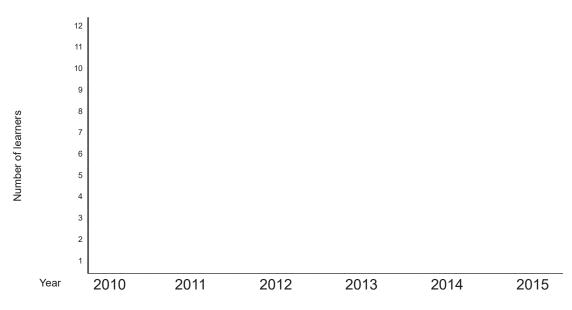
Number of	Number of Learners that Ran 5 000 m from 2010-2015						
2010							
2011							
2012							
2013							
2014							
2015							
Key 😳 :	= 4 learners						

- 3. Bar graph: We are now going to look at all the data together and compare different totals of runners that took part in the 5 000 m race.
 - A bar graph has a heading to describe the data that is presented in the bar.
 - A bar graph has a horizontal line (the x-axis) and a vertical line (the y-axis) that normally meet each other at 0 (zero).
 - The x-axis has the names of the categories of data (like the year or the grade).
 - The y-axis has the number value that goes with the categories.
 - In a bar graph the bars do not touch each other and have a space between bars.



a. Number of Grade 4 learners who ran 5000m per year from 2010 - 2015

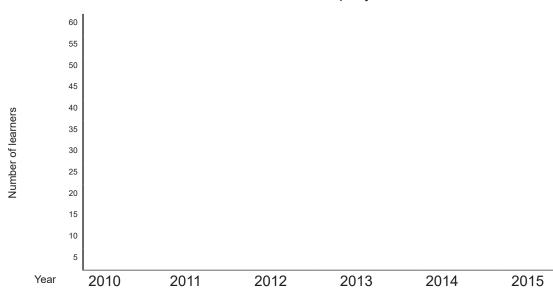
b. Number of Grade 5 learners who ran 5000m per year from 2010 - 2015



c. In the following bar graph we are going to present the total number of learners who ran 5 000 m per year from 2010-2015. The numbers are too big to give a line for each runner, so we give a line for each 5 runners. Firstly, calculate the total for every year.

Total:	Gr. 7: 12	Gr. 6: 8 ;	Gr. 5: 5 ;	Gr. 4: 2 ;	2010:	•
Total:	Gr. 7: 13	Gr. 6: 9 ;	Gr. 5: 7 ;	Gr. 4: 3 ;	2011:	•
Total:	Gr. 7: 16	Gr. 6: 11 ;	Gr. 5: 8 ;	Gr. 4: 4 ;	2012:	•
Total:	Gr. 7: 16	Gr. 6: 12 ;	Gr. 5: 8 ;	Gr. 4: 4 ;	2013:	•
Total:	Gr. 7: 18	Gr. 6: 15 ;	Gr. 5: 10	Gr. 4: 5 ;	2014:	•
Total:	Gr. 7: 21	Gr. 6: 17 ;	Gr. 5: 10 ;	Gr. 4: 6 ;	2015:	•

Now present the total number of learners that ran from 2010-2015 in a bar graph.



Total number of learners who ran 5000m per year from 2010 - 2015

- 4. Pie chart:
 - Data can be represented in the form of a circle that is cut into sectors (slices of the 'pie')
 - There should always be a heading which shows clearly what data is being represented
 - Each sector should be clearly labelled OR a key on the side should show what each sector represents

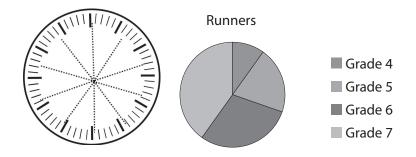
Example: The total number of learners.....

If we divide the circle into 10 portions, we can give the Grade 4s one portion, Grade 5s two portions, Grade 6s three portions and Grade 7s four portions.

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-7

Teaching tip:

An easy way to understand the division of a circle in 10 portions, is by thinking of a clock which is divided into 60 minutes. That means that there are ten times six portions marked on a clock. Every 6 minutes is one tenth of



the circle, like in the picture below.

5. Making predictions: We can see how things went from 2010 to 2015: the number of runners went up each year, from 27, to 32, to 38, to 41, 46, to 56. In future, we may guess that the number in 2016 will be more. That is to make a reasonable statistical prediction. A good guess would be to say that we think the number can be more than 60 in 2016.

Learners need to be able to answer questions from a given pie chart.

For example, from the above chart learners should be able to say which grade had the most or least learners or be able to see that the Grade 4 and Grade 7 learners make up half of the total learners and so on.

Once learners have covered and practiced all the basic skills, they should do a project where they work through the entire data cycle.

They can develop a question, collect data then organise it before representing the data in a suitable format and describing their findings.

There is a good resource for Grades 4 to 7 at this address: http://academic.sun.ac.za/mathed/malati/3primdat.pdf

More work on a data set

1. We are going to work on another data set now to learn new skills and concepts.



Example: The following is a data set of the Grade 5 mathematics marks of last year. There were 39 learners in the class.

68%; 42%; 72%; 36%; 58%; 16%; 92%; 55%; 81%; 78%; 58%; 66%; 36%; 59%; 37%; 53%; 90%; 72%; 82%; 58%; 54%; 33%; 49%; 51%; 29%; 40%; 66%; 45%; 54%; 78%; 74%; 44%; 49%; 58%; 52%; 59%; 63%; 27%; 85%

2. Ordering a data set: Often we need to order a list of numbers to work with it properly. We write the numbers from the lowest to the highest, or in ascending order.



Teaching tip:

Cross out the numbers as you write them down, to make sure you do not write a number twice, or skip a number.

3. Finding the mode: The mode is the number that appears most in the

data set. If there is no number that appears more than once, the data set has no mode. If there is more than one number that appears the same number of times (the most) then the data set has more than one mode. Find the mode of the above data set.

TOPIC 9: NUMERIC PATTERNS INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area 'Patterns, Functions and Algebra' an area which is allocated 10% of the total weight shared by the five content areas in Grade 5.
- This unit covers the relationships between input and output values, involving more complex rules, including patterns with a constant difference and a constant ratio.
- The purpose of this unit is to apply calculation skills, and recognise the effect on numbers as a result of specific operations, within various number patterns.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE		
LOOKING BACK	CURRENT	LOOKING FORWARD		
Investigate and extend numeric patterns according to rules	 Investigate and extend numeric patterns looking for relationships/rules 	 Investigate and extend numeric patterns looking for relationships and rules 		
• Find a constant difference in patterns	• Find constant difference in a numeric pattern	 Find a constant difference in a numeric pattern 		
Find a constant ratio in a numeric pattern	 Find a constant ratio in a numeric pattern 	 Find a constant ratio in a numeric pattern 		
Describe relationships in words and diagrams	 Describe relationships in words, diagrams, tables 	 Describe relationships in words, diagrams and tables 		
• Create and describe own	• Complete flow diagrams with	• Describe rules in general terms		
patterns	two actions or a double rule	• Complete flow diagrams with two		
Complete flow diagrams	• Understand that the input	actions or a double rule		
with two actions or a double rule	value is derived from the inverse operations than those	 Understand that the input value is derived from the inverse operations 		
• Understand the effect of	of the rule	than those of the rule		
inverse operations in input/ output values				

GLOSSARY OF TERMS

Term	Explanation/diagram
Number pattern	A number pattern is a list of numbers that follow a certain rule.
Sequence	A sequence is an ordered list of numbers which form a pattern. We can call a number pattern a sequence.
Term	Term is the position of a number in the sequence, for example, in 2, 5, 8, 11, 14, 17, the number 8 is in the third position from the first number, and it is therefore term 3.
Input value	In a number pattern where the numbers relate to each other, you do something to the number you choose [the input value] to produce the second number [the output value].
Rule	The rule of the pattern is a description of what calculations we do with the input number to find the output number.
Output value	The output value is the answer that we calculate when we apply the rule to the input value.
Flow diagram	A flow diagram is a visual way to write a number pattern, with the input values to the left, the rule in the middle and the output values to the right.
Inverse operations	Inverse operations are opposite operations that undo each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.
Table for a number pattern	A way of representing a number pattern in a table

SUMMARY OF KEY CONCEPTS

Introduction

There are two ways in which we can look at pattern in a sequence:

- We look at how a number in a sequence relates to the next number(s) in the sequence. Example: 2, 5, 8, 11, 14... (each following number is three more than the previous number). Each number in a sequence has its own position. Number 8 in this sequence is in the third position. The term is the position of a number in the sequence, for example, 8 in this sequence is in the third position from the first number, and it is therefore term 3.
- 2. We look at how an input number relates to an output number. We take a number, and we do something to that number to produce the second number. The number we take, (eg 5) is the input value, we operate on the number (eg x 3 - 2), and we find the output number (13).

Number patterns

A number pattern is a list of numbers or a sequence of numbers that follow a certain pattern, for every following number. We can call a number pattern a sequence.



Example: 2, 5, 8, 11, 14, 17, ... is a sequence. It starts at 2 and increases by 3 every time.

Input value

The input number can be any number: the counting numbers, any whole number, a common fraction, a decimal fraction or zero, like $\frac{1}{2}$; 5; 0; 0,3; ... or any of millions of options.

The rule of the pattern

What we do to our input number (adding, subtracting, multiplying or dividing) each time, we call the rule of the number pattern.

Output value

For all the numbers that we choose, we do the same and we get the output value which is the answer that we calculate when we apply the rule to the input value.

Example:

Input value	Rule	Output value
2	<u>x 3 – 1</u>	5
1/3	<u>x 3 – 1</u>	0
8	<u>x 3 – 1</u>	23

Tabling a number pattern

A way of representing a number pattern is in a table, as follows:



Example:

2; 5; 8; 11... (Rule: Starting at two, three is added to each following number)

Term]	2	3	4	5	
Number in the number pattern	2	5	8]]		

Different types of number patterns

Teaching tip:

The first four questions to ask in understanding a number pattern, are:

- "Do I add the same number each time?" (Ascending: constant difference)
- "Do I subtract the same number each time?" (Descending: constant difference)
- "Do I multiply by the same whole number each time?" (Ascending: constant ratio)
- "Do I divide by the same whole number each time?" (Descending: constant ratio)

(however, it is important to note that when dividing by a number it is the same as multiplying by its inverse. For example, divide 2 is the same as multiplying by $\frac{1}{2}$)

- 1. Number patterns with a constant difference
- a. Ascending patterns formed through addition:



For example: 4; 7; 10;... "The pattern starts at 4 and is increased by 3 (+3) each time"

- b. Descending patterns formed through subtraction:
- For example: 84; 81; 78;... "The pattern starts at 84 and is decreased by 3 (- 3) each time."
 - 2. Number patterns with a constant ratio
 - a. Ascending patterns formed through multiplication:



For example: 3; 6; 9;... "The pattern starts at 3 and becomes 3 more each time" The rule of the pattern is: Each term is multiplied by three (x 3)

b. Descending patterns formed through division:



For example: 144; 72; 36;... "The pattern starts at 144 and becomes 2 times less each time" (divide 2)

3. Number patterns with neither a constant difference, nor a constant ratio



Example 1:

4; 5; 8; 13; 20; ... "The pattern starts at 4 and becomes 1 more the first time, 3 more the second time, 5 more the third time. Each time the next odd number is added."



Example 2:

2; 3; 5; 7; 11; 13; 17;...

"All the numbers in the pattern are prime numbers."



Example 3:

1; 1; 2; 3; 5; 8; 13;... "The first and second number are added to form the third number; the second and third number are added to form the fourth number; the third and fourth number are added to form the fifth number... and so on."

Number Patterns and Functions

1. A function does not have term numbers, but input numbers, which can be all numbers we know, even fractions and zero. There is a rule for functions too, like this:

Input number	0	1/4	7	10
	↓	\downarrow	\downarrow	¥
Output number	3	31⁄2	17	23

The rule is: multiply by 2 and add three (x 2 + 3). We can apply the rule to any input number.

Input value		Rule	Output value	
•	8	x 2 + 3	= 19	
•	1/2	x 2 + 3	= 4	
•	0	x 2 + 3	= 3	

2. In a flow diagram it would look like this:

Input value	Rule	Output value		
8			19	
1/2	x2 –	+3	4	
0			3	

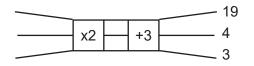
3. We can also represent the function in a table:

Rule: Output value= input value x 2 + 3

Input value	8	1/2	0		
Output value	19	4	3		

Inverse operations

If we have the output value and the rule of a number pattern, we can get the input number by doing the inverse operations of the rule (from right to left in the flow diagram).



 $19 - 3 \rightarrow 16 \div 2 \rightarrow 8$ $4 - 3 \rightarrow 1 \div 2 \rightarrow \frac{1}{2}$ $3 - 3 \rightarrow 0 \div 2 \rightarrow 0$

TOPIC 10: MULTIPLICATION

INTRODUCTION

- This unit runs for 7 hours.
- It is part of the Content Area 'Numbers, Operations and Relationships' an area which is allocated 50% of the total weight shared by the five content areas in Grade 5.
- This unit covers various strategies to multiply at least three digit numbers by two digit numbers.
- The purpose of this unit is revision of the work done in Term 2 and to recognise the commutative, associative and distributive properties of number.

SEQUENTIAL TEACHING TABLE

GRADE 4 INTERMEDIATE PHASE	GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Multiply at least 2 digit- by two digit numbers	 Multiply at least 3 digit- numbers by 2 digit numbers 	 Multiply at least 4 digit by three digit numbers
 Multiply using estimation, doubling and halving, 	 Multiply using estimation, doubling and halving, building up and breaking down, rounding off and compensating Round off and estimate up to at least 99 999 	• Multiply whole numbers with or without brackets
building up and breaking down, rounding off and compensating		 Multiply using estimation, doubling and halving, building up
		and breaking down, rounding off
• Round off and estimate up to at least 9 999		and compensating and using the standard vertical column algorithm
		• Round off and estimate up to at least 999 999

Term	Explanation / Diagram		
Multiplication	A short way of adding more than one of the same number together. Example: 4 + 4 + 4 + 4 + 4 + 4 + 4 = 28 or 7 x 4 = 28		
Multiples	A number made up by multiplying two other numbers.		
Factors	A factor is a whole number that will divide exactly into another number without a remainder. The factors of a number are those numbers that were multiplied to make that number.		
Distributive Property of Multiplication over Addition	If we multiply a number by numbers that are added together, it is the same as multiplying the number by each of the other numbers. Example: $5[3 + 2] = 15 + 10 = 25$		
Terminology (words) Used in Multiplication Equations or Calculation	9 $x4 = 36$ $\downarrow \qquad \downarrow \qquad \downarrow$ Multiplicand x multiplier = Product The product is the answer to a multiplication sum.		
Halving	To divide a number into two equal parts, which is the same as dividing the number by two.		
Doubling	To multiply a number by two, or to add the same number to it, so that the answer is twice as many as the number.		
Rounding off Symbol	When one number is not exactly equal to, or the same as another number, we use the symbol \approx to indicate that it is approximately, or almost the same as the other when we round off or estimate.		

SUMMARY OF KEY CONCEPTS

Introduction



We can make up a multiple by multiplying two other numbers. Example: 28 is the seventh multiple of 4, since 7 x 4 = 28. 28 is also the fourth multiple of 7, since 4 x 7 = 28.



Note: The number itself is its own first multiple: 7 is the first multiple of 7, since 7 x 1 = 7

Multiplication of at least a 3 digit number by a 2 digit number.

1. For the 'break-down' strategies, there are three options: we can see the multiplier as the sum, the difference or the product of other numbers.



Example: We want to multiply by 28:

- 28 is the sum of two numbers:
- 28 = 20 + 8 (20 and 8 are terms of 28)
- 28 is the product of two numbers: $28 = 7 \times 4$ (7 and 4 are factors of 28)

28 is the difference between two numbers: 28 = 30 - 2
 (28 is one of the terms of 30)

Use terms of the multiplier to multiply
a. Sum of the termsUse factors of the multiplier to multiply
b. Product of factors 468×28 468×28 $= 468 \times [20 + 8]$ $= 468 \times 7 \times 4$ $= [468 \times 20] + [468 \times 8]$ Distributive property $= 468 \times 7 \times 4$ = 9 360 + 3 744 $= 3276 \times 4$ = 13 104= 13 104

2. In the third strategy, we see the multiplier as the difference between two numbers. We are still working with terms, but in a way where a number is the result of subtraction. This method is called rounding up and compensating.

Using terms of the multiplier to multiply: c. Difference of a number and a term	Teaching tip: Be careful that learners understand completely what they are doing:	
468 x 28	28 is closer to 30, so we think of 28 as $30 - 2$.	
= 468 x [30 - 2] = [468 x 30] - [468 x 2]	We put it in brackets to see this is 28. We multiply 468 by 30 but we know it is too much. We have to	
= 14 040 - 936	subtract two times 468 from that to make sure we	
= 13 104	actually multiplied by 28.	

3. We can double and halve in some cases to multiply, but when we multiply larger numbers, it works well only in cases where one of the numbers is multiple of at least 8 or 16.



Example: 288 x 35

Halving	Doubling			
288	35			
144	70			
72	140			
36	280			
18	560			
9	1120			
9 x 1120 = 10 080				

Rounding Off Symbol

When we round off or estimate, one number is not exactly equal to, or the same as another number so we cannot use the = symbol. Then we use the symbol \approx which looks a little like, but is not the same. This indicates that it is approximately, or almost the same as the other number.



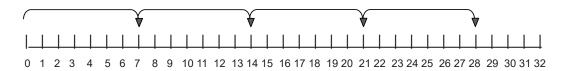
For example:

1234 rounded to the nearest 100 1234 \approx 1230

Multiples and multiplication by 0

Teaching tip:

Explain this multiplication fact with a number line:



7 is the first multiple of 7; 14 is the second multiple of 7; 21 is the third multiple of 7; 28 is the fourth multiple of 7

Multiplication by 1

One multiplied by, or divided into a number does not change that number: one is the identity element for multiplication and division. Investigate the product of odd and even numbers

- 1. Even x even = _____ (example: 4 x 6 = 24)
- 2. Odd x even = ____ (example: 5 x 6 = 30)
- 3. Odd x odd = _____ (example: 5 x 7 = 35)
- 4. Even x odd = _____ (example: 4 x 7 = 28)

Word problems using multiplication

When we have a word problem, it is a good strategy to set up a number sentence and to decide on the strategy that we are going to use to do the calculation.



Example:

The spectators' seats at the school soccer field can take 872 people. Each Saturday there was a match, the seats were at full capacity. If there were 29 matches, how many people in total were at the matches?

Number sentence: $872 \times 29 = \square$

Strategy: Rounding up and compensating, because 29 is almost 30

Factors of numbers

We can find the factors of numbers by dividing the number until it can divide no more.



Example:

840						
2	420					
2	2	210				
2	2	2	105			
2	2	2	3	35		
2	2	2	3	5	7	
840 = 2 x 2 x 2 x 3 x 5 x 7						

