Grade 6

MATHEMATICS CONTENT BOOKLET: TARGETED SUPPORT

Term 3

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers,
This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?
In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education and to help the DBE reach the NDP goals.

The NECT has successfully brought together groups of relevant people so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?
One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers. The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this embedding process.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

Contents

INTRODUCTION: THREE PRINCIPLES OF TEACHING MATHEMATICS

PRINCIPLE 1: TEACHING MATHEMATICS DEVELOPMENTALLY

What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which determine whether children are ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract, comes from Piaget (1969). We adopted a refined version of that idea though, which works very well for mathematics teaching, namely a "concrete-representational-abstract" classification (Miller & Mercer, 1993).
- It is not possible in all cases or for all topics to follow the "concrete-representational-abstract" sequence exactly, but at the primary level it is possible in many topics and is especially valuable in establishing new concepts.
- This classification gives a tool in the hands of the teacher to develop children's mathematical thinking but also to fall back to a previous phase if it is clear that the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phase.

How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- Where applicable, the initial concrete way of teaching and learning the concept is suggested and educational resources provided at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- In most cases pictures (semi-concrete) and/or diagrams (semi-abstract) are provided, either at the clarification of terminology section, within the topic or lesson plan itself or at the end of the lesson plan or topic as an educational resource.
- In all cases the symbolic (abstract) way of teaching and learning the concept, is provided, since this is, developmentally speaking, where we primarily aim to be when learners master mathematics.

PRINCIPLE 2: TEACHING MATHEMATICS MULTI-MODALLY

What is multi-modal teaching and what are the benefits of such an approach?

- We suggest a rhythm of teaching any mathematical topic by way of "saying it, showing it and symbolising it". This approach can be called multi-modal teaching and links in a significant way to the developmental phases above.
- Multi-modal teaching includes speaking about a matter verbally (auditory mode), showing it in a picture or a diagram (visual mode) and writing it in words or numbers (symbolic mode).
- For multi-modal teaching, the same learning material is presented in various modalities: by an explanation using spoken words (auditory), by showing pictures or diagrams (visual) and by writing words and numbers (symbolic).
- Modal preferences amongst learners vary significantly and learning takes place more successfully when they receive, study and present their learning in the mode of their preference, either auditory, visually or symbolically. Although individual learners prefer one mode above another, the exposure to all three of these modes enhance their learning.

How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the "say it" or auditory mode.
- The pictures and diagrams provide suggestions for the "show it" mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the "symbol it" or symbolic mode of representation.

PRINCIPLE 3: SEQUENTIAL TEACHING

What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths the approach to teaching needs to be systematic. Teaching concepts out of sequence can lead to difficulties in grasping concepts.
- Teaching in a systematic way (according to CAPS) allows learners to progressively build understandings, skills and confidence.
- A learner needs to be confident in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- Ongoing review and reinforcement of previously learned skills and concepts is of utmost importance.
- Giving learners good reasons for why we learn a topic and linking it to previous knowledge can help to remove barriers that stop a child from learning.
- Similarly, making an effort to explain where anything taught may be used in the future is also beneficial to the learning process.

How can this approach be implemented practically?

If there are a few learners in your class who are not grasping a concept, as a teacher, you need to find the time to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again. This could cause difficulties when trying to keep on track and complete the curriculum in the time stipulated.

Some topics have a more generous time allocation in order to incorporate investigative work by the learners themselves. Although this is an excellent way to assist learners to form a deeper understanding of a concept, it could also be an opportunity to catch up on any time missed due to remediating and re-teaching of a previous topic. With careful planning, it should be possible to finish the year's work as required.

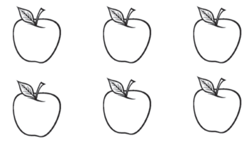
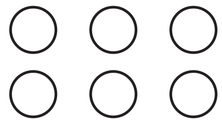
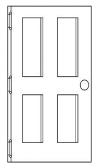
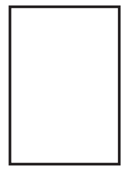
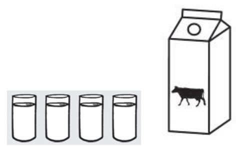
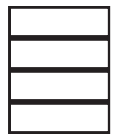
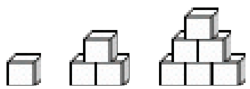
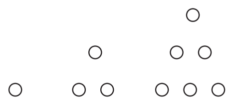
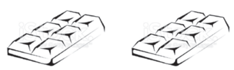
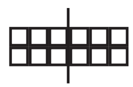
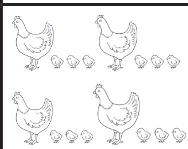
Another way to try and save some time when preparing for a new topic, is to give out some revision work to learners prior to the start of the topic. They could be required to do this over the course of a week or two leading up to the start of the new topic. For example, in Grade 8, while you are teaching the Theorem of Pythagoras, the learners could be given a homework worksheet on Area and Perimeter at Grade 7 level. This will allow them to revise the skills that are required for the Grade 8 approach to the topic.

What does this look like in the booklet?

At the beginning of each topic, there will be a SEQUENTIAL TEACHING TABLE, that details:

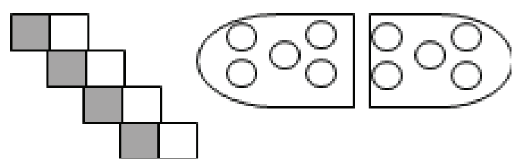
- The knowledge and skills that will be covered in this grade
- The relevant knowledge and skills that were covered in the previous grade or phase (looking back)
- The future knowledge and skills that will be developed in the next grade or phase (looking forward)

THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

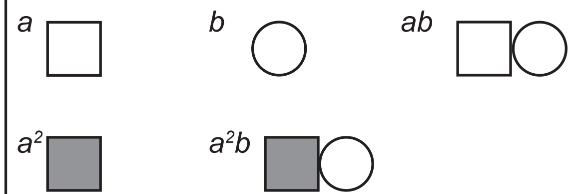
CONCRETE: IT IS THE REAL THING		REPRESENTATIONAL: IT LOOKS LIKE THE REAL THING		ABSTRACT: IT IS A SYMBOL FOR THE REAL THING	
Mathematical topic	**Real or physical For example:**	**Picture**	**Diagram**	**Number (with or without unit)**	**Calculation or operation, general form, rule, formulae**
Counting	Physical objects like apples that can be held and moved			6 apples	$2 \times 3 = 6$ or $2+2+2=6$ or $\frac{1}{2}$ of $6 = 3$ or $\frac{2}{3}$ of $6 = 4$
Length or distance	The door of the classroom that can be measured physically			80 cm wide 195 cm high	Perimeter: $2L + 2W = 390 + 160$ $= 550cm$ Area: $L \times W = 195 \times 80$ $= 15\,600cm^2$ $= 1.56m^2$
Capacity	A box with milk that can be poured into glasses			1 litre box 250 ml glass	$4 \times 250ml = 1\,000ml$ $= 1$ litre or 1 litre $\div 4 = 0.25$ litre
Patterns	Building blocks			1; 3; 6...	$n\,(n+1)$ 2
Fraction	Chocolate bar			6 12	6 = 1 12 2 or $\frac{1}{2}$ of 12 = 6
Ratio	Hens and chickens		* *** * *** * *** * ***	4:12	$4 : 12 = 1 : 3$ Of 52 fowls ¼ are hens and ¾ are chickens, ie 13 hens, 39 chickens
Mass	A block of margarine			500g	$500g = 0{,}5$ kg or calculations like 2 ½ blocks = 1,25kg

Teaching progresses from concrete -> to -> abstract. In case of problems, we fall back <- to diagrams, pictures, physically.

MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

Examples	SPEAK IT: to explain the concept • Essential for introducing terminology in context • Supports learning through the auditory pathway • Important to link mathematics to everyday realities	SHOW IT: to embody the idea • Essential to assist storing and retrieving concepts • Supports understanding through the visual pathway • Important to condense a variety of information into a single image	SYMBOL IT: to enable mathematising • Essential to assist mathematical thinking about concepts • Supports the transition from situations to mathematics • Important to expedite calculation and problem solving
FP: Doubling and halving	"To double something, means that we make it twice as much or twice as many. If you got R50 for your birthday last year and this year you get double that amount, it means this year you got R100. If Mom is doubling the recipe for cupcakes and she used to use 2½ cups of flour, it means she has to use twice as much this time." "To halve something, means that we divide it into two equal parts or share it equally. If I have R16 and I use half of it, I use R8 and I am left with R8. If we share the 22 Astro's in the box equally between the two of us, you get eleven, which is one half and I get eleven, which is the other half."	1. Physical objects: Example: Double 5 beads Halve 12 beads 2. Pictures: Example: Double  Halve  3. Diagrams: 	7 + 7 = □ 7 + □ = 14 △ + ○ = 14, but □ + □ = 14 2 times 7 = 14 double of 7 is 14 14 – 7 = 14 – □ = 7 14 divided by 2 = 7 14 halved is 7

IP: Geometric patterns	"If we see one shape or a group of shapes that is growing or shrinking a number of times, every time in the same way, we can say it is forming a geometric pattern. If we can find out how the pattern is changing every time, we can say we found the rule of the sequence of shapes. When we start working with geometric patterns, we can describe the change in normal language. Later we see that it becomes easier to find the rule if there is a property in the shapes that we can count, so that we can give a number value to each , or each term of the sequence" "You will be asked to draw the next term of the pattern, or to say how the eleventh term of the pattern would look, for example. You may also be given a number value and you may be asked, which term of the pattern has this value?"	o o o o oo oo o oo ooo Draw the next term in this pattern. T1 T2 T3 T4 o o oo o oo ooo o oo ooo oooo Describe this pattern. What is the value of the 9th term of this pattern (T9)? o o oo o oo ooo o oo ooo oooo o oo ooo oooo ooooo To draw up to the ninth term of this pattern, is a safe but slow way. It is even slower to find out by drawing, which term has a value of 120 for example. One is now almost forced to deal with this problem in a symbolic way.	Note how important it is to support the symbolising by saying it out: 1; 3; 6… 1; 3; 6; 10… 1; 3; 6;10;15 Inspecting the terms of the sequence in relation to their number values: T1: 1 = 1 The value of term 1 is 1 T2: 3 = 1+2 The value of term 2 is the sum of two consecutive numbers starting at 1 T3: 6 = 1+2+3 The value of term 3 is the sum of three consecutive numbers starting at 1 T4: 10 = 1+2+3+4 The value of term 4 is the sum of four consecutive numbers starting at 1 T5: 15 = 1+2+3+4+5 The value of term 5 is the sum of five consecutive numbers starting at 1 T9: 45 = 1+2+3+4+5+6+7+8+9 The value of term 9 is the sum of nine consecutive numbers starting at 1 We can see that the value of term n is the sum of n number of consecutive numbers, starting at 1.

SP: Grouping the terms of an algebraic expression	*“We can simplify an algebraic expression by grouping like terms together. We therefore have to know how to spot like terms. Let us say we have to sort fruit in a number of baskets and explain the variables or the unknowns in terms of fruits. Try to visualise the following pictures in your mind:”*	Although not in a real picture, a mind picture is painted, or a mental image to clarify the principle of classification: • Basket with green apples (a) • Basket with green pears (b) • Basket with green apples and green pears (ab) • Basket with yellow apples (a2) • Basket with yellow apples and green pears (a2b) Or in diagrammatic form 	$4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a - 2ab + 2a^2b + a^2b$ $= -3a + 5a - a + 4b + 4b - 2ab - 2ab - a^2 + 3a^2b + 2a^2b + a^2b$ $= a + 8b - 4ab - a^2 + 6a^2b$

TOPIC 1: MASS

INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area, 'Measurement' which counts for 15% in the final exam.
- The unit covers revision of skills covered in Grade 4 and 5 with some extension of the learner's understanding of the measurement of mass.
- Although learners have been working with mass calculations in the earlier parts of the Intermediate Phase, they must be able to work with larger numbers and perform conversions with confidence by the end of Grade 6.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Practical work to estimate, measure, record, compare and order mass of 3D objects • Measuring instruments include bathroom scales, kitchen scales and balances • Units used are grams and kilograms • Calculations involve converting between units, whole units and fractions • Problem solving in mass contexts	• Practical work to estimate, measure, record, compare and order mass of 3D objects • Measuring instruments include bathroom scales, kitchen scales and balances, analogue and digital • Units used are grams and kilograms • Calculations involve converting between units, whole units, fractions and decimals to two places • Problem solving in mass contexts	• The topic is not included in the Grade 7 curriculum

GLOSSARY OF TERMS

Term	Explanation / Diagram
Mass and weight	**Mass** is the measure of the amount of matter in an object. It does not matter how big the object is, or how much its volume is. **Weight** is the force with which gravity pulls on the mass of an object. On the Moon, where gravity is weak, the pulling force is weak and an object's **weight** is less, because gravity is weaker there, but the mass of the object is still the same. On the surface of planet Earth, the mass and weight of an object have the same reading.
Balance scales	An instrument that has a balanced beam and two pans. When the pans contain exactly the same mass, the beam is in balance. To find the mass of an object in one pan, standard measures of mass are put in the other, until the pans are at the same height.
Digital scales	An instrument to measure mass that shows the amount in units of mass electronically by displaying the number in a small window.
Analogue scales	Analogue scales have round dials, where a pointer moves clockwise according to the mass of the object. There are markings with equal spaces in between the numbers to indicate amounts.
Estimate	To judge the mass of an object without using scales. One needs some idea of the mass of common objects to be able to estimate.
Conversion	To switch from one unit of mass (for example gram) to another unit of mass (for example kilogram). Example: Convert 2 300 gram to kilogram: 2 300 g = 2.3 kg

SUMMARY OF KEY CONCEPTS

Introduction

The measurement skills required by CAPS in Grade 6 are

- Estimating mass in standard measurement units
- Using instruments practically to measure mass in standard units
- Recording mass readings in standard units, in whole- and decimal numbers
- Comparing the mass of different objects
- Converting between standard units of mass
- Doing calculations of mass up to two decimals
- Solving problems in mass contexts

Estimating Mass in Standard Measurement Units

Estimating requires that learners will be able to make a good judgement of the mass of an object, without using a scale. For learners to be able to estimate mass, they need to know the mass of some everyday objects to refer to.

1. Gram (g) is a standard unit to measure mass, which is the same all over the world. A small paperclip has a mass of about 1 gram. A 10c coin has a mass of 2 grams; a 50c coin has a mass of 5 grams. A R5 coin has a mass of almost 10 grams.

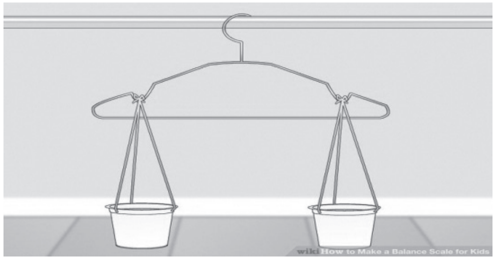
2. Ten R5 coins together have a mass of 95 grams, so if we add a 50c coin, they have a mass of exactly 100 grams altogether.
3. Kilogram (kg) is a standard unit that we use to measure mass. It is 1 000 times more than a gram. A pile of 1 000 paper clips has a mass of 1 kilogram. One litre water has a mass of one kilogram (of course without the mass of the container, just the water).
4. A 500g brick of margarine is also a handy way to get an idea of a half kilogram.

5. The mass of boys of average build can also be used as a very general guideline, always keeping individual differences in mind:
   - The average mass of a boy of 1 year old is approximately 10 kg
   - The average mass of a boy of 6 years old is approximately 20 kg
   - The average mass of a boy of 10 years old is approximately 30 kg
   - The average mass of a boy of 12 years old is approximately 40 kg

Using Instruments Practically to Measure Mass in Standard Units

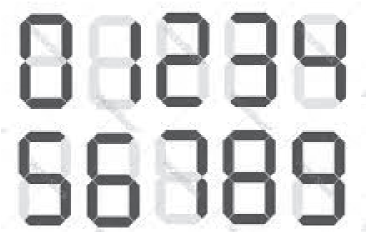
It is required by CAPS that learners in Grade 6 will be able to measure the mass of objects practically, using at least three types of scales.

1. **Balance scales** is an instrument that has a balanced beam and two pans. When the pans contain exactly the same mass, the beam is in balance. To find the mass of an object, one can place it in one pan and put standard measures of mass in the other, until the pans are at the same height. One can make one's own balance with a clothes hanger and margarine tubs on strings like the one below.



2. **Analogue scales** have round dials, where a pointer moves clockwise according to the mass of the object. There are markings with equal spaces in between the numbers to indicate specific values. Learners must be able to read the unmarked lines in between the marked lines. In the first example of an analogue bathroom scale below, each unmarked line represents 1kg. In the second example of an analogue kitchen scale, each unmarked line represents 10g.

3. **A digital scale** is an instrument to measure mass that immediately shows the amount in standard units like kilograms or grams. Bathroom scales usually measure in kilograms and kitchen scales measure in grams. Digital scales show the numbers in a display window, without dials and pointers like on an analogue scale. In the third picture below, one can see that there are seven possible line segments (that can be darkened or left light) in the display window of a digital scale and for each digit different line segments are made darker to create the digit. The digit 8 requires all seven segments to be darkened whereas the digit 7 only requires three.



Recording Mass Readings in Standard Units, in Whole- and Decimal Numbers

Using whatever instrument available, learners have to record their readings or findings. They can use a multipurpose table to record estimates, measures, comparisons and ordering:

Object	Estimated mass	Measured mass	Comparing and ordering the mass of different objects
Apple 1  Apple 2  Apple 3  Apple 4 			Compare the mass of the four apples and order them in ascending order:
Banana whole  Banana peeled 			Compare the mass of a whole banana and a peeled banana and find the mass of the peel:
2 boys 6 years old 2 girls 6 years old 2 boys 10 years old 2 girls 10 years old 2 boys 12 years old 2 girls 12 years old			Compare the mass of boys and girls at each different age. Compare the mass of boys at different ages. Compare the mass of girls at different ages. Compare your findings with the average mass of boys above.

Converting Between Standard Units of Mass

In Grade 6, conversions are done between the two units of mass, kilogram and gram. The basic fact needed to do these conversions, is that 1 kg = 1 000 g. In Grade 5 conversions have been done, giving the kilogram and gram values, but in Grade 6 the grams are reflected as a decimal fraction of a kilogram. A table such as the following can help learners to make the transition to decimal fractions of a kilogram:

Gram (g)	Kg and g	Kilogram (kg)	Round off to the nearest kilogram
Example: 1 230 g	1 kg 230 g	1,23 kg	1 kg
		76,54 kg	
	32 kg 10 g		
890 g			
		7,25 kg	
	5 kg 5 g		
13 579 g			

Teaching tip:
To convert kilogram to gram, add zeros up to three decimal places before multiplying by 1 000, for example to convert 4,5 kg to gram, write 4,500 kg which equals 4 500 g.

Solution:

Gram (g)	Kg and g	Kilogram (kg)	Round off to the nearest kilogram
Example: 1 230 g	1 kg 230 g	1,23 kg	1 kg
76 540 g	76 kg 540 g	76,54 kg	77 kg
32 010 g	32 kg 10 g	32,01 kg	32 kg
890 g	0 kg 890 g	0,89 kg	1 kg
7 250 g	7 kg 250 g	7,25 kg	7 kg
5 005 g	5 kg 5 g	5,005 kg	5 kg
13 579 g	13 kg 579 g	13,579 kg	14 kg

Topic 1 Mass

Doing Calculations of Mass up to Two Decimals

Calculations of mass requires a solid understanding of conversions, of the relation between common fractions and decimal fractions and of the order of operations. The following are some examples:

1. Calculate the following by first converting to one unit of mass and giving your answer in kg:

   $1550g + (\frac{7}{10}kg - \frac{1}{2}kg)$

   Solution: $1550g + (700g - 500g) = 1550g + 200g = 1350g = 1,35kg)$

2. Estimate by rounding off and then calculate the exact answer: $588g \times 62g$

   (Solution: $588g \approx 600g; 62 \approx 60; 600g \times 60 = 36\ 000g$ or $36kg$)

   $588g \times 62 = 36456g = 36,456kg$

3. Calculate 34% *of* $3kg$

   (Solution: $\frac{34}{100} \times \frac{3\ 000}{1}g = 34 \times 30g = 1\ 020g = 1,02kg$)

Solving Problems in Mass Contexts

Each textbook contains a multiple of examples of this kind. What follows below is a fun way of solving problems in mass contexts:

Gift packs of 2,5 kg, 3,4 kg and 1,725 kg each have to be made up with any of the articles below in each pack. Try to come as close to the above weights by combining any of the following articles (you can pick more than one of a type if you wish):

 250 g	 80 g	 125 g	 140 g	 35 g	 170 g
 340 g	 800 g	 200 g	 125 g	 10 g	 127 g

TOPIC 2: WHOLE NUMBERS

INTRODUCTION

- This unit runs for 1 hour.
- It is part of the Content Area, 'Numbers, Operations and Relationships' which counts for 50% in the final exam.
- The unit covers counting, ordering, comparing, representing, and place value of 9 digit numbers.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Order, compare and represent numbers to at least 6-digit numbers • Represent odd and even numbers to at least 1000 • Recognize the place value of digits in whole numbers to at least 6 digit numbers. • Round off to the nearest 5, 10, 100 and 1 000	• Order, compare and represent numbers to at least 9-digit numbers • Represent prime numbers to at least 100 • Recognize the place value of digits in whole numbers to at least 9-digit numbers • Round off to the nearest 5, 10, 100 and 1 000	Revise the work done in Grade 6: • Order, compare and represent numbers to at least 9-digit numbers • Recognise and represent prime numbers to at least 100 • Round off numbers to the nearest 5, 10, 100 or 1 000

GLOSSARY OF TERMS

Term	Explanation / Diagram
Whole Numbers	The numbers in the set {0, 1, 2, 3, ...} are called whole numbers. Whole numbers are counting numbers including zero.
Place Value	The value of the digit depends on its position in the number. In 65 314 728, the 2 is in the 'tens' position, so it shows a value of 20.
Rounding Off	**Rounding** is a method used to change numbers into more 'friendly' numbers which are easier to work with particularly if needing to do a calculation mentally. **Rounding** involves either increasing or decreasing a number to the next digit by writing a number as an approximate, understood to be "about", "almost" or "closest to" a multiple of a given number, like 5. We can round numbers to the nearest multiple of five, or to the nearest multiple of ten, or to the nearest multiple of hundred and so on. We can round up to the next multiple or round down to the previous multiple. We indicate that we have rounded by using the symbol ≈. 34 ≈ 35 if we rounded (up) to the nearest multiple of 5.
Digit	A single character used in a numbering system. In the decimal system digits are 0 to 9.
Inverse Operations	Inverse operations are **opposite** operations that undo each other. Addition and subtraction are inverse operations. Multiplication and division are inverse operations.
Order Of Operation	BODMAS When there are multiple operations in the same sum there is a particular order in which calculations have to be performed: • Brackets (parts of a calculation inside brackets always come first). • Orders (numbers involving powers or square roots). • Division and Multiplication (from left to right). • Addition and Subtraction (from left to right).
Estimation	The best guess arrived at after considering all the information given in a problem. To estimate is to find an answer that is very close to the exact answer
Million	A million is 1 000 000 or 1000 thousands

SUMMARY OF KEY CONCEPTS

Counting in millions, ten millions and hundred millions

1. There are many ways to practice counting and it is important to have learners count in the given interval such as millions, not always starting with the first multiple or with zero.
2. Counting can be done orally at the start of a lesson or grids can be used. Learners can write number sets where they work with these counting sets in various forms.
3. Examples:

   Counting in millions:

   1 000 000, 2 000 000, 3 000 000…

   1 340 000, 2 340 000, 3 340 000…

   Counting in ten millions:

   10 000 000, 20 000 000, 30 000 000…

   1 000 000, 11 000 000, 21 000 000…

   12 300 000, 22 300 000, 32 300 000…

   Counting in hundred millions:

   100 000 000, 200 000 000, 300 000 000…

   102 000 000, 202 000 000, 302 000 000…

4. Learners must be able to count forwards and backwards. If learners are finding this difficult, use place value tables and charts to help them see how to order the numbers correctly.

Numbers up to one hundred million in words and in numeric form

	HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
Digits	5	6	5	3	1	4	7	2	8
What the digit means in terms of its position	This represents 5 hundred millions	This represents 6 ten millions	This represents 5 millions	This represents 3 hundred thousands	This represents 1 ten thousand	This represents 4 thousands	This represents 7 hundreds	This represents 2 tens	This represents 8 ones or units
Numeric	500 000 000	60 000 000	5 000 000	300 000	10 000	4000	700	20	8
How you would say it	Five hundred million	Sixty million	Five million	Three hundred thousand	Ten thousand	Four thousand	Seven hundred	Twenty	Eight
What the value of each digit is in the number	The digit 5 has a value of 500 000 000	The digit 6 has a value of 60 000 000	The digit 5 has a value of 5 000 000	The digit 3 has a value of 300 000	The digit 1 has a value of 10 000	The digit 4 has a value of 4000	The digit 7 has a value of 700	The digit 2 has value of 20	The digit 8 has a value of 8

1. In Numerals: 565 314 728
2. In Words:
   Five hundred and sixty five million, three hundred and fourteen thousand, seven hundred and twenty-eight.
3. Expanded Form:
   500 000 000 + 60 000 000 + 5 000 000 + 300 000 + 10 000 + 4 000 + 700 + 20 + 8
4. Identify the place value of each digit in a number up to hundred millions (9 digits)

565 314 728

The digit 5 is in the **hundred million** place
The digit 6 is in the **ten million** place
The digit 5 is in the **million** place
The digit 3 is in the **hundred thousand** place
The digit 1 is in the **ten thousand** place
The digit 4 is in the **thousand** place
The digit 7 is in the **hundred** place
The digit 2 is in the **tens** place
The digit 8 is in the **unit /ones** place

Teaching Tip:
Learners often misread or write numbers incorrectly when they have zeros. This can be prevented by working with place value tables emphasising the importance of zeros as place holders in the number. Learners know that if hundred millions are mentioned there will be at least 9 digits in the number.

Compare numbers within hundred millions (9 digits):

1. Learners compare and order numbers according to their value working with place value tables. Writing numbers directly below each other is a good strategy that will help them make accurate comparisons.
2. Smaller than: <
3. Example: Which number is smaller, 467 237 981 or 467 230 600?
   When comparing numbers look at the value of each digit starting from the left and gradually moving to the right of the number.

0 thousands is **smaller** than **7** thousands. So, 467 23**0** 600 is **smaller** than 467 23**7** 981.

HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
4	6	7	2	3	7	9	8	1
4	6	7	2	3	0	6	0	0

This is written as 467 23**0** 600 < 467 23**7** 981

4. Greater than: >
5. Example: Which number is greater, 300 712 935 or 300 712 846?

HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
3	0	0	7	1	2	9	3	5
3	0	0	7	1	2	8	4	6

A table makes it easier to work out which number is bigger/smaller than another.
Work from left to right.

If they are the same, continue to compare until the values of the digits are not the same.
The values of the digits in the hundreds place are not the same.

9 hundred is **greater** than **8** hundred.
So 300 712 **9**35 is **greater** than 300 712 **8**46.

This is written as 300 712 **9**35 > 300 712 **8**46

6. Arrange the numbers from smallest to biggest (ascending order)
7. Example: 324 688, 32 468, 3 246 880, 324 560 004
   Look at which number has the least digits and this will become the first number. It will help learners to use a place value table and write numbers underneath each other.

   Answer: 32 468, 324 688, 3 246 880, 324 560 004

8. Arrange the numbers from biggest to smallest (descending order)

   324 688, 32 468, 3 246 880, 324 560 004

   Look at which number has the **most** digits. This becomes the first number in the sequence and is then followed by the next biggest number and so on.

   Answer: 324 560 004, 3 246 880, 324 688, 32 468

9. Summary: We write $a < b$ to indicate that a is smaller than b and $b > a$ to indicate that b is bigger than a. The three symbols $\neq$, $<$ and $>$ are symbols for inequalities, where the left side is not equal to the right side. It is only when the left side and the right side are equal that we use = which indicates equality.

Rounding off

The following strategy can be used for rounding up or rounding down:

1. Decide which digit is the last one you need to keep, which is the digit in the place you are asked to round off to.
2. Leave that digit the same if the next digit is less than 5.

Example: Round 74 to the nearest multiple of 10

We want to keep the 7 in the 10s place
The next digit (4) is less than 5, so no change is needed to 7.
74 ≈ **70**
74 gets rounded down

3. Increase the digit by 1 if the next digit is 5 or more

Example: Round 86 to the nearest multiple of 10

We want to keep the 8 in the 10s place
The next digit is 6 which is more than 5, so increase the 8 by 1 to 9
86 ≈ **90**
86 gets rounded up

4. Learners must be able to round to the nearest multiple of 5, 10, 100 or 1 000.
5. Rounding to the nearest multiple of 5 was learned in Grade 5, but needs to be revised in Grade 6. Round up to the next multiple of 5 or down to the previous multiple of 5 as follows:
   0, 1, 2<|>3, 4, **5**, 6, 7<|>8, 9, **10**, 11, 12<|>13, 14, **15**, 16, 17<|>23, 24, **25**, 26, 27<|>...

TOPIC 3: ADDITION AND SUBTRACTION

INTRODUCTION

- This unit runs for 8 hours.
- It is part of the Content Area, 'Numbers, Operations and Relationships' which counts for 50% in the final exam.
- The unit covers addition and subtraction of numbers up to 6 digits.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Describe, order, compare whole numbers up to at least 6 digit numbers • Round off to 5, 10, 100 and 1 000 • Represent odd and even numbers to at least 10 000 • Add and subtract whole numbers of at least 5 digits • For addition and subtraction use strategies of building up and breaking down; number lines; rounding off and compensating; using addition and subtraction as inverse operations; adding and subtracting in columns	• Describe, order, compare whole numbers up to at least 9 digit numbers • Round off to 5, 10, 100, 1 000, 100 000, 1 000 000 • Represent prime numbers to at least 100 • Add and subtract whole numbers of at least 6 digits • For addition and subtraction use strategies of building up and breaking down; number lines; rounding off and compensating; using addition and subtraction as inverse operations; adding and subtracting in columns; using a calculator	• Revise adding and subtracting whole numbers of at least 6 digits • For addition and subtraction use strategies of rounding off and compensating; estimation; adding and subtracting in columns; using a calculator

GLOSSARY OF TERMS

<table>
<tr><th>Term</th><th>Explanation / Diagram</th></tr>
<tr><td>Place Value</td><td>The position of a number shows its value.
Numbers up to a hundred million
<table><tr><td>HM</td><td>TM</td><td>M</td><td>HTh</td><td>TTh</td><td>Th</td><td>H</td><td>T</td><td>U</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></tr></table></td></tr>
<tr><td>Odd and Even Numbers</td><td>Odd numbers start at one and include all numbers that are not divisible by 2.
Even numbers start at two and include all numbers that are divisible by 2.
Zero is neither odd nor even.</td></tr>
<tr><td>Ordering Numbers</td><td>Arranging numbers according to their magnitude, either ascending or descending.</td></tr>
<tr><td>Comparing Number</td><td>To observe different numbers and judge which is bigger and which is smaller.</td></tr>
<tr><td>Commutative
Property</td><td>It does not matter in which order we add numbers (15 + 16 = 16 + 15).
This does not apply for subtraction.</td></tr>
<tr><td>Associative Property</td><td>In addition, the grouping does not matter 15 + (3 + 5) = (15 + 3) + 5.
This does not apply for subtraction.</td></tr>
<tr><td>Distributive property of multiplication over addition and subtraction</td><td>If we multiply a number by numbers that are added together, it is the same as multiplying the number by each of those numbers and then adding them.

Example: 3(4 + 5) = 3 x 4 + 3 x 5 = 12 + 15 = 27

The same goes for the distributive property of multiplication over subtraction.

Example: 6 x (4 - 2)
or also written as 6(4 - 2) = 6 x 4 - 6 x 2 = 24 - 12 = 12</td></tr>
<tr><td>0 – Additive Property</td><td>Adding or subtracting zero to a number leaves the number unchanged.</td></tr>
<tr><td>1 – Multiplicative Property</td><td>Multiplying or dividing by one leaves a number unchanged.</td></tr>
<tr><td>Inverse
+ and –
x and ÷</td><td>Addition and subtraction are the opposite operations of which one can undo the other. Addition can be checked by subtraction.
25 + 67 = 92 and 92 -25 = 67

Multiplication and division are the opposite operations of which one can undo the other. Multiplication can be checked by division
8 x 9 = 72 and 72 ÷ 8 = 9</td></tr>
</table>

SUMMARY OF KEY CONCEPTS

Addition and Subtraction

1. Grouping numbers in addition
   Look for pairs of numbers that add up to 10, 100 or 1 000
2. Example: 6 + 5 + 2 + 5 + 4 + 8 = 30

because	6 + 4	= 10
	5 + 5	= 10
	2 + 8	= 10
	all together	= 30

3. Adding up to 6 digits

   Expanded vertical column method
   Addition and subtraction started with horizontal expansion of numbers. Now learners have to write these expanded numbers underneath each other (vertically) in preparation for the vertical column method. They should also be encouraged to work from right to left (from the units backwards), to prepare them for the standard algorithm of adding and subtracting in vertical columns.

37 633	= 30 000	+ 7 000	+ 600	+ 30	+3
50 762	= 50 000	+	+ 700	+ 60	+ 2
+62 873	= 60 000	+ 2 000	+ 800	+ 70	+ 3
	= 140 000	+ 9 000	+ 2 100	+ 160	+ 8

Grouping ten thousands, thousands, hundreds, tens and units together:
=15 000 + 1 000 + 200 + 60 + 8 = 16 268

a. The vertical column method which should be used in Grade 6

   34 305 + 26 779 + 884 =

$$
\begin{array}{rccccc}
 & {}^{1}3 & {}^{1}4 & {}^{1}3 & {}^{1}0 & 5 \\
+ & 2 & 6 & 7 & 7 & 9 \\
+ & & & 8 & 8 & 4 \\
\hline
 & 6 & 1 & 9 & 6 & 8 \\
\hline
\end{array}
$$

   Remind learners of the importance of writing units directly underneath each other and tens directly underneath each other and so on.

4. Subtraction with up to 6 digits:

   a. Subtraction using the expanded vertical column method

   38 342 = 30 000 + 8000 + 300 + 40 +2
   - 3 231 = 3000 + 200 + 30 +1
   = 30 000 + 5000 + 100 + 10 + 1
   = 35 111

   b. The standard algorithm or the vertical column method of subtraction that is used in Grade 6

   Subtract 634 684 from 818 743

	$^{7}\not{8}$	$^{11}\not{1}$	8	$^{6}\not{7}$	$^{13}\not{4}$	$^{1}3$
-	6	3	4	6	8	4
	1	8	4	0	5	9

   Again, it is important that learners write units directly underneath each other, tens directly underneath each other, and so on.

Learners also need plenty of practice subtracting from numbers that contain zeros.

Examples:
9 0 0 0 0 0
- 6 8 4 9 5

	$^{8}\not{9}$	$^{9}\not{0}$	$^{9}\not{0}$	$^{9}\not{0}$	$^{9}\not{0}$	$^{1}\not{0}$
-		6	8	4	9	5
	8	3	1	5	0	5

Teaching Tip:
It is important to encourage learners to estimate so that they can determine the relevance of their answer.

Properties of numbers:

1. Commutative Property
   The order of numbers in addition can change yet the answer remains the same:
   4 + 2 = 6 and 2 + 4 = 6
   a + b = b + a
   but this law does not work for subtraction

2. Associative Property
   The grouping of numbers in addition may change, but the answer remains the same (the numbers can be associated in any way):
   (7+ 3) + 5 = 15 or 7 + (3 + 5) = 15
   (a + b) + c = a + (b + c)
   but this law does not work for subtraction

3. Distributive Property
   The number before the brackets applies to both the numbers in the brackets - it "distributes" multiplication over addition or subtraction inside the brackets:
   a(b + c) = (a x b) + (a x c)
   a(b - c) = (a x b) – (a x c)
4. 0 – Additive Property
   Adding or subtracting a zero leaves a number unchanged
   8 + 0 = 8 and 27 - 0 = 27
   t + 0 = t and t - 0 = t

Teaching Tip:
Some learners find it difficult when they round off to ensure they round the correct digit so they end up rounding every digit. It is helpful to have learners circle digits that must be rounded in a colour so they highlight the fact that they are only looking if that digit needs to change.

TOPIC 4: VIEWING 3D OBJECTS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area, 'Space and Shape' which counts for 15% in the final exam.
- The unit deals with the various views of the same object and how the object is observed from various points of view or positions.
- In Grade 6 learners start sketching the view of the object from various points of view or positions, and not only as they themselves view it.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Link the position of the viewer relative to side- and top views of single everyday objects • Link the position of the viewer relative to the view of groups of everyday objects • Link the position of the viewer relative to the view of everyday scenes • Link an object or scene to the plan or map of the object or scene	• Link the position of the viewer relative to the view of single everyday objects and collections of objects • Link the position of the viewer relative to the view of composite everyday objects • Link the position of the viewer relative to the view of everyday scenes • Link an object or scene to the plan or map of the object or scene	• The topic is not taken further into the Senior Phase, however the skill of visual imagery acquired in the viewing of objects, becomes a basic and crucial skill in Transformational Geometry.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Viewpoint	The position from which you view an object or item
Position	The place from where the viewer is looking at the object or item. The viewer's location relative to the object or item they are looking at
Sketches	Rough drawings showing a 2D side of a 3D object
Point of view	This is the point from which the viewer is viewing the object or item
Perspective	The way that a 3D object is represented or drawn as a 2D picture, giving the impression of three dimensions, especially of depth.

SUMMARY OF KEY CONCEPTS

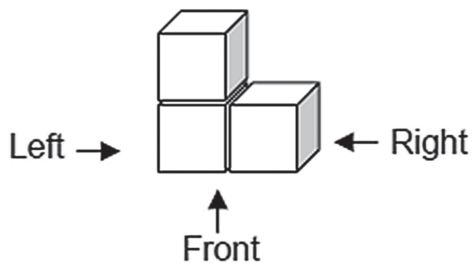
Different view points

1. Learners should have a good understanding of the position from which an object is being viewed.

Teaching Tip:
Have learners stand around the same object and describe what they are seeing from their various perspectives.

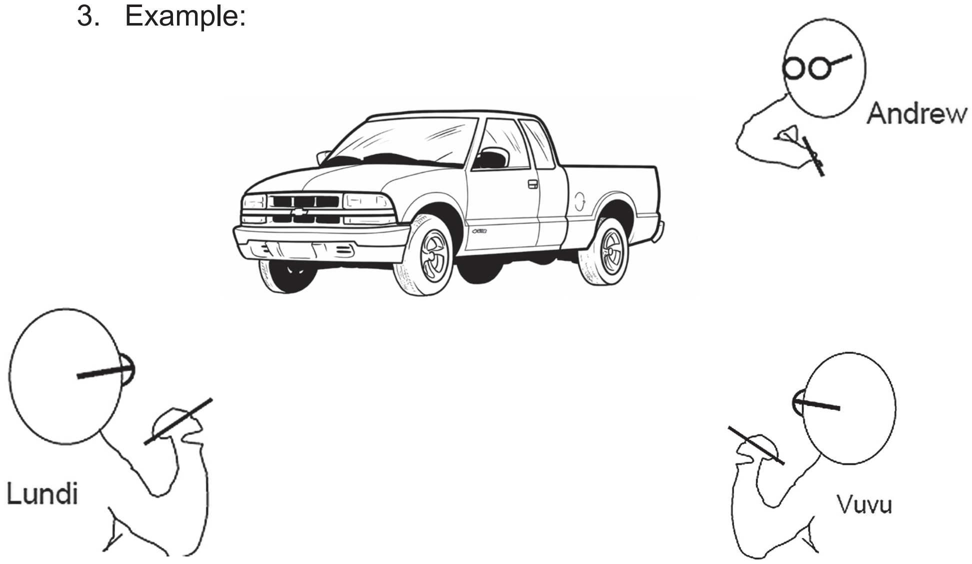
2. Learners must understand that position is relative to the object.

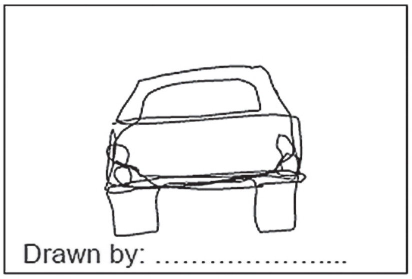
3. Example:

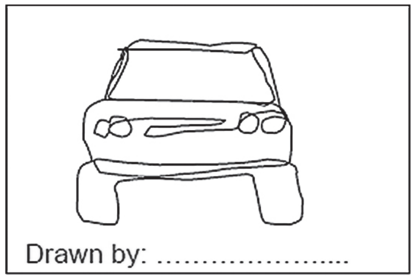


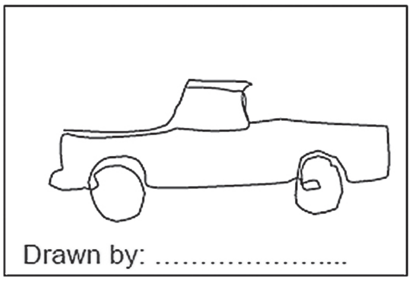
Drawing an object from different viewpoints

1. Learners should be able to sketch an object from a variety of viewpoints.
2. Learners should also be able to match a sketch to a viewer's perspective based on the viewer's position.
3. Example:

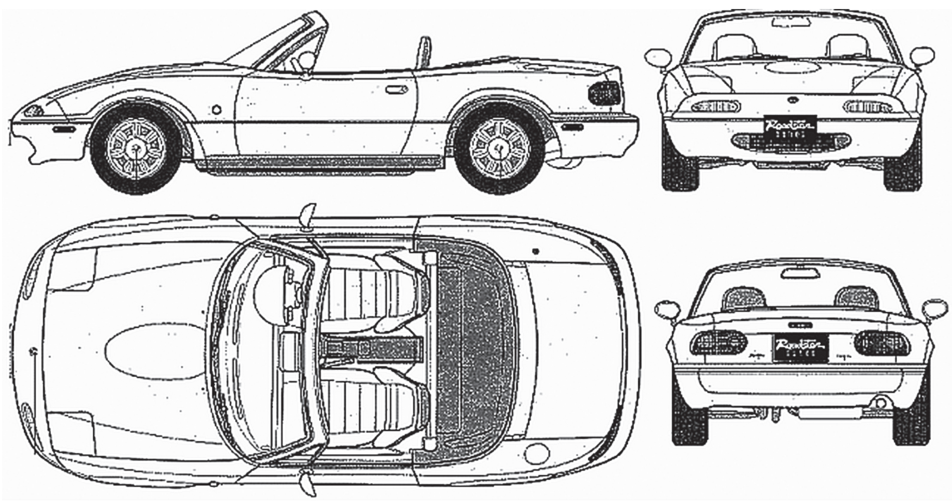




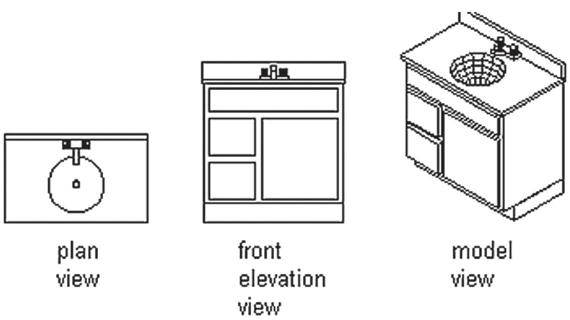




Further examples of different views of one object:



The car is viewed from the side, the front, the top and the back.



Teaching Tip:
Putting up posters with pictures like the above will encourage learners to think about everyday objects and the view from different perspectives.

TOPIC 5: PROPERTIES OF 2D SHAPES

INTRODUCTION

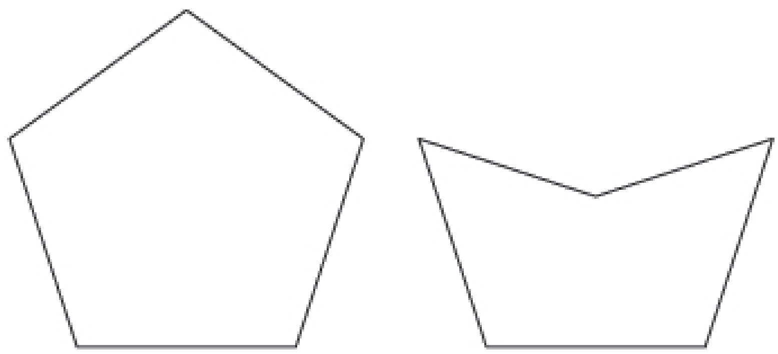
- This unit runs for 4 hours.
- It is part of the Content Area, 'Space and Shape' which counts for 15% in the final exam.
- The unit covers revision of the content covered in Term 1 relating to space and shape but introduces the use of a compass to draw circles and draw patterns of circles.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Range of 2D shapes includes circles, squares, rectangles, regular and irregular polygons, triangles, pentagons, hexagons and heptagons
• Identify and name regular and irregular 2D shapes in the environment and in geometric settings
• Describe and create 2D shapes with straight and curved sides. Determine the number and length of sides, angles in shapes, (right angles, acute angles and obtuse angles)
	• Range of 2D shapes expands to include parallelograms and octagons in addition to circles, squares, rectangles, regular and irregular polygons, triangles, pentagons, hexagons and heptagons
	• Identify similarities and differences between rectangles and parallelograms
	• Range of angles expands to include straight- and reflex angles and a revolution in addition to right angles, acute angles and obtuse angles
	• Draw circles using compasses and create patterns with circles
		• Describe, sort, name and compare triangles according to their sides and angles, focusing on:
		  • equilateral triangles
		  • isosceles triangles
		  • right-angled triangles
		• Describe, sort, name, compare quadrilaterals in terms of:
		  • length of sides
		  • parallel and perpendicular sides
		  • size of angles (right-angles or not)
		• Describe and name parts of a circle
		• Knowledge of the properties of 2D shapes forms the basis of the concepts of congruency and similarity

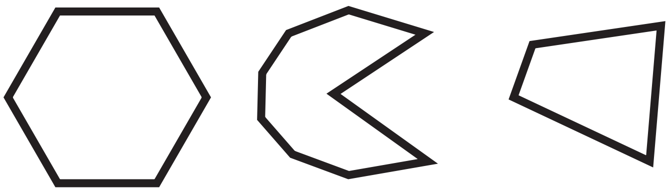
GLOSSARY OF TERMS

Term	Explanation / Diagram
2D shape	2-D shapes consist of more than one line and exists on a plane. The picture of a square on paper is a 2-D shape.
Polygon	2-D shape that has at least 3 straight sides such as a triangle, quadrilateral etc.
Revolution	Is a 360° angle
Circle	A **circle** is a shape with all points on its circumference the same distance from its centre. A **circle** is named by its centre.
Compass	A **compass** is a drawing instrument used for drawing circles and arcs. It has two legs, one with a point and the other with a pencil or lead.
Regular Shapes	A regular shape means that all the angles are equal in size and all the sides are of equal length. An equilateral triangle and a square are both examples of regular shapes.
Irregular Shapes	A polygon that does not have all sides equal and the angles are also not always equal.
Convex and Concave	A convex polygon has no angles pointing inwards. A concave polygon has a side or sides that cave in.  *convex polygon* *concave polygon*

SUMMARY OF KEY CONCEPTS

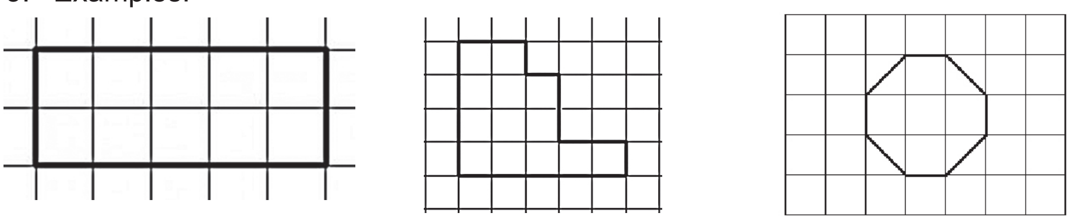
Revision of content from Term 1

1. Learners must be able to identify, name, describe and compare a variety of 2D shapes according to their sides, interior angles and shape. They also need to be able to tell regular from irregular 2D shapes.



Draw 2D shapes on grid paper

1. Learners must be able to draw a variety of shapes accurately on grid paper.
2. These shapes should have straight edges so that learners can accurately copy the shape using the grid paper lines as reference.
3. Examples:

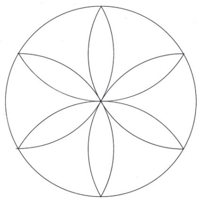


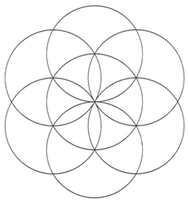
Draw circles using a pair of compasses

1. To draw a circle with a compass:
    - Open the compass to the distance required for the radius of the circle to be drawn.
    - Make sure that the hinge at the top of the compass is tight, so that it does not slip.
    - Insert and tighten the hold for the pencil also, so it does not slip.
    - Check that the pencil is aligned with the compass's needle.
    - Place the needle in the appropriate spot on the paper and place the pencil on the paper. Ensuring that both the needle and the pencil remain fixed on the paper, hold the compass carefully and turn the top in order for the pencil to draw the circle.

Circle patterns using a pair of compasses

1. Learners must be able to make some basic circle patterns. Patterns with interlocking circles are the simplest types to start with.



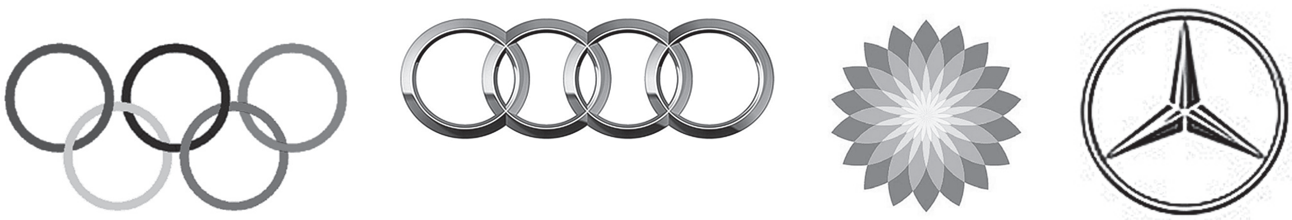


   a. For the first pattern, keep the compass at the same width as when the circle was drawn. Put the sharp point of the compass on any point on the circumference of the circle and draw an arc starting and ending on the circumference. Note that the arc will run through the centre of the circle. Put the compass (at the same width) on one of the two points and draw a similar arc, and so on until six petals have formed, all connected at the centre of the circle.

   b. For the second pattern, do exactly the same as for the first pattern, only now continue drawing a full circle and not only an arc. This pattern is an extension of the first pattern.

Many well-known logos consist of circle patterns.

Real-life examples of circle patterns:



There is a resource available at the end of the book that can be used to develop learner's understanding of 2D shapes.

The worksheet should be copied then each square cut out. Learners should work in groups to match the squares of information that go together.

TOPIC 6: TRANSFORMATIONS

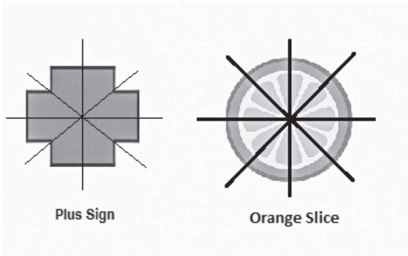
INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area, 'Space and Shape (Geometry)' which counts for 15% in the final exam.
- The unit covers transformations of 2D shapes. This includes symmetry, translation, rotation, reflection and enlargement or reduction of 2D shapes.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Recognise, draw and describe lines of symmetry in 2D shapes • Use transformations to build composite 2D shapes by tracing and moving 2D shapes by rotation, by translation or by reflection • Use transformations to tessellate patterns with 2D shapes • Observe and recognise symmetry and transformations in nature and in the environment • Use reflection, rotation and translation • Describe patterns	• Continue the work and concepts learned in Grade 4 and 5 • Transform 2D shapes through reflection, translation, rotation, enlargement and reduction • Use transformations to describe shapes in the world, in nature and from our cultural heritage • Describe transformations in terms of reflection, rotation, translation, enlargement and reduction	• Recognize, describe and perform translations, reflections and rotations with geometric figures and shapes on squared paper • Identify and draw lines of symmetry in geometric figures • Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size

GLOSSARY OF TERMS

Term	Explanation / Diagram
Transformation	A change in the direction, position, orientation or size of an object or shape
Rigid Transformation	Transformations in which size and shape are preserved (that means they do not change)
Translation	A transformation in which a shape moves left, right, up or down
Object	The original shape before a transformation
Image	The resulting shape after the object has undergone transformation
Reflection	A mirror image of a shape
Glide Reflection	A reflection that has simultaneously undergone translation
Line Of Symmetry	This is the line that divides the object exactly in half where the result is that one half is an identical mirror image of the other half
Vertical Symmetry	A vertical axis dividing the shape in half in such a way that the two sides of the shape are an exact mirror image of each other
Horizontal Symmetry	A horizontal axis dividing the shape in half in such a way that the two sides of the shape are an exact mirror image of each other
Radial Symmetry	Symmetry about a central axis 
Rotation	A transformation in which a shape is rotated or turned to give the resulting image
Point Of Rotation	The point around which a shape is rotated to deliver a desired image
Order Of Rotational Symmetry	The number of positions a shape can be rotated where the resulting image is identical to the original object
Enlargement	A transformation where the shape of the object is maintained but the size is proportionally increased.
Reduction	A transformation where the shape of the object is maintained but the size is proportionally decreased.
Centre Of Enlargement / Reduction	The point from which an enlargement takes place or the point towards which a reduction occurs
Scale Factor	The number of times the image is larger or smaller than the original object
Tessellation	A pattern of congruent (identical) shapes and images where there would be no gaps or overlaps
Congruent	Absolutely identical to each other

SUMMARY OF KEY CONCEPTS

Use transformations to describe patterns

1. Learners describe patterns by discussing the shapes they see in the pattern and how they would transform that shape if they wanted to extend the pattern.

Note: In Grade 6, learners are no longer required to draw tesselating patterns, only to describe them. However, it is sometimes easier to describe once the physical act of drawing has been done. If this assists your learners, then encourage them to draw.

2. The pattern seen on the honeycomb is a tesselation pattern of hexagons. This pattern is made by translating the hexagon.
3. Example:

4. The pattern seen on the bead bracelet looks like a tessellation pattern of triangles. This pattern is made by reflecting the triangle and then translating it either one position to the right and one down, or one position to the right and one up, depending on the axis of reflection.
5. Example:

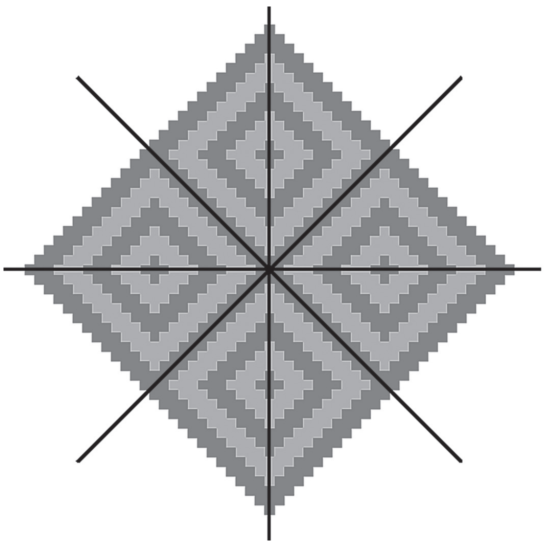
Use symmetry to describe patterns

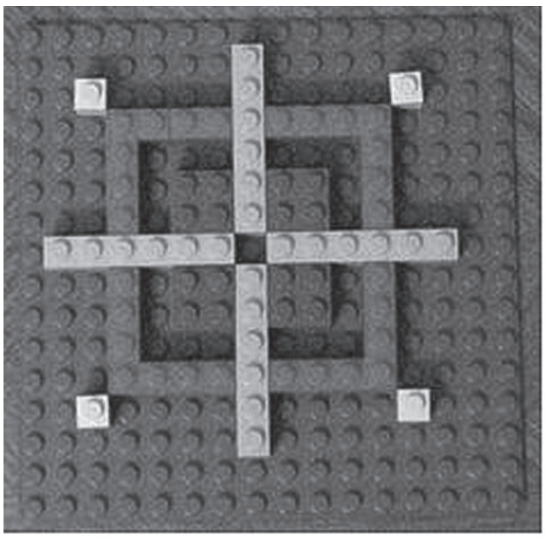
1. Learners identify symmetry in patterns.
2. It is useful to link the process of making or copying patterns with the descriptions of patterns from nature, modern everyday life and our cultural heritage. Often the geometrical process you use to make a copy of the pattern is not the same as the original process used to make the pattern. Bees do not tessellate with hexagons to make a honeycomb, but if learners tessellate with a hexagon, they can make a pattern that looks similar to the pattern they see in the honeycomb.

Teaching Tip:
Throughout the Intermediate Phase learners have identified and drawn lines of symmetry. Learners must be able to identify these with ease and describe the symmetry and the influence it has on the patterns.

Learners can also build their own symmetrical patterns with building blocks.

3. Example:





Transformations will be covered again in Term 4. An overview of the entire topic is provided in that booklet.

TOPIC 7: TEMPERATURE

INTRODUCTION

- This unit runs for 1 hour.
- It is part of the Content Area, 'Measurement' which counts for 15% in the final exam.
- The unit covers the recording, calculation and solution of problems involving temperature.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Understand the range within which temperature occurs in our everyday lives • Estimate and compare different temperatures • Measure and record temperature with thermometers • Read temperature in degrees Celsius • Solve problems in a temperature context	• Understand the range within which temperature occurs in our everyday lives • Estimate and compare different temperatures • Measure and record temperature with analogue and digital thermometers • Read temperature in degrees Celsius • Solve problems in a temperature context	• The topic as such is not covered in Grade 7. • The concept of temperature is applied in various mathematical problems, especially to illustrate calculations with integers. and also in data handling and graphing data.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Temperature	This is the formal measurement of how hot or cold something or someone is.
Minimum temperature	The coldest temperature in a given period of time, for example a day or a year
Maximum temperature	The hottest temperature in a given period of time, for example a day or a year
Convert	To change the units from one standard unit to another
Digital thermometer	A temperature measuring instrument with an electronic display that shows whole numbers and decimals
Analogue thermometer	The original type of instrument for measuring temperature that has a graduated scale that shows the temperature as the mercury rises in the tube behind the scaled markings.

SUMMARY OF KEY CONCEPTS

Temperature

1. People and animals feel when it is cold or hot in their environment or when an object is cold or hot. We can feel it with our hands or feet, or just “feel it” in our bodies. That is subjective – what I think is a hot bath, may not feel so hot for another person. What I think is cold tea, may still be warm for someone else. We use words like “freezing”, “cold”, “just right”, “warm”, “hot”, and “burning” to describe what we feel.
2. Temperature is an objective measure of cold or heat. That means it is actually how hot or cold it is. We can measure temperature with an instrument that makes it possible to attach a number value to the temperature to say exactly how cold or how hot something is. That does not depend on our feeling but on the instrument.
3. To measure temperature, we use something in nature that is always reacting the same to cold or to heat. One such material is the fluid metal mercury, which expands or takes up a bigger volume when it is hot, and shrinks or takes up a smaller volume when it is cold. When it is cold, the mercury in a measured tube shrinks and goes lower down in the tube. We say the temperature is low or it is cold. When the mercury becomes warmer, it expands and rises in the tube. Then the temperature is higher, because the volume of the mercury went up to a higher number on the tube.

The Celsius scale

1. Degrees Celsius (°C) is the standard unit on the Celsius measurement scale where the temperature of melting ice is 0°C and that of boiling water is 100°C.
2. The Celsius scale also goes below 0°C. We count above zero 1, 2, 3, and so on, and we count below zero in minus one (-1), minus two (-2), minus three (-3) and so on.

Thermometer

1. A thermometer is an instrument for measuring and indicating temperature in numbers. Normally it consists of a narrow, sealed glass tube marked with numbers and has a bulb at one end that contains mercury or alcohol which moves up along the tube as it expands when it heats up, and moves down as it shrinks when it cools down.
2. A measuring instrument for temperature is marked in the number of degrees Celsius.

3. The thermometer tube is marked according to its purpose. If it is used to measure baking, it may go up to 400°C. If it is for measuring body temperature, it usually goes from 35°C to 45°C.

Body temperature

1. Body temperature is the degree of heat that a human- or animal body keeps on the inside of the body. In degrees Celsius, the normal human body temperature is 37°C .
2. Our bodies work hard to keep its temperature somewhere between 36°C and 38°C. When it is cold outside, the body works on the inside to make it warmer. When it is hot outside, we sweat to cool us down. The body always has to keep a balance.
3. The way we feel heat or cold is often what we feel in the air around us, which may differ a lot from our body temperature. It may be very cold like 12°C outside, but our body temperature stays round about 37°C; it may be very hot, like 42°C outside, but our body temperature stays round about 37°C. Our skin on the outside is cooler than our body on the inside, because that is the part that gives off heat to the environment.

Freezing point

1. The temperature at which a liquid turns into a solid when it cools down, is called that liquid's freezing point.
2. Different liquids freeze at different temperatures. In the metric measuring system for temperature, (degrees Celsius, or °C) we use water's freezing point as zero point. Water freezes at a temperature of 0°C and turns into solid ice. Sweet cream freezes below 0°C, so its freezing point is negative or below the freezing point of water.
3. Alcohol freezes at a much lower temperature as water, namely at -80°C and benzene freezes at a higher temperature than the freezing point of water, namely at 5.5°C.
4. When the frozen (solid) liquid is heated up again, it melts back to liquid when it reaches the same temperature. Therefore, the freezing point and the melting point of a liquid are the same, and in water it is 0°C.

Boiling point

1. The temperature at which a liquid turns into a gas when it is heated up, is called that liquid's boiling point.
2. Different liquids boil at different temperatures. In the metric measuring system water's boiling point is the 100 mark. Water boils at 100°C and turns into steam.
3. Alcohol boils at a lower temperature than water, namely at about 80°C. Mercury (the fluid metal in many thermometers) boils at the extremely high temperature of 357°C.
4. When the gas is cooled down again, it returns back to liquid or we say it condensates.

Maximum and minimum temperature

1. Minimum temperature is the lowest temperature that the environment reaches in a given period, like a day or a month or a year. Maximum temperature is the highest.
2. The highest temperature ever recorded on earth was 58°C in Libya. The lowest was -89°C in Antarctica. In South Africa our maximum temperature may reach a high of 45°C and our minimum temperature in certain regions may reach a low of -15°C .
3. Deserts have extreme temperatures which can range from a maximum of 36°C during the day to a minimum of 6°C during that same night.

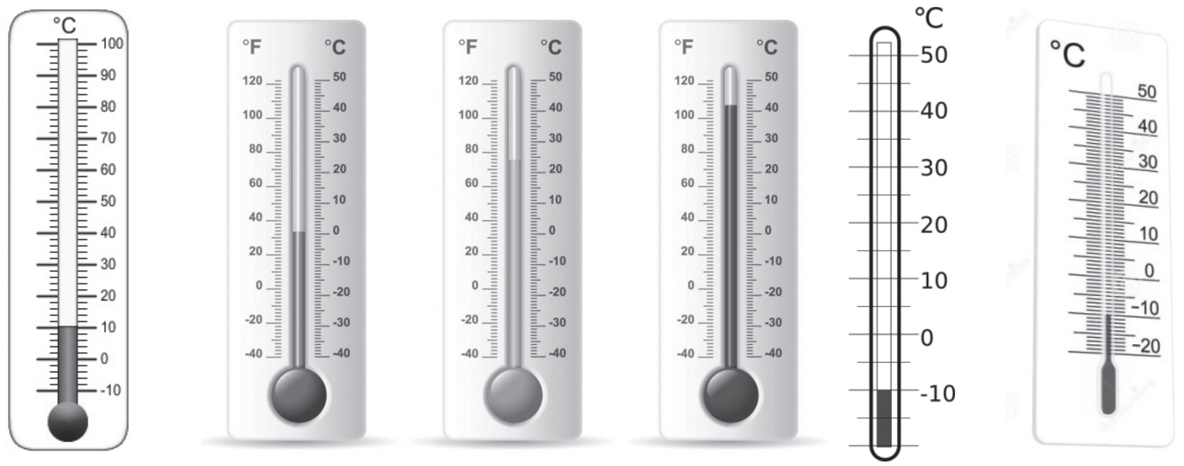
Feeling temperature

There are a few pointers that we can use to help us estimating temperature:

- The temperature of an ice cube in our hands is 0°C
- The water from a fridge is about 5°C
- When the temperature is below 15°C, would likely wear three layers of clothes
- When the temperature is between 15°C and 20°C, would likely wear two layers of clothes
- The temperature of your skin on the outside is about 35°C
- Our hands feel as if they are burning if the temperature of the water is about 50°C
- Food at a temperature of 75°C will not burn your mouth
- Food at a temperature of 85°C will burn your mouth

Some temperature practice

Do the readings on the thermometers below. (Some have a Fahrenheit scale next to the Celsius scale, which is a different measurement unit used in some countries.)



__________ __________ __________ __________ __________ __________

1. What is the difference between a maximum temperature of 24°C and minimum 8°C?

   Answer: 16°C

2. What is the difference between a maximum temperature of 13°C and minimum -2°C?

   Answer: 15°C

3. What is the difference between a maximum temperature of 0°C and minimum -8°C?

   Answer: 8°C

Note: the difference is given as 8 °C and not as -8°C

TOPIC 8: PERCENTAGES

INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area, 'Numbers, Operations and Relationships' which counts for 50% in the final exam.
- The unit introduces a new concept to learners. It covers a basic introduction to percentages as fractions of a whole number.
- Learners have previously worked with decimal and common fractions and this section completes the requirement for leaners to find equivalence between each of these similar concepts.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• In common fractions there is a special focus on tenths and hundredths to lay the basis for decimals and percentages.	• Find percentages of whole numbers
	• Recognize equivalence between
	  • common fraction and decimal fraction forms of the same number
	  • common fraction and percentage forms of the same number
	  • decimal fraction and percentage forms of the same number
	• Understand data expressed in percentages
		• Find percentages of whole numbers
		• Calculate the percentage of part of a whole
		• Calculate percentage increase or decrease of whole numbers
		• Solve problems in contexts involving percentages
		• Recognize equivalence between common fraction, decimal fraction and percentage forms of the same number
		• Solve problems that involve whole numbers, percentages and decimal fractions in financial contexts such as:
		• profit, loss and discount
		  • budgets
		  • accounts
		  • loans
		  • simple interest

GLOSSARY OF TERMS

Term	Explanation / Diagram
Fraction	**a.** A fraction is a part or parts of a whole that has been shared equally into a number of pieces. **b.** b. A fraction can also be a part of a number of things that have been divided into equal groups.
Decimal	A fraction whose denominator is a power of ten and which is expressed only by writing its numerator in digits to the right of a decimal comma following a whole number
Percentages	A number or rate that is expressed as a certain number of parts of something divided into 100 parts. The word can be broken down to realise where it comes from - 'per' means divide and 'cent' is part of the word century meaning 100.
Equivalence	When something has the same value as its counterpart.
Conversion	To change one format into another without changing the value, hence maintaining equivalence

SUMMARY OF KEY CONCEPTS

Revising Decimal Fractions with Two Decimal Places

The decimal number system uses a base of ten and ten digits (0 to 9) to indicate amounts. These digits are arranged in places where they are worth that number times any power of ten. To find the value of the whole numbers, the digit is multiplied by 10, 100, 1 000 etc. For decimal fractions of the whole numbers, the digit is divided by 10, 100, 1 000 etc.

Thousands ☐ x 1 000	Hundreds ☐ x 100	Tens ☐ x 10	Units ☐ x 1	tenths ☐ ÷ 10	hundredths ☐ ÷ 100
8	3	6	4,	9	7
= 8 x 1 000 = 8 000	= 3 x 100 = 300	= 6 x 10 = 60	= 4 x 1 = 4	= 9 ÷ 10 = $\frac{9}{10}$	= 7 ÷ 100 = $\frac{7}{100}$

Read: Eight thousand three hundred and sixty-four comma nine seven

Making the Transition from Common Fractions and Decimal Fractions to Percentage

Learners learned decimal fractions based on the understanding of common fractions. They learn percentages based on their understanding of common fractions and decimal fractions.

Teaching Tip:
It is important to keep in mind that the denominator in percentage is 100. This can be remembered as follows: Rands are the whole numbers and cents are the decimal fractions of rands when a rand has been divided in 100 parts. Cent has the same meaning of 100 as in "century", which is 100 years. Percentage is a fraction where only the numerator and the "percent" or "per cent" or "per 100" symbol (%) is written, without writing the denominator of 100.

Examples:

- Write 25% as a common fraction and as a decimal fraction
  ($25\% = \frac{25}{100} = 0,25$)
- Write 6% as a common fraction and as a decimal fraction
  ($6\% = \frac{6}{100} = 0,06$)
- Write $\frac{78}{100}$ as a decimal fraction and as percentage
  ($\frac{78}{100} = 0,78 = 78\%$)
- Write $\frac{3}{100}$ as a decimal fraction and as percentage
  ($\frac{3}{100} = 0,03 = 3\%$)
- Write 0,49 as a common fraction and as percentage
  ($0,49 = \frac{49}{100} = 49\%$)
- Write 0,08 as a common fraction and as percentage
  ($0,08 = \frac{8}{100} = 8\%$)

Finding a Percentage of a Whole Number

Again, learners understand percentage of whole numbers based on their understanding of fractions of whole numbers, only this time the fraction has 100 as its denominator.

Examples:

- How many is $\frac{2}{5}$ of 35 oranges?
  ($\frac{2}{5}$ of 35 oranges is 14 – we x 2 and ÷ 5)
- Express $\frac{2}{5}$ as a fraction with 100 as denominator and as percentage
  ($\frac{2}{5} = \frac{40}{100} = 40\%$)
- How many is 40% of 35 oranges?
  (40% of 35 oranges is 14 – we x 40 and ÷ 100)
- How many is 33% of 600?
  ($\frac{30}{100}$ of $600\,[\times 600$ and $\div 100]\ \therefore 33\%$ of $600 = 198$)
- What percentage is 45 apples of 180?
  ($\frac{45}{180} = \frac{1}{4} = \frac{25}{100} = 25\%$)

  * Another way to find the same answer is by multiplying the fraction by 100, like this: ($\frac{45}{180} \times \frac{100}{1} = \frac{4500}{180} = 25\%$)

How We Use Percentage in the Real World

- Zinzi has $\frac{12}{20}$ for her mathematics test. Her teacher writes 60% next to the mark out of 20. How did she calculate that?
  (Solution: $\frac{12}{20} \times \frac{100}{1} = \frac{1200}{20} = 60\%$)
- Thabo reads in the window of a shop: 30% OFF ON ALL MARKED PRICES. He likes a pair of shoes which is marked R400. How much less than R400 will he pay?
  (Solution: 30% of $R400 = \frac{30}{100} \times \frac{400}{1} = \frac{12000}{100} = R120$)
- The principal has to report to the DBE how many learners were absent each day. On Tuesday 2 May 2017, 5% of the learners were absent. There are 440 learners in the school. How many learners were absent?
  (Solution: 5% of 440 learners $= \frac{5}{100} \times \frac{440}{1} = \frac{2200}{100} = 22$ learners were absent)
- We understand data with our knowledge of percentage. Below is a table of language statistics in South Africa (note that the percentages are not always whole numbers):

Language name		Speakers as a 1st language	
English	**Endonym**	**Count**	**Of population**
Zulu	isiZulu	11,587,374	22.7%
Xhosa	isiXhosa	8,154,258	16.0%
Afrikaans	Afrikaans	6,855,082	13.5%
English	English	4,892,623	9.6%
Northern Sotho	Sesotho sa Leboa	4,618,576	9.1%
Tswana	Setswana	4,067,248	8.0%
Sesotho	Sesotho	3,849,563	7.6%
Tsonga	Xitsonga	2,277,148	4.5%
Swati	siSwati	1,297,046	2.5%
Venda	Tshivenda	1,209,388	2.4%
Ndebele	isiNdebele	1,090,223	2.1%
SA Sign Language		234,655	0.5%
Other languages		828,258	1.6%
Total		**50,961,443**	**100.0%**

A table similar to this can be discussed at length with learners asking them questions to check their understanding of percentages as well as allow them to practice reading information from tables. This skill is useful in the following topic, Data Handling.

TOPIC 9: DATA HANDLING

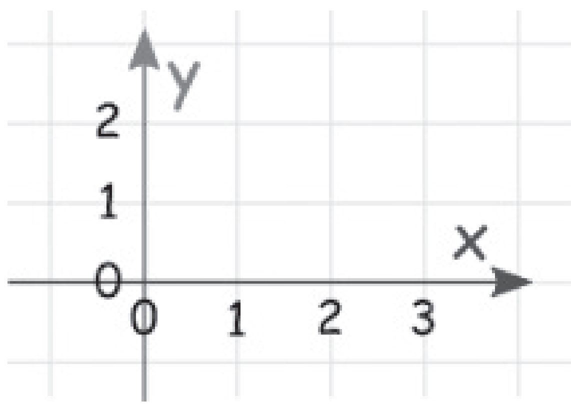
INTRODUCTION

- This unit runs for 9 hours.
- It is part of the Content Area, 'Data Handling' which counts for 10% in the final exam.
- The unit covers the Collecting, Organising, Representing, Analysing, Interpreting and Reporting of data.
- This topic is the recommended project topic in Grade 6.
- Learners have previously mastered some of the basic skills required in this topic, however in Grade 6 these skills are extended to include: data expressed in percentages, data collection through questionnaires, double bar graphs, pie charts and finding the mode and median of a data set.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Collect and record data through tally-marks • Represent data in words, bar graphs, pictograms (one-to-one correspondence) and pie charts • Answer questions related to data categories • Critically read and interpret data • Summarise data verbally and written	• Collect and arrange data from smallest to largest • Represent data in words, pictograms (many-to-one correspondence) and bar graphs • Answer questions related to data categories, sources and contexts • Read and interpret data • Summarise data verbally and written, draw conclusions and make predictions • Determine the mode and median of the data set	• Collect data by identifying appropriate sources and with questionnaires • Organize and summarise data, using tallies, tables and stem-and-leaf displays • Distinguish between samples and populations • Group data in intervals • Represent data in single and double bar graphs, histograms and pie charts • Interpret data, draw conclusions, make predictions and identify sources of error and bias • Analyse and critique data ito scales, categories, intervals, sources, contexts, central tendencies and dispersion • Determine the range, mean, mode and median of the data set • Make inferences about probability

GLOSSARY OF TERMS

Term	Explanation / Diagram
Investigate	To collect data based on a question or several questions in order to draw a conclusion
Compare	To look at the data critically especially when collected from different sources and looking for common factors as well as differences
Questionnaire	A set of questions used in the investigation that delivered the data to be used in the data cycle
Population	The whole group from which a sample is taken. Example: you ask 100 randomly chosen people at a football match what their main job is. Your sample is 100, but the **population** is all the people at that match, which may be 12 549.
Sample	A selection taken from a larger group (the "population") so that you can examine it to find out something about the larger group. Example: you ask 100 randomly chosen people at a football match what their main job is. Your sample is the 100, while the population is all the people at that match, which may be 12 549.
Bias	When certain groups are excluded in the collection of data so that the results are manipulated to deliver a desired result. Example: In a survey at school about the preference for pink and green Fizzers the investigator only surveyed girls.
x-axis and y-axis	The reference lines drawn on a graph from which you can find values. The x-axis runs horizontally and the y-axis runs vertically. 
Mode	This is the number that appears the most in the data set.
Mean	This is the average – this is the sum of the data divided by the number of data.
Median	This is the number in the middle, if data is arranged from lowest to highest / highest to lowest.

Term	Explanation / Diagram
Range	This is the difference between the largest and the smallest numbers of the data.
Tally	This is an easy way to add up data when you are collecting information from your data source.  5 The 4 lines represent 4 items counted and the 5th is a cross through the 4.
Bar graph	A bar graph is a way of showing data that uses horizontal or vertical rectangular bars.
Double Bar Graph	A bar graph where more than one data set is being compared.
Pie graph	A pie chart or pie graph is a circular chart divided into sectors, each sector showing the relative size of each value.
Pictographs **One-to-one pictograph** **Many-to-one pictograph**	A pictograph uses pictures or symbols to represent an amount of data. The key for a pictograph tells the number that each picture or symbol represents, for example Each picture of an animal represents one animal Each picture of an animal represents ten animals
Data	A collection of facts, such as numbers, words, measurements, observations or even just descriptions of things. A data set is a collection of numbers or values that relate to a particular subject. For example, the test scores of each student in a particular class is a data set.

SUMMARY OF KEY CONCEPTS

Complete a data cycle using personal context

1. The complete data cycle includes posing a question, collecting, organising, representing, analysing, interpreting data and reporting on the data.
2. Learners work through the whole data cycle using contexts that relate to themselves, their class, their school or their family.

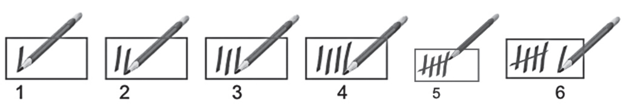
Collecting and ordering data

1. Data Collection
   Data can be collected in different ways. The simplest way is observation.
2. **Example:**
   You would like to know how many children are buying bread and milk from the local shop.

   Stand at the door of the local shop and look at the number of children leaving with bread and milk in their packets.
3. Data can be collected by doing surveys.

   To do this:
   Survey people (through questionnaires, opinion polls, etc.) or observe phenomena (like pollution levels in a river, or favourite colours).
4. Here are four steps to conduct a survey through a questionnaire:
   - Draw up the questions
   - Ask the questions
   - Tally the results
   - Present the results

5. Data can be recorded by using tally marks.
   This is how you record the items:



tally 6, 7, 8 and 9, just when you get to 10 then you mark the 6-9 off with a line through like you did for 5.

6. When doing a simple survey, tally each person's answers:

Yellow	||||	4
Red	卌	5
Blue	卌 |	6
Green	|	1
Pink	||||	4

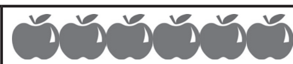
Order the data for your own purpose, either from smallest to largest or from largest to smallest. So blue has the most with 6, then red with 5, followed by pink and yellow with 4 each and lastly green with 1.

Types of graphs

1. Pictograph: many-to-one correspondence
   This type of graph uses images to represent what is being compared. This is a many-to-one correspondence as one picture of an apple represents 10 apples.

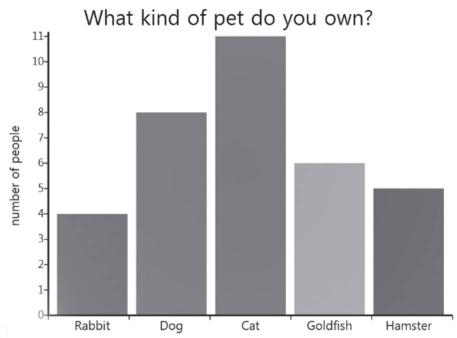
APPLES PICKED AT JOZI FARM

Month  = 10 apples

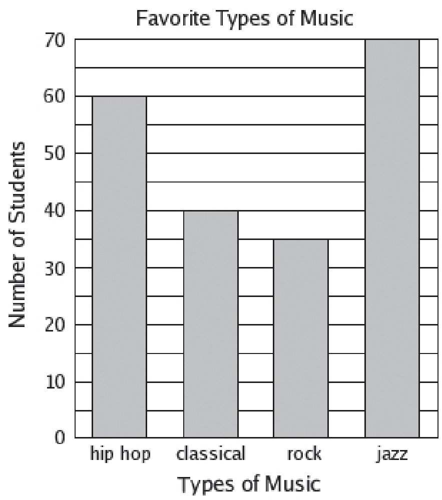
January	
February	
March	
April	

This needs to be discussed with learners. Many learners may make the mistake of not taking note of the key which states that one apple represents 10 apples.

2. Bar graph



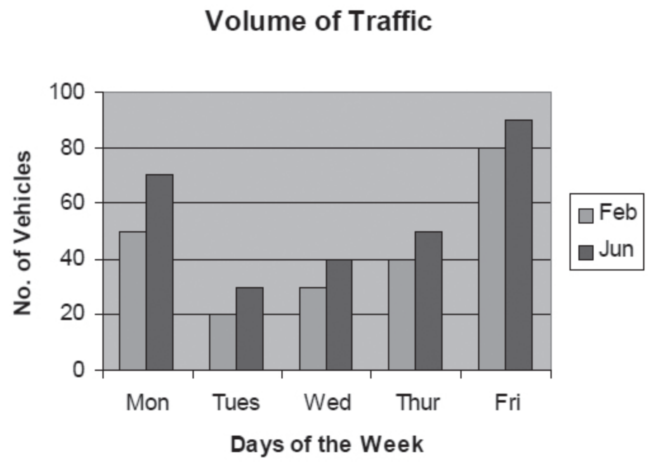
This is a single bar graph and only looks at a single set of data. The survey was only about the type of pet, but if one was doing the survey about two aspects, like whether the specific type of pet is male or female, one would have two sets of data in each category. There would then be male rabbits and female rabbits in the first category, male dogs and female dogs in the second category, and so on.



To prepare the plane for graphing data on a single bar graph, one needs to do the following:

- Decide on a suitable heading that describes which data is displayed in the graph and write it on top of the graph plane.
- Give a space on the x-axis for each category that was investigated, in this case four types of music. Write down the category for each space below the x-axis.
- Look at the maximum number that you want to graph, in this case 70, and see how you can sub-divide your available vertical space to accommodate that.
- Decide on the intervals, in this case two lines are used for 10 outcomes. This graph is numbered in intervals of ten. Write the number values to the left of the y-axis.

The bar graph below is known as a double or compound bar graph:



It represents represents a comparison of two sets of data from two months. The data is compared directly to determine a trend and this makes it easier for learners to analyse the data by comparison.

To prepare the plane for graphing data on a double bar graph, one needs to do the following:

- Decide on a suitable **heading** that describes which data is displayed in the graph and write it on top of the graph plane.
- Give a double space on the x-axis for each **category** that was investigated, with its two sub-categories, in this case Monday in February and Monday in June; Tuesday in February and Tuesday in June, and so on. Write down the category for each space below the x-axis and create a key on the side, showing what shade the first and second bar will be.
- Look at the **maximum number** that you want to graph, in this case 90, and see how you can sub-divide your available vertical space to accommodate that.
- Decide on the **intervals**, in this case one line is used for 20 vehicles. This graph is numbered in intervals of twenty. Write the number values to the left of the y-axis.

3. Pie chart, circle diagram or pie graph:
Below is a data set of the total number of learners per grade who ran 5 000 m from 2010-2015. We want to set up a pie chart or a pie graph to display this data. Firstly, we have to calculate the total for each grade.

• 2010:	Gr. 4: 2;	Gr. 5: 5;	Gr. 6: 8;	Gr. 7: 12	total: 27
• 2011:	Gr. 4: 3;	Gr. 5: 7;	Gr. 6: 9;	Gr. 7: 13	total: 32
• 2012:	Gr. 4: 4;	Gr. 5: 8;	Gr. 6: 11;	Gr. 7: 16	total: 39
• 2013:	Gr. 4: 4;	Gr. 5: 8;	Gr. 6: 12;	Gr. 7: 16	total: 40
• 2014:	Gr. 4: 5;	Gr. 5: 10;	Gr. 6: 15;	Gr. 7: 18	total: 48
• 2015:	Gr. 4: 6;	Gr. 5: 10;	Gr. 6: 17;	Gr. 7: 21	total: 54
Total	_______	_______	_______	_______	______

We can display the data in the form of a circle that is cut up in sectors, called a pie chart, a pie graph or a circle diagram. A pie graph is useful to answer questions like the following:

- "Of all the learners who ran the 5 000 m race, what portion of the learners were in Grade 4??"
- "Of all the learners who ran the 5 000 m race, what portion of the learners were in Grade 5?"
- "Of all the learners who ran the 5 000 m race, what portion of the learners were in Grade 6?"
- "Of all the learners who ran the 5 000 m race, what portion of the learners were in Grade 7?"

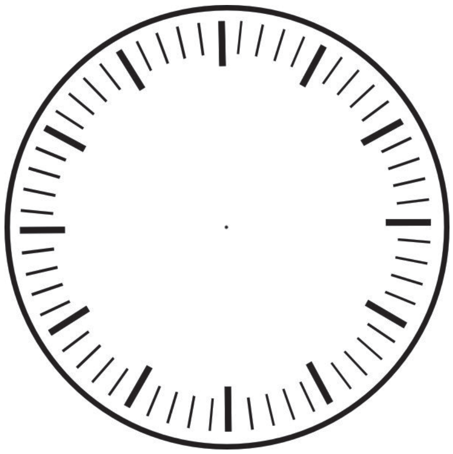
The total number of learners who ran the race, was 240. We can say:

- Grade 4: 24 out of the 240, which is a fraction (portion) of 24/240 or 1/10 (simplified).
- Grade 5: 48 out of the 240, which is a fraction (portion) of 48/240 or 2/10 (simplified).
- Grade 6: 72 out of the 240, which is a fraction (portion) of 72/240 or 3/10 (simplified).
- Grade 7: 96 out of the 240, which is a fraction (portion) of 96/240 or 4/10 (simplified).

If we divide the circle in 10 portions, and colour the portions differently,

- The Grade 4's will get one portion or 1/10,
- The Grade 5's will get two portions or 2/10,
- The Grade 6's will get three portions or 3/10 and
- The Grade 7's will get four portions or 4/10.

Teaching tip:
An easy way to understand the division of a circle in 10 portions, is with a clock which is divided in 60 minutes. That means that there are ten times six portions marked on a clock. Every 6 minutes is one tenth of the circle, like in the picture below.



Making Predictions Based on Data

When one has a data set like the one above, we can see how things went from 2010 to 2015. The number of runners went up from 27, to 32, to 39, to 40, 48, to 54. There was not one year that the numbers went down. If we think of the future, we have no reason to think that the number in 2016 will be fewer. We have reason to think it can still be more. That is to make a reasonable statistical prediction. A good guess would be to say that we think the number can be 60 or more in 2016.

Answering Questions about the Data Source

In data and statistics, it is always important to ask where data comes from, or what the source of the data is. Some sources of data can be trusted more than other sources. Rate the following sources of data for the data set that we are working on, by saying which data source you trust most and which data source you trust least:

a. There was a camera up at the Finish of the race for the past four years from 2012 to 2016 and all learners who finished, can be seen on the video recordings.
b. The sports coach did not keep a written record of the number of runners, but he was sure that he could remember how many learners ran the race every year. This is how he recorded the numbers on the list.
c. Each year from 2010 to 2015, the sports coach used the same type of table to tally the number of runners for each grade that ran the big annual race.
d. The sports coach asked the Grade 7 learners to tell him about how many learners ran the race each year as far as they could remember.

Ordering a data set

The following is a data set of the Grade 6 mathematics marks of last year. There were 40 learners in the class, and here are their marks in percentage:
68%; 42%; 72%; 36%; 58%; 16%; 92%; 55%; 81%; 78%; 58%; 66%; 36%; 59%; 37%; 64%; 53%; 90%; 72%; 82%; 58%; 54%; 33%; 49%; 51%; 29%; 40%; 66%; 45%; 54%; 78%; 74%; 44%; 49%; 58%; 52%; 59%; 63%; 27%; 85%
For quite a few purposes in data handling, we need to order a list of numbers before we can work with the data properly. We usually order a data set by writing all the numbers from the lowest to the highest, or in ascending order.
Teaching tip: Cross out the numbers as you write them down, to make sure you do not write a number twice, or skip a number.

_____ _____ _____ _____ ____ _____ _____ _____ _____ _____ _____
_____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____
_____ _____ _____ _____ _____ _____ _____ _____ _____ _____ _____
_____ _____ _____ _____ _____ _____ _____

Finding the Mode and the Median of a Data Set

1. The mode is the number that appears most in the data set. If there is no number that appears more than once, the data set has no mode. If there is more than one number that appears the same number of times (the most) then the data set has more than one mode. Find the mode of the above data set.

   (Solution: 58 appears the most, four times, in the data set. The mode is 58)
2. The median is the middlemost value in an ordered data set.
   a. A set with an odd number of values has a middlemost value, like in the following example of a data set: 2; 3; 5; 6; 8; 10; 11; 11; 13. In this data set we can read off the median, which is 8.
   b. A data set with an even number of values does not have one middlemost value, therefore we have to look at the two values in the middle, like in this example of a data set: 2; 3; 5; 6; 8; 10; 11; 11; 13; 14. In this data set we have to calculate the median by adding the two middle values and dividing the sum by 2. The median of this data set is (8 + 10) ÷ 2 = 9. The median of the above data set is (58 + 58) ÷ 2 = 58

Developing Critical analysis skills

1. Learners compare graphs on the same topic but where data has been collected from different groups of people, at different times, in different places or in different ways.
2. Here learners will be able to discuss the differences between the graphs.
3. The aim is also for learners to become aware of factors that can impact on the data.

Teaching Tip:

This topic is best consolidated as a project that is continuously grown during class until the final product and analysis is completed. Learners will best understand the various components and their interrelated structure by seeing the process run its course.

TOPIC 10: NUMERIC PATTERNS

INTRODUCTION

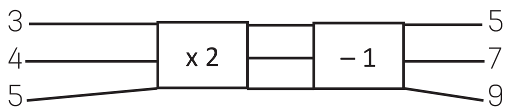
- This unit runs for 5 hour.
- It is part of the Content Area, 'Patterns, Functions and Algebra' which counts for 10% in the final exam.
- The unit covers the use of flow diagrams and with a deeper focus on finding the rule than in Term 1.
- This topic lays the foundation for a clear and definite understanding of algebra in the senior and FET phase and expands the learners' knowledge base from merely substitution to determining a working rule that connects input and output values.

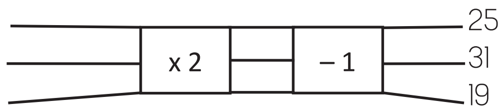
SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Investigate and extend numeric patterns looking for relationships/rules
• Find constant difference in a numeric pattern
• Find a constant ratio in a numeric pattern
• Describe relationships in words, diagrams, tables
• Complete flow diagrams with two actions or a double rule
• Understand that the input value is derived from the inverse operations than those of the rule
	• Investigate and extend numeric patterns looking for relationships and rules
	• Find a constant difference in a numeric pattern
	• Find a constant ratio in a numeric pattern
	• Describe relationships in words, diagrams and tables
	• Describe rules in general terms
	• Complete flow diagrams with two actions or a double rule
	• Understand that the input value is derived from the inverse operations than those of the rule
		• Investigate and extend numeric patterns looking for relationships between numbers
		• Find rules for patterns not limited to sequences involving a constant difference or ratio
		• Describe and justify the general rules for observed relationships between numbers in own words
		• Determine input and output values or rules for patterns using:
		  ◦ flow diagrams
		  ◦ tables
		  ◦ formulae
		• Determine, interpret and justify equivalence of different descriptions of the same relationship or rule presented:
		  ◦ verbally
		  ◦ in flow diagrams
		  ◦ in tables
		  ◦ by formulae
		  ◦ by number sentences

GLOSSARY OF TERMS

<table>
<tr><th>Term</th><th>Explanation/diagram</th></tr>
<tr><td>Number pattern</td><td>A number pattern is a list of numbers that follow a certain pattern.
Example: 2, 5, 8, 11, 14, 17, … is a number pattern. It starts at 2 and increases by 3 for each consecutive term.</td></tr>
<tr><td>Sequence</td><td>A sequence is an ordered list of numbers which form a pattern. We can call a number pattern a sequence.</td></tr>
<tr><td>Term</td><td>Each number in a sequence has its own position. Number 8 in the above sequence is in the third position.</td></tr>
<tr><td>Input value</td><td>In a functional number sequence we start with any number, which is the input value and when we have operated on that number, we find the output value (See the flow diagram below).</td></tr>
<tr><td>Rule</td><td>The rule of the pattern is a description of the relationship between the numbers in a sequence. We can describe this relationship in words, in a table or in a flow diagram.</td></tr>
<tr><td>Output value</td><td>When we look at how two numbers relate to each other, the output value is the answer that we calculate when we apply the rule to the input value. The output value depends on which input number we apply the rule to. (See the flow diagram below).</td></tr>
<tr><td>Flow diagram</td><td>A flow diagram is a visual (and a shorter) way to write a number pattern, with the input values to the left, the rule in the middle and the output values to the right.
Input value Rule Output value

</td></tr>
<tr><td>Inverse operations</td><td>Inverse operations are opposite operations that undo each other. From the output value, we can get the input value by doing the inverse operations of the rule (from right to left in the flow diagram).

Input value Rule Output value


25 + 1 → 26 ÷ 2 → 13
31 + 1 → 32 ÷ 2 → 16
19 + 1 → 20 ÷ 2 → 10</td></tr>
<tr><td>Table for a number pattern</td><td>A way of representing a number pattern is in a table, as follows: Example: 2; 5; 8; 11;…
Rule: Starting at two, three is added to each following number. The term (x) is the input number and the value (y) is the output number.

<table><tr><td>Term (x)</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Value (y)</td><td>2</td><td>5</td><td>8</td><td>11</td></tr></table></td></tr>
</table>

SUMMARY OF KEY CONCEPTS

Introduction

In a number pattern we look at how one number in the sequence relates to the next number in the sequence. Our task is to investigate this relationship and to formulate a rule for the relationship. A number pattern always has a rule according to which its terms are formed.

Example
2, 5, 8, 11, 14… (starting at two, each following number increases by three)
Each number in a sequence has its own position. Number 8 in this sequence is in the third position and is called Term 3.

Describing Different Types of Number Patterns in Words

The questions to ask in understanding a number pattern, are:

- "Do I add the same number each time?" (Ascending: constant difference)
- "Do I subtract the same number each time?" (Descending: constant difference)
- "Do I multiply by the same whole number each time?" (Ascending: constant ratio)
- "Do I divide by the same whole number each time?" (Descending: constant ratio)
- Do none of the above apply to the pattern?

1. Number patterns with a constant difference
   a. Ascending patterns formed through addition:

      Example:
      4; 7; 10;...
      "The pattern starts at 4 and becomes three more each time"

   b. Descending patterns formed through subtraction:

      Example:
      84; 81; 78;...
      "The pattern starts at 84 and becomes three less each time"

2. Number patterns with a constant ratio

   a. Ascending patterns formed through multiplication:

      Example:
      3; 6; 9;...
      "The pattern starts at 3 and becomes 3 more each time"

   b. Descending patterns formed through division:

      Example:
      144; 72; 36;...
      "The pattern starts at 144 and becomes 2 times less each time"

3. Number patterns with neither a constant difference, nor a constant ratio

   Example:
   4; 5; 8; 13; 20; ... "The pattern starts at 4 and becomes 1 more the first time, 3 more the second time, 5 more the third time. Each time the next odd number is added."

   Example:
   2; 3; 5; 7; 11; 13; 17;... "All the numbers in the pattern are prime numbers."

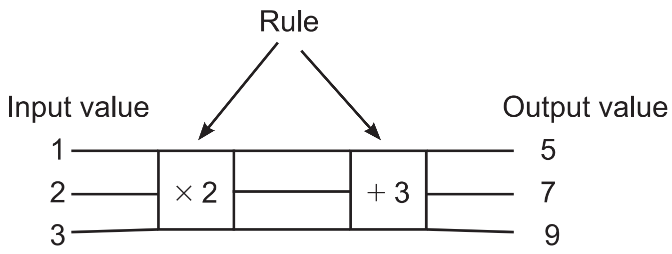
   Example:
   1; 1; 2; 3; 5; 8; 13;... "The first and second number are added to form the third number; the second and third number are added to form the fourth number... and so on."

Presenting a Pattern in a Flow Diagram

A flow diagram is a visual way to write a number pattern, with the input values to the left, the rule in the middle and the output values to the right.

Example:
If the input numbers of a pattern are 1, 2 and 3 and the rule is multiply by 2 and add 3, represent this in a flow diagram then fill in the output numbers.

Solution:



Tabling a number pattern

A way of representing a number pattern is in a table, as follows:
Example: Present the number pattern in a table and extend it to Term 5: 2; 5; 8; 11...

Term	1	2	3	4	5		
The value of the term in the number pattern	2	5	8	11			

Finding the General Rule for an Ascending Pattern with a Constant Difference

This is probably the most difficult concept for learners to do. The following steps may be helpful:

a. Study the pattern by looking at the starting number and considering the constant difference between terms. Say what you see in words:

   Example:
   2, 5, 8, 11, 14… "Starting at two, each following number increases by three"

b. The constant difference (+ 3) helps us to find the first part of the rule, in this case x 3. We will need to multiply 1 by three to link to the first term and 2 by three to link to the 2nd term and so on.

c. See how much the constant difference differs from the value of the first term. In this case the value of the first term is one less than the constant difference, which helps us to find the second part of the rule, – 1.

d. Apply this rule to all the terms to test it:

   1 x 3 – 1 = 2; 2 x 3 – 1 = 5; 3 x 3 – 1 = 8... etc

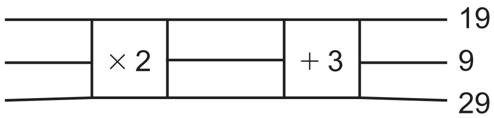
In Grade 6 learners describe the general rule (Tn, where n represents any term) for the pattern in the following way: Using the example above, Tn = n x 3 – 1.

If learners can master finding the rule for this type of number pattern in the above way, they may try to find the rule for other types of patterns through trial and improvement.

Inverse operations

If we have the output value and the rule of a number pattern, we can get the input number by doing the inverse operations of the rule (from right to left in the flow diagram).

Example:



Solution:

- 19 – 3 ⟶ 16 ÷ 2 ⟶ 8
- 9 – 3 ⟶ 6 ÷ 2 ⟶ 3
- 29 – 3 ⟶ 26 ÷ 2 ⟶ 13

TOPIC 11: LENGTH

INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area, 'Measurement' which counts for 15% in the final exam.
- The unit covers the measuring and recording of lengths using various instruments. The change from Grade 5 to 6 is that decimals are introduced.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Measure 2D shapes and 3D objects formally in standard units of length • Estimate, compare, order and record formal measurements • Use and discern standard units of length: millimetre (mm), centimetre (cm), metre (m) and kilometre (km) • Use measuring instruments rulers, metre sticks, tape measures, trundle wheels • Solve problems in context involving length • Convert between mm and cm; between cm and m; and m and km, including fractions of units	• Measure 2D shapes and 3D objects formally in standard units of length • Estimate, compare, order and record formal measurements • Use and discern standard units of length: millimetre (mm), centimetre (cm), metre (m) and kilometre (km) • Use measuring instruments rulers, metre sticks, tape measures, trundle wheels • Solve problems in context involving length • Convert between any units of length including mm, cm, m and km, including fractions and decimal fractions of units	• Length is not covered as a topic on its own in the Senior Phase, however it is used amongst others, in perimeter and area measurement and calculations.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Length	A one-dimensional measurement along a line which indicates the distance between two points.
Measuring Instruments	A device or a system that is used to measure a physical property, in this case, length. The instrument is usually calibrated or marked in intervals of standard units, in this case units of length.
Trundle Wheel	The trundle wheel is a measuring device for length. If the circumference of a trundle wheel is one metre, it measures one metre in one turn or rotations. If it only makes half a rotation, it has measured 50 cm. It is an easy way to find a rough distance and is often used to measure out sports fields or tracks. 
Odometer	An instrument in a car that measures the distance that the car travels.
Conversion	Changing a unit of measurement to a different but equal unit of measurement. Example: 1 cm = 10 mm 100 cm = 1 m
Estimate	Judging something (length in this case) without measuring or calculating it. Estimation is based on knowledge and experience about that which is estimated.
Standard unit of length	A single standard distance from one point to another, that is used the same across the world and bears a specific name. The metric length unit is metre and this standard length is multiplied by powers of ten or divided by powers of ten to get longer and shorter standard units of length: 100 metres = 1 kilometre $\frac{1}{100}$ metre = 1 centimetre $\frac{1}{1000}$ metre = 1 millimetre

SUMMARY OF KEY CONCEPTS

Measurement Facts to Know

1. Metre is our standard unit for measuring length. We use the letter 'm', meaning metre. The prefixes k (kilo-), c (centi-) and m (milli-) are used to show multiples or fractions of the standard unit.
   a. When a metre is divided in a hundred parts, the small parts are called centimetres (cm). One metre is the same length as hundred centimetres (1 m = 100 cm).
   b. When a metre is divided in a thousand parts, the small parts are called millimetres (mm). One metre is the same length as thousand millimetres (1 m = 1 000 mm).
   c. When a metre is multiplied by a thousand, the large length is called a kilometre (km). One kilometre is the same length as thousand metres (1 km = 1 000 m).
   d. When a centimetre is divided in ten parts, the small parts are called millimetres (mm). One centimetre is the same length as ten millimetres (1 cm = 10 mm).

We use a metric measuring system for length, just as we use for numbers:

Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths
1 000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
kilometre	hectometre	decametre	metre	decimetre	centimetre	millimetre
m x 1000			m		m ÷ 100	m ÷ 1000

Comparing lengths

In Grade 6 learners measure in standard units of length and convert these lengths to the same unit to be able to compare them.

Example:
Measure the circumference of the wheels of five bicycles at school with a string or a measuring tape in centimetres. Round off the circumferences to the nearest cm. Record the exact circumferences and the rounded numbers. Convert the exact circumference to metres including decimals to two places.

Number	Wheel circumference in cm	Rounded to nearest cm	Converted to m
Example	187.5 cm	188 cm	1.88 m
1			
2			
3			
4			
5			

Estimate length

We can use some handy approximates in everyday life to make better judgements of length:

a. The width of one's pinky finger is approximately one centimetre

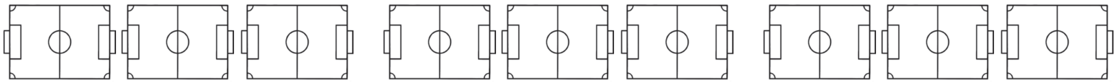
b. A pencil line is approximately one millimetre wide

c. An exercise book is approximately 20 cm wide and 30 cm long

d. The width of five exercise books next to each other is approximately one metre (about 5 cm more than a metre)

e. Learners can innovate their own approximates, compare the length of a pen (14 cm) with a new pencil (18 cm)

f. A soccer field is approximately 110 meters long. Nine soccer fields next to each other make approximately a kilometre.



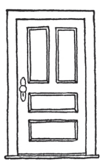
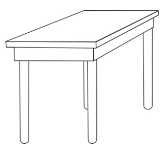
g. They can put seven Bic pens tip to end to make approximately one metre, etc.

Measurement Instruments

Learners must choose the correct instruments for the measuring of given lengths, from a ruler, a tape measure, a metre stick, a trundle wheel and the odometer of a car. Their most available instrument is the ruler and we have to make much out of the ruler.

Example:

Combined Assignment: Measuring Length in Real Life

Object	Estimate	Instrument	Record	Conversion
Example: Width of a chair 	50 cm	Ruler	50 cm	50 cm = 0,5 m
Height of your classroom door 	in cm			cm to m
Perimeter (length around) a table 	in m			m to cm

Topic 11 Length

Calculating and solving problems with units of length

1. Add 750 cm, 1,2 m and 2 231 cm and give the answer using metre including decimals.

   Step 1: Convert all lengths to the same unit
   Step 2: Use any addition strategy to add the length
   Step 3: Convert back to metres including decimals

   (Solution: 750 cm + 1 200 cm + 2 231 cm = 4 181 cm = 41,81 m)

2. Subtract 12 500 m from 21km and give answer in km including decimals.

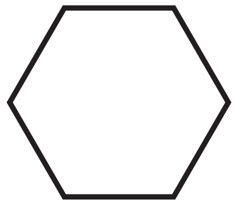
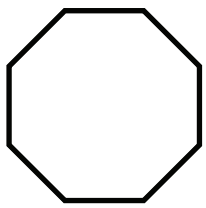
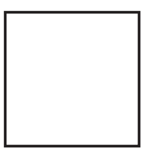
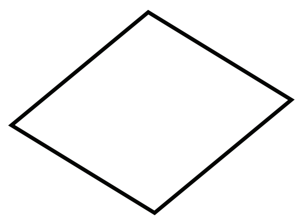
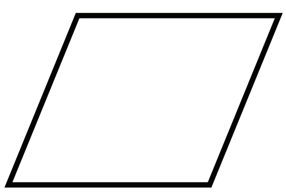
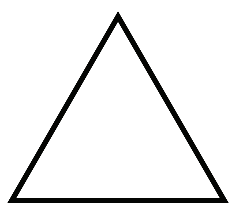
   Step 1: Convert all lengths to the same unit
   Step 2: Use any subtraction strategy to find the difference in length
   Step 3: Convert back to kilometre including decimals

   (Solution: 21 000 m – 12 500 m = 8 500 m = 8,5 km)

3. Beauty has four pieces of red material from which she has to make bandanas for a sports day. She has 12,54 m; 2,9 m; 1 850 cm and 923 cm. She needs 48m of material to make all the bandanas. How much more material does she need? Give your answer in metres including decimals.

   (Solution: 1 254 cm + 290 cm + 1 850 cm + 923 cm = 4 317 cm = 43,17 m; 48 – 43,17 = 4,83 m)

EDUCATOR RESOURCES

Shape	Name	Sides?	Angles?
	opposite sides are equal & parallel	6 equal sides	2 pairs of parallel & equal sides
4 equal angles	Square		4 equal angles
3 equal angles	Equilateral Triangle	3 equal sides	Rectangle
2 pairs of equal angles	4 equal sides & 2 pairs of parallel sides		2 pairs of equal angles
	4 equal sides & 2 pairs of parallel sides	Parallelogram	
Regular Octagon	Regular Hexagon	Rhombus	6 equal angles
8 equal sides		8 equal angles	

Notes

Notes

Notes