Grade 8

MATHEMATICS CONTENT BOOKLET: TARGETED SUPPORT

Term 3

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers,
This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?
In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education and to help the DBE reach the NDP goals.

The NECT has successfully brought together groups of relevant people so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?
One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers. The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this embedding process.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

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INTRODUCTION:
THREE PRINCIPLES OF TEACHING MATHEMATICS

PRINCIPLE 1: TEACHING MATHEMATICS DEVELOPMENTALLY

What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which determine whether children are ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract, comes from Piaget (1969). We adopted a refined version of that idea though, which works very well for mathematics teaching, namely a "concrete-representational-abstract" classification (Miller & Mercer, 1993).
- It is not possible in all cases or for all topics to follow the "concrete-representational-abstract" sequence exactly, but at the primary level it is possible in many topics and is especially valuable in establishing new concepts.
- This classification gives a tool in the hands of the teacher to develop children's mathematical thinking but also to fall back to a previous phase if it is clear that the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phase.

How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- Where applicable, the initial concrete way of teaching and learning the concept is suggested and educational resources provided at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- In most cases pictures (semi-concrete) and/or diagrams (semi-abstract) are provided, either at the clarification of terminology section, within the topic or lesson plan itself or at the end of the lesson plan or topic as an educational resource.
- In all cases the symbolic (abstract) way of teaching and learning the concept, is provided, since this is, developmentally speaking, where we primarily aim to be when learners master mathematics.

PRINCIPLE 2: TEACHING MATHEMATICS MULTI-MODALLY

What is multi-modal teaching and what are the benefits of such an approach?

- We suggest a rhythm of teaching any mathematical topic by way of “saying it, showing it and symbolising it”. This approach can be called multi-modal teaching and links in a significant way to the developmental phases above.
- Multi-modal teaching includes speaking about a matter verbally (auditory mode), showing it in a picture or a diagram (visual mode) and writing it in words or numbers (symbolic mode).
- For multi-modal teaching, the same learning material is presented in various modalities: by an explanation using spoken words (auditory), by showing pictures or diagrams (visual) and by writing words and numbers (symbolic).
- Modal preferences amongst learners vary significantly and learning takes place more successfully when they receive, study and present their learning in the mode of their preference, either auditory, visually or symbolically. Although individual learners prefer one mode above another, the exposure to all three of these modes enhance their learning.

How can this approach be implemented practically?

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What does this look like in the booklet?

Throughout the booklets, within the lesson plans for the Foundation Phase and within the topics at the Intermediate/Senior Phase, suggestions are made to implement this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the “say it” or auditory mode.
- The pictures and diagrams provide suggestions for the “show it” mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the “symbol it” or symbolic mode of representation.

PRINCIPLE 3: SEQUENTIAL TEACHING

What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths the approach to teaching needs to be systematic. Teaching concepts out of sequence can lead to difficulties in grasping concepts.
- Teaching in a systematic way (according to CAPS) allows learners to progressively build understandings, skills and confidence.
- A learner needs to be confident in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- Ongoing review and reinforcement of previously learned skills and concepts is of utmost importance.
- Giving learners good reasons for why we learn a topic and linking it to previous knowledge can help to remove barriers that stop a child from learning.
- Similarly, making an effort to explain where anything taught may be used in the future is also beneficial to the learning process.

How can this approach be implemented practically?

If there are a few learners in your class who are not grasping a concept, as a teacher, you need to find the time to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again. This could cause difficulties when trying to keep on track and complete the curriculum in the time stipulated.

Some topics have a more generous time allocation in order to incorporate investigative work by the learners themselves. Although this is an excellent way to assist learners to form a deeper understanding of a concept, it could also be an opportunity to catch up on any time missed due to remediating and re-teaching of a previous topic. With careful planning, it should be possible to finish the year's work as required.

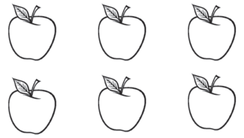
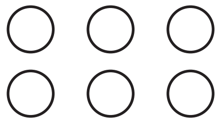
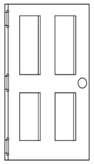
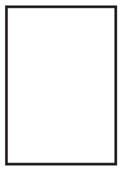
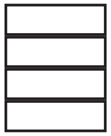
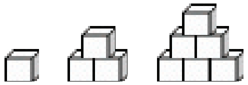
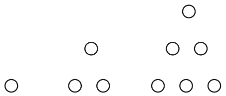
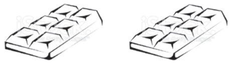
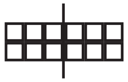
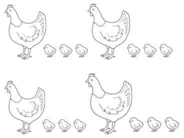
Another way to try and save some time when preparing for a new topic, is to give out some revision work to learners prior to the start of the topic. They could be required to do this over the course of a week or two leading up to the start of the new topic. For example, in Grade 8, while you are teaching the Theorem of Pythagoras, the learners could be given a homework worksheet on Area and Perimeter at Grade 7 level. This will allow them to revise the skills that are required for the Grade 8 approach to the topic.

What does this look like in the booklet?

At the beginning of each topic, there will be a SEQUENTIAL TEACHING TABLE, that details:

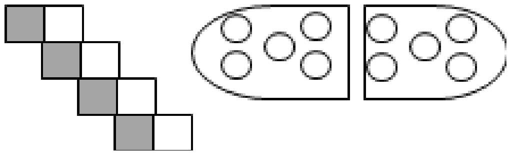
- The knowledge and skills that will be covered in this grade
- The relevant knowledge and skills that were covered in the previous grade or phase (looking back)
- The future knowledge and skills that will be developed in the next grade or phase (looking forward)

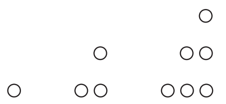
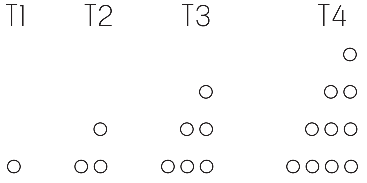
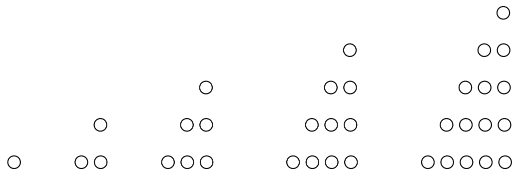
THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

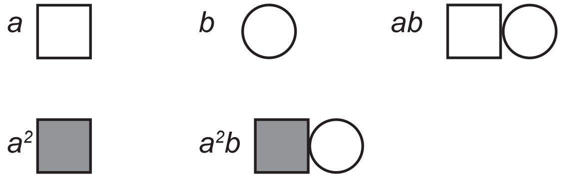
CONCRETE: IT IS THE REAL THING		REPRESENTATIONAL: IT LOOKS LIKE THE REAL THING		ABSTRACT: IT IS A SYMBOL FOR THE REAL THING	
Mathematical topic	Real or physical For example:	Picture	Diagram	Number (with or without unit)	Calculation or operation, general form, rule, formulae
Counting	Physical objects like apples that can be held and moved			6 apples	$2 \times 3 = 6$ or $2+2+2=6$ or $\frac{1}{2}$ of $6 = 3$ or $\frac{2}{3}$ of $6 = 4$
Length or distance	The door of the classroom that can be measured physically			80 cm wide 195 cm high	Perimeter: $2L + 2W = 390 + 160$ $= 550cm$ Area: $L \times W = 195 \times 80$ $= 15\,600cm^2$ $= 1.56m^2$
Capacity	A box with milk that can be poured into glasses			1 litre box 250 ml glass	$4 \times 250ml = 1\,000ml$ $= 1$ litre or 1 litre $\div$ 4 $= 0.25$ litre
Patterns	Building blocks			1; 3; 6...	n (n+1) 2
Fraction	Chocolate bar			6 12	6 = 1 12 2 or $\frac{1}{2}$ of 12 = 6
Ratio	Hens and chickens		* *** * *** * *** * ***	4:12	4 : 12 = 1 : 3 Of 52 fowls ¼ are hens and ¾ are chickens, ie 13 hens, 39 chickens
Mass	A block of margarine			500g	500g = 0,5 kg or calculations like 2 ½ blocks = 1,25kg

Teaching progresses from concrete -> to -> abstract. In case of problems, we fall back <- to diagrams, pictures, physically.

MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

Examples	**SPEAK IT: to explain the concept**	**SHOW IT: to embody the idea**	**SYMBOL IT: to enable mathematising**
	• Essential for introducing terminology in context • Supports learning through the auditory pathway • Important to link mathematics to everyday realities	• Essential to assist storing and retrieving concepts • Supports understanding through the visual pathway • Important to condense a variety of information into a single image	• Essential to assist mathematical thinking about concepts • Supports the transition from situations to mathematics • Important to expedite calculation and problem solving
FP: Doubling and halving	"To double something, means that we make it twice as much or twice as many. If you got R50 for your birthday last year and this year you get double that amount, it means this year you got R100. If Mom is doubling the recipe for cupcakes and she used to use 2½ cups of flour, it means she has to use twice as much this time." "To halve something, means that we divide it into two equal parts or share it equally. If I have R16 and I use half of it, I use R8 and I am left with R8. If we share the 22 Astro's in the box equally between the two of us, you get eleven, which is one half and I get eleven, which is the other half."	1. Physical objects: Example: Double 5 beads Halve 12 beads 2. Pictures: Example: Double  Halve  3. Diagrams: 	7 + 7 = □ 7 + □ = 14 △ + ○ = 14, but □ + □ = 14 2 times 7 = 14 double of 7 is 14 14 – 7 = 14 – □ = 7 14 divided by 2 = 7 14 halved is 7

IP: Geometric patterns	"If we see one shape or a group of shapes that is growing or shrinking a number of times, every time in the same way, we can say it is forming a geometric pattern. If we can find out how the pattern is changing every time, we can say we found the rule of the sequence of shapes. When we start working with geometric patterns, we can describe the change in normal language. Later we see that it becomes easier to find the rule if there is a property in the shapes that we can count, so that we can give a number value to each , or each term of the sequence" "You will be asked to draw the next term of the pattern, or to say how the eleventh term of the pattern would look, for example. You may also be given a number value and you may be asked, which term of the pattern has this value?"	 Draw the next term in this pattern.  Describe this pattern. What is the value of the 9th term of this pattern (T9)?  To draw up to the ninth term of this pattern, is a safe but slow way. It is even slower to find out by drawing, which term has a value of 120 for example. One is now almost forced to deal with this problem in a symbolic way.	Note how important it is to support the symbolising by saying it out: 1; 3; 6… 1; 3; 6; 10… 1; 3; 6;10;15 Inspecting the terms of the sequence in relation to their number values: T1: 1 = 1 The value of term 1 is 1 T2: 3 = 1+2 The value of term 2 is the sum of two consecutive numbers starting at 1 T3: 6 = 1+2+3 The value of term 3 is the sum of three consecutive numbers starting at 1 T4: 10 = 1+2+3+4 The value of term 4 is the sum of four consecutive numbers starting at 1 T5: 15 = 1+2+3+4+5 The value of term 5 is the sum of five consecutive numbers starting at 1 T9: 45 = 1+2+3+4+5+6+7+8+9 The value of term 9 is the sum of nine consecutive numbers starting at 1 We can see that the value of term n is the sum of n number of consecutive numbers, starting at 1.

SP: Grouping the terms of an algebraic expression	*“We can simplify an algebraic expression by grouping like terms together. We therefore have to know how to spot like terms. Let us say we have to sort fruit in a number of baskets and explain the variables or the unknowns in terms of fruits. Try to visualise the following pictures in your mind:”*	Although not in a real picture, a mind picture is painted, or a mental image to clarify the principle of classification: • Basket with green apples (a) • Basket with green pears (b) • Basket with green apples and green pears (ab) • Basket with yellow apples (a^2) • Basket with yellow apples and green pears (a^2b) Or in diagrammatic form 	$4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a - 2ab + 2a^2b + a^2b$ $= - 3a + 5a - a + 4b + 4b - 2ab - 2ab - a^2 + 3a^2b + 2a^2b + a^2b$ $= a + 8b - 4ab - a^2 + 6a^2b$

TOPIC 1: COMMON FRACTIONS

INTRODUCTION

- This unit runs for 7 hours.
- It is part of the content area, ‘Number, Operations and Relations’ and counts for 25 % in the final exam.
- The unit covers revision of all operations on fractions which includes working with mixed numbers.
- Calculations with percentages are also part of this section.
- It is important to note that learners need to feel confident with fraction work to include multiplication and division (invert and multiply).
- Fractions are necessary in everyday life because everything does not fit into a perfectly defined number. Tasks such as cooking, building, sewing and manufacturing use fractions for measurements that could potentially change the outcome of the product. Fractions are also involved with business and the monetary system.
- Although learners have been working with fractions for many years they still seem to struggle with the concept. Try and spend time speaking in fractions to them. For Example: I have a cake to share between 5 people, how much of the cake would each person get etc
- Use visual examples where possible, particularly for the learners struggling
- Practice as much as possible – if the textbook used does not have enough exercises in, re-do some, find some more or make up a worksheet of your own.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/ FET PHASE
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Describing and ordering fractions	• Describing and ordering fractions	• Ordering fractions
• Calculations with fractions	• Calculations with fractions	• Calculations with fractions
• Solving problems	• Solving problems	• Solving problems
• Percentages	• Percentages	• Percentages
• Equivalent forms	• Equivalent forms	• Equivalent forms

GLOSSARY OF TERMS

Term	Explanation / Diagram
Fraction	A number representing a part of a whole. For example: $\frac{3}{4}$ means 3 parts of 4. In this case the whole was split up into 4 parts.
Denominator	The top part of a fraction. In the fraction . $\frac{12}{47}$ the numerator is 12.
Numerator	The bottom part of a fraction. In the fraction . $\frac{3}{5}$ the denominator is 5.
Equivalent fractions	Fractions that have the same denominators. This is necessary in order to add or subtract fractions.
Mixed Number	A fraction made up of a whole number and a common fraction. For example $2\frac{1}{2}$ (this means 2 + $\frac{1}{2}$) This can be written as an improper fraction.
Common Fraction	A fraction in which the numerator and denominator are both integers. For example $\frac{3}{5}$ or $\frac{12}{5}$ The second example can be written as a mixed number.
Factor	A number that can divide into another number without a remainder. For example: 5 is a factor of 10 and 2 is a factor of 20
Multiple	The result of multiplying a number by the natural numbers (1, 2, 3....) For example: The multiples of 5 are 5, 10, 15, 20.....
Multiplication	Repeated addition (a shorter way to add many of the same number) Example: 3 + 3 + 3 + 3 = 4 x 3 (you are adding 4 lots of 3 together)
Lowest Common Denominator (LCD)	The lowest possible whole number that 2 or more denominators can be changed into in order to add or subtract them. For example: The LCD of 3 and 4 is 12
Reciprocal	To reciprocate a fraction means to turn a fraction upside down. For example: the reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$ This is used when dividing fractions.
Square	The number formed when multiplying a number by itself. For example: The square of 3 is 9 (3 x 3 = 9)
Square root	The opposite operation to squaring a number. For example: The square root of 25 is 5 (because 5 x 5 = 25)
Cube	The number formed when multiplying a number by itself and then again For example: The cube of 3 is 27 (3 x 3 x 3 = 27)
Cube root	The opposite operation to cubing a number. For example: The cube root of 125 is 5 (because 5 x 5 x 5 = 125)
Percentage	A number shown as a fraction of 100 25% = $\frac{25}{100}$
Percentage Increase	The percentage by which a certain amount has increased (gone up)
Percentage Decrease	The percentage by which a certain amount has decreased (gone down)

SUMMARY OF KEY CONCEPTS

Simplified Fractions

Answers to all fraction questions should always be in their simplest form.

1. In order to get a fraction into its simplest form, we need to find a whole number that can divide exactly into both the numerator and denominator.

Teaching Tip:
As this means we need to find the highest common factor of both numbers, learners may need reminding how to do that. Knowing their times tables is essential for simplifying fractions quickly and accurately.

2. Once you have divided both the numerator and denominator by the highest common factor, the fraction will be in its simplest form. These two fractions will be equivalent fractions.
3. For example: $\frac{4}{20} = \frac{1}{5}$ (4 was the factor that went into both 4 and 20)

Addition and Subtraction of Fractions

1. In order to add or subtract fractions they MUST have the same denominator. This makes them ‘like terms’ (which was covered in Algebra earlier in the year) which then means you can add or subtract them.

Teaching Tip:

Discuss why we need to find a common denominator to add or subtract – ask about adding one half to one third and show (using paper cut up if necessary) why it is so much easier to add them when they are both called “sixths”. Just hearing three sixths plus two sixths is usually very easy for most learners.

Use this opportunity to remind learners of when they added and subtracted like terms in the Algebra section in Term 1. Needing a common denominator is doing the same thing – ensuring that like terms are being added or subtracted.

2. If mixed numbers are involved it is best to first change them into improper fractions.

3. For example:

		Explanation:
1.	$\frac{7}{9}+\frac{1}{6}$	Find LCD (in this case 18 as 6 and 9 can both go into 18)
	$=\frac{14}{18}+\frac{3}{18}$	Change the fractions into equivalent fractions (whatever you multiplied the original denominator with to get the LCD, also needs to be multiplied by the numerator)
	$=\frac{17}{18}$	
2.	$1\frac{1}{3}+5\frac{3}{4}$	Change mixed numbers to improper fractions
	$=\frac{4}{3}+\frac{23}{4}$	Find LCD and change numerators to make equivalent fractions
	$=\frac{16}{12}+\frac{69}{12}$	
	$=\frac{85}{12}$	There is no need to change this back into a mixed number, unless the question has said to do so.
3.	$5\frac{1}{3}-2\frac{3}{7}$	Change mixed numbers to improper fractions
	$=\frac{16}{3}-\frac{17}{7}$	Find LCD and change numerators to make equivalent fractions
	$=\frac{112}{21}-\frac{51}{21}$	
	$=\frac{61}{21}$	
4.	Subtract $4\frac{1}{3}\ \textit{from}\ 10\frac{2}{5}\ \textit{then}\ \textit{add}\ 1\frac{1}{2}$	Whatever comes after the word from must always go first.
	$=10\frac{2}{5}-4\frac{1}{3}+1\frac{1}{2}$	
	$=\frac{52}{5}-\frac{13}{3}+\frac{3}{2}$	
	$=\frac{312}{30}-\frac{130}{30}+\frac{45}{30}$	
	$=\frac{227}{30}$	

Finding fractions of whole numbers

It is important that this concept is developed. Dividing 21 by 3 is the same as establishing the size of each of the 3 equal groups made from 21. This would be 7 (in each). If we wanted to find $\frac{2}{3}$ of 21 it would follow the same logic but the answer would need to be 2 of those (2 x 7 in this case). The following steps need to be done with the above concept in mind.

1. To find fractions of whole numbers, change the ‘of’ to ‘x’ (multiply). Then put the whole number over 1 to make it more clear where the numerators and denominators are.

2. For example:

$$\begin{aligned}&\text{Find } \frac{2}{3} \text{ of } 21\\ &\ \frac{2}{3}\times\frac{21}{1}\\ &=\frac{2}{1}\times\frac{7}{1}\\ &=14\end{aligned}$$

If learners prefer to find one third of 21 (7) then multiply that number by 2, allow them to do this. Learners finding their own method can be very useful to them in understanding a concept more clearly.

Multiplication of Fractions

As with the previous concept, it is important that the real meaning of multiplying fractions is understood. Note that the more this idea is engaged with in the classroom, the more the connections will be recognised in the shortcuts presented in the following steps.

1. When multiplying fractions, the main rule is to multiply all numerators with numerators and all denominators with denominators.
2. You DO NOT need a common denominator like in addition and subtraction.
3. Change all mixed numbers into improper fractions before starting to multiply. Once this has been done, check if you can simplify. Any numerator can be simplified by any denominator – you just need to find a whole number that goes into both without a remainder (Highest Common Factor).

4. For Example:

1.	$\frac{2}{3}\times\frac{1}{4}$	Here we are being asked to find 2 thirds of 1 quarter. If each quarter was divided into 3 equal parts, each of those parts would become 1 twelfth of the whole. 2 of these parts would then be 2 twelfths. Check to see if any of the numerators can simplify with any of the denominators
	$=\frac{1}{3}\times\frac{1}{2}$	2 goes into 2 and 4
	$=\frac{1}{6}$	[The simplified version of 2 twelfths]
2.	$3\frac{1}{2}\times 2\frac{3}{14}$	Here we are being asked to find three and a half parts of 2 and 3 fourteenths. Change the mixed numbers to improper fractions
	$=\frac{7}{2}\times\frac{31}{14}$	Check to see if any of the numerators can simplify with any of the denominators
		7 goes into 7 and 14
	$=\frac{1}{2}\times\frac{31}{2}$	
	$=\frac{31}{4}$	

Teaching Tip:
In primary school learners are encouraged to write their answers as mixed numbers. In high school, improper fractions are acceptable (and preferable) as an answer, provided they are in their simplest form. Point this out to learners.

Division of Fractions

It is important to ensure learners understand what division of fractions really means. Use whole numbers to demonstrate first. For example, for 12 ÷ 3 we can explain that one meaning of this is ‘how many threes are there in 12’ with the answer being: 4 threes. Therefore, 12 ÷ 3 = 4.

The division of fractions can be seen in the same way: $\frac{2}{3} \div \frac{3}{4}$ actually means how many $\frac{3}{4}$ are there in $\frac{2}{3}$

1. When you divide fractions, it is the same as multiplying by its reciprocal. (This is sometimes known as ‘tip and times’)

Teaching Tip:
It is important that learners understand that dividing any number (not just fractions) is the same as multiplying by its reciprocal. Show this by having a short discussion using whole numbers. For example, 6 divided by 2 (is 3) will be the same as 6 multiplied by $\frac{1}{2}$. 10 ÷5 will give the same answer as 10× $\frac{1}{5}$ which is the same as $\frac{1}{5}$ of 10

2. Reciprocal: A fraction turned upside down
3. For Example:

1.	$\frac{2}{3} \div \frac{3}{4}$	Change the ÷ to × and reciprocate the fraction
	$= \frac{2}{3} \times \frac{4}{3}$	Proceed as you would for multiplication including checking if any simplifying can occur.
	$= \frac{8}{9}$	
2.	$3\frac{5}{7} \div 1\frac{1}{3}$	Change the mixed numbers to improper fractions. Keep the divide sign at this stage – don’t try and reciprocate yet.
	$= \frac{26}{7} \div \frac{4}{3}$	Change the ÷ to × and reciprocate the fraction
	$= \frac{26}{7} \times \frac{3}{4}$	Check to see if any of the numerators can simplify with any of the denominators. (2 goes into 26 and 4)
	$= \frac{13}{7} \times \frac{3}{2}$	
	$= \frac{39}{14}$	

Comparing Fractions

1. In order to compare fractions, they need to have the same denominators.
2. For example: Which is the bigger number, $\frac{7}{9}\, or \,\frac{2}{3}$?

   The Lowest Common denominator (LCD) of 9 and 3 is 9 (Both 9 and 3 can go into 9 perfectly)
   We now need to make equivalent fractions and both must have a denominator of 9.

   $$\frac{7}{9} \qquad\qquad \frac{2}{3}$$

   $$= \frac{6}{9}$$

3. These can now easily be compared and we can see that $\frac{7}{9}$ is bigger than $\frac{6}{9}$ which in turn means that $\frac{7}{9}$ is bigger than $\frac{2}{3}$.

Equivalent Fractions

1. These are fractions that represent the same part of a whole.
2. For example:
   a. $\frac{5}{10} = \frac{1}{2}$
   b. 5 parts of a whole that has been divided into 10 parts will be the same size as 1 part of a whole that has been divided into 2 parts.

Teaching Tip:
Using a poster (or a few posters) similar to the one provided in the resources at the end of this topic would be useful to have displayed during the teaching of both common fractions and decimal fractions.

Squares and Cubes of Fractions

Teaching Tip:
Learners have already spent time squaring, cubing, square rooting and cube rooting when Integers was done in Term 1. Remind them of this and start this part of common fractions by doing some mental maths involving integers on their own. Squaring or cubing integers will lead to squaring or cubing fractions as the numerator and denominator can be looked at separately to assist learners in doing the calculation.

1. When squaring or cubing fractions, remember that you are really multiplying.
2. For example: $\left(\frac{2}{3}\right)^2 = \frac{2}{3}\times\frac{2}{3} = \frac{4}{9}$
3. But – you can use a shortcut and just square the numerator AND denominator
$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$$
(the middle step here would not be necessary)
4. The rules for cubing are the same.
$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$$
(Remember: $2^3=2\times2\times2$ NOT 2×3)

Square roots and Cube Roots of Fractions

1. All mixed numbers MUST be changed to improper fractions.
2. Examples:

$$\sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} = \frac{4}{5}$$

$$\sqrt{1\frac{7}{9}} = \sqrt{\frac{16}{9}} = \frac{4}{3}$$

$$\sqrt[3]{\frac{27}{8}} = \frac{3}{2}$$

Solving problems

Solving problems involving real life are always important so that learners can see the relevance of needing to learn about fractions. Learners need to see for themselves that being skilled in working with fractions will in fact be useful to them.
Types of problems need to cover a variety of fraction work. These should include finding fractions of whole numbers as well as the four basic operations and working with mixed numbers.

For example: Mary earned R250 during her school holidays. She spent $\frac{2}{5}$ of the money on a gift for her grandparents and $\frac{1}{3}$ of what was left on a dress. What was the cost of the dress? How much did Mary have left?

Teaching Tip:
Encourage learners to visualise the situation to make it more concrete in their minds. If learners are struggling they could even make it concrete by using bits of paper to represent the money in this situation. Also discuss the process that will need to be followed in order to arrive at the answer.

Solution: First, we need to calculate what Mary spent on the gift and find how much was left over:

$\frac{2}{5}$ of $R250$ (remember that 'of' means multiplication and always put whole numbers over 1 when dealing with fractions)

$= \frac{2}{5} \times \frac{250}{1}$

$= \frac{2}{1} \times \frac{50}{1}$

$= 100$ (amount spent on the gift)

Amount left over: $\mathrm{R}250 - \mathrm{R}100 = \mathrm{R}150$

Secondly, we need to find what she spent on the dress:

$\frac{1}{3} \times \frac{150}{1}$

$= \frac{1}{1} \times \frac{50}{1}$

$= \mathrm{R}50$

Finally we can find how much is left over:

$\mathrm{R}150 - \mathrm{R}50 = \mathrm{R}100$

Teaching Tip:
Do not start with a question similar to this one. The textbook will have more basic examples leading up to this kind. Ask learners to come up with their own question that shows how fractions are used in their own or their family's lives. Choose a few of these questions to do on the board with the class.

Percentages

1. A percentage is a fraction with '100' as the denominator but is written as a whole number with the percentage sign next to it.
2. For example:

$$\frac{25}{100} = 25\%$$

Teaching Tip:
Take the time to discuss the word 'percentage' with learners. 'Per' means to divide. As in km/h (kilometres per hour) – this means kilometres were divided by hours to give, for example, 60km/h which is the speed. 'cent' is derived from the word century which represents 100. Hence, percentage means 'divide by 100'.

3. To find a percentage of a whole number: multiply the percentage by the amount.

   Example: Find 15% of 400 (Remember that OF is the same as multiply)

$$= \frac{15}{100} \text{ of } 400$$

(It is best to write whole numbers as a fraction by putting it over 1)

$$= \frac{15}{100} \times \frac{400}{1}$$

$$= \frac{15}{1} \times \frac{4}{1}$$

$$= 60$$

4. To calculate the percentage of part of a whole:
   The most common example that learners will relate to for this is to change a test mark into a percentage.
   For example: Find the percentage if a test result is 14 out of 20

$$\frac{14}{20} \times 100$$

$$= \frac{14}{20} \times \frac{100}{1}$$

$$= \frac{14}{1} \times \frac{5}{1}$$

$$= 70\%$$

If the units are not the same, it is essential to first make them the same before doing the calculation.

For example: Write 3mm of 6cm as a percentage.
This is the same as 3mm of 60mm

$$\frac{3}{60}\times100$$

$$=\frac{3}{60}\times\frac{100}{1}$$ (3 went into itself once and into 60, 20 times; then 20 went into itself once and into 100, 5 times)

$$=\frac{1}{1}\times\frac{5}{1}$$

$$= \quad 5\%$$

5. To increase an amount by a given percentage:
Find the percentage of the amount, then add it to the original amount.
For Example: Increase R85 by 12%
12% of R85

$$=\frac{12}{100}\times\frac{85}{1}$$

$$=\frac{12}{20}\times\frac{17}{1}$$ (5 divided into 100 and 85)

$$=\frac{3}{5}\times\frac{17}{1}$$ (4 divided into 12 and 20)

$$=\frac{51}{5}$$

As this is a question about money, the fraction needs to be changed into a decimal.
= 10,2
Money must always have 2 decimal places so R10,20 is the increase amount.
R85 + R10,20 = R95,20

6. To decrease an amount in a given percentage:

   Find the percentage of the amount, then subtract it from the original amount.
   For example: Decrease R250 by 15%

$$
\begin{aligned}
&15\,\%\text{ of R250}\\
&= \frac{15}{100}\times\frac{250}{1}\\
&= \frac{15}{2}\times\frac{5}{1} \qquad \text{(50 divided into 100 and 250)}\\
&= \frac{75}{2}\\
&= 37{,}50\\
&\text{R250} - \text{R37{,}50} = \text{R212{,}50}
\end{aligned}
$$

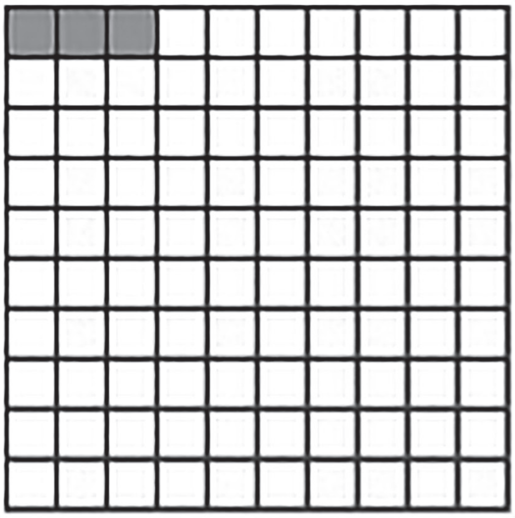
All of the above examples are the types of questions that could be asked when linking percentages to problem solving. As mentioned previously, it is important that learners recognise the importance of why they have to learn about percentages and how it can be useful to them in their own lives.

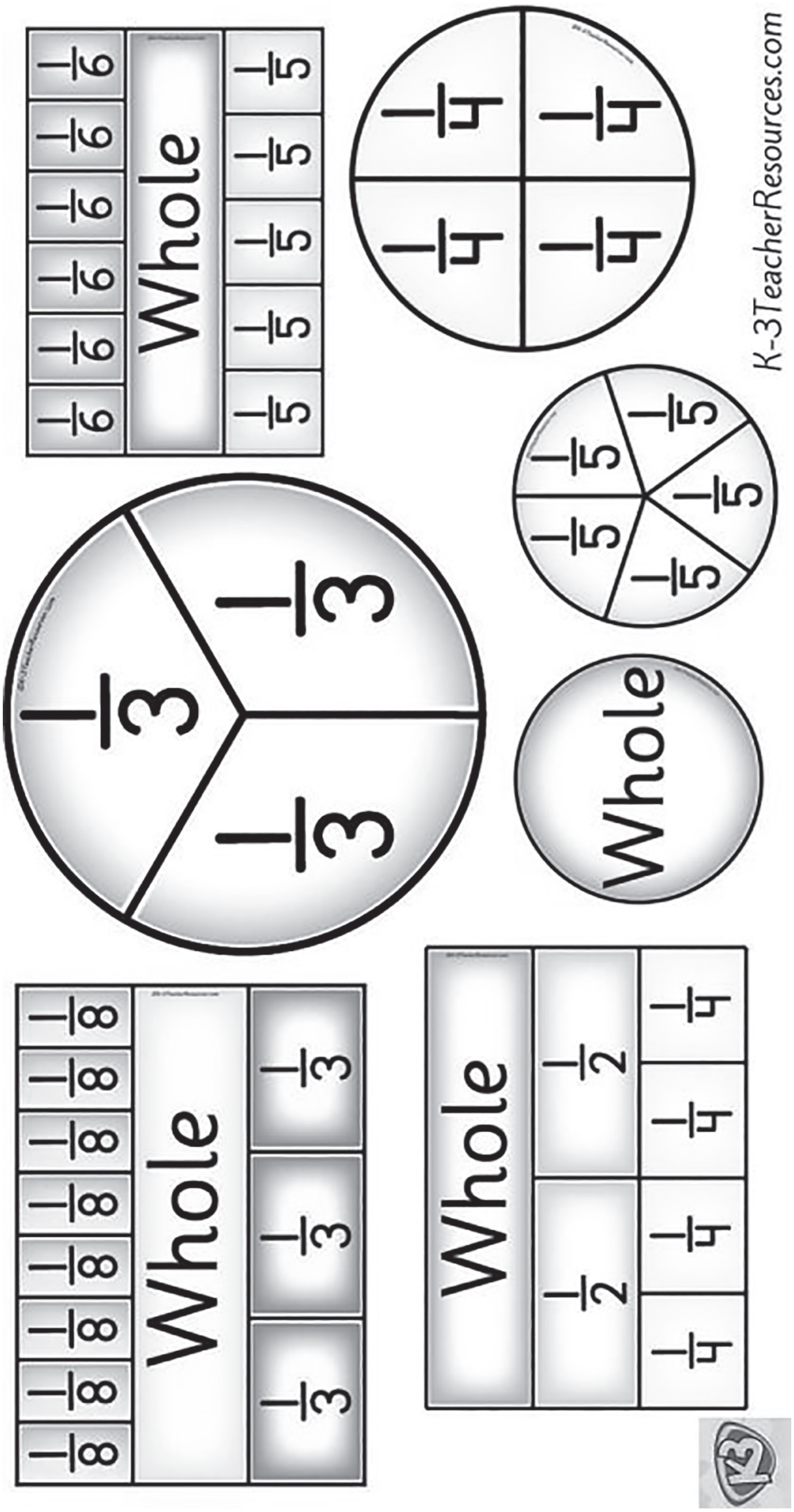
TOPIC 1: RESOURCES

$$\frac{3}{100}$$

0.03

3%

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TOPIC 2: DECIMAL FRACTIONS

INTRODUCTION

- This unit runs for 6 hours
- It is part of the content area, ‘Number, Operations and Relations’ and counts for 25 % in the final exam.
- The unit covers all operations on decimal fractions as well as multiplication of decimals with more than one decimal place.
- It is important to note that historically learners do not feel confident with decimal fractions and time needs to be taken to assist learners in overcoming this difficulty that is perceived by them.
- The purpose of needing a good knowledge of decimal fractions is that they are used on such a regular basis daily. The most common of these being when dealing with money. Times in athletics and measurements are also regularly used.
- As with common fractions, learners tend to also struggle with decimal fractions. The more they practice the more confident they should become.
- Always encourage learners to estimate answers before doing the calculation in order for them to check if their answer makes sense. Placing the decimal in the wrong place is a common error and by first estimating then checking if their answer is reasonable can help to eliminate this.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Ordering and comparing decimal fractions • Calculations with decimal fractions • Calculation techniques • Solving problems • Equivalent forms	• Ordering and comparing decimal fractions • Calculations with decimal fractions • Calculation techniques • Solving problems • Equivalent forms	• Ordering and comparing decimal fractions • Calculation techniques • Solving problems • Equivalent forms

GLOSSARY OF TERMS

Term	Explanation / Diagram
(Decimal) fraction	A number representing a part of a whole. For Example: 0.3 means 3 parts of 10 and 0.03 means 3 parts of 100
Divisor	A number by which you divide For Example: In 10 ÷ 2 = 5, 2 is the divisor
Dividend	A number which is divided by another number For Example: In 10 ÷ 2 = 5, 10 is the dividend
Quotient	The answer of a division operation For Example: In 10 ÷ 2 = 5, 5 is the quotient
Ascending	Starting at the smallest and going up to the largest
Descending	Starting at the largest and going down to the smallest
Units	Numbers with only one digit (1 – 9)
Tenths	One part of ten equal parts
Hundredths	One part of one hundred equal parts
Thousandths	One part of one thousand equal parts

SUMMARY OF KEY CONCEPTS

Decimal fractions are common fractions written in a different way.
The Decimal System is a place-value system. The digits 0 – 9 are used and the position in which they are placed determines their value.

For example: In the number 29,43

2 → Tens (2 x 10 = 20)

9 → Units (9 x 1 = 9)

4 → tenths (4 x $\frac{1}{10}$ = $\frac{4}{10}$ or 0,4)

3 → hundredths (3 x $\frac{1}{100}$ = $\frac{3}{100}$ or 0,03)

29,43 = 20 + 9 + $\frac{4}{10}$ + $\frac{3}{100}$

OR = 20 + 9 + $\frac{43}{100}$

29,43 can also be written as $\frac{2943}{100}$

If a digit moves to the left (if we were using the example above 294,3), its value is increased by a factor of 10 (29,43 x 10 = 294,3).
If a digit moves to the right (if we were using the example above 2,943), its value is decreased by a factor of 10 (29,43 ÷ 10 = 2,943).
An increase of 10 = 10 times as much.
A decrease of 10 = one tenth as much.

Zeroes at the back (or end) of a decimal fraction do not change the value of a fraction.
For example: 3,2 = 3,20 = 3,200 etc

Decimal fractions (unlike common fractions) only work with denominators of 10, 100, 1000 and so on (Powers of 10).

Converting between decimal fractions and common fractions

Decimal fractions to common fractions

- Check how many digits there are to the right of the decimal place
- 1 digit means a denominator of 10 will be used
  2 digits means a denominator of 100 will be used…….and so on
- Place the digits to the right of the decimal place over the correct denominator according to the rule above.
- Simplify the fraction

For example:

1. 0,3 (1 digit to the right so the denominator will be 10)

$= \frac{3}{10}$

2. 4,3 (1 digit to the right so the denominator will be 10, but watch out for the whole number)

$= \frac{43}{10}$

$= 4\frac{3}{10}$ (check if simplification is possible)

3. 0,16 (2 digits to the right so the denominator will be 100)

$= \frac{16}{100}$

$= \frac{4}{25}$

Common fractions to decimal fractions

It is important to remember that a common fraction is just a division operation.

Example:

$\frac{20}{10} = 20 \div 10 = 2$

$\therefore \frac{1}{2} = 1 \div 2$

To convert $\frac{1}{2}$ into a decimal: (although you should really know this one immediately)

- Change the fraction into a division operation
- Use short (or long) division and put the divisor on the outside and the dividend on the inside.
- Because you are dividing a bigger number into a smaller number, insert a decimal place after the dividend and add as many zeroes as needed as you divide.
- The answer (quotient) must have a decimal sign directly below the one placed next to the dividend.

For example:
1 ÷ 2

$2\ \underline{\lfloor 1 \qquad\qquad}$

$2\ \underline{\lfloor 1\ ,\ 0 \qquad}$

$2\ \underline{\lfloor 1\ ,\ 0 \qquad}$
$\quad\ \ 0\ ,\ 5$

$\therefore\ \frac{1}{2}\ =\ 0,5$

Try and follow this example:

$\frac{5}{8} = 5 \div 8$

$8\ \underline{\lfloor 5{,}^{5}0^{2}0^{4}0 \qquad}$
$\quad\ \ 0{,}6\ 2\ 5$

$\frac{5}{8} = 0,625$

Another way of changing common fractions to decimal fractions is to make equivalent fractions from the common fraction but at the same time make a denominator of 10 or 100 or 1000 etc.

For example:

$\frac{4}{25}\ =\ \frac{16}{100}\ =\ 0,16$

Note that this is the method encouraged in CAPS and it is particularly good if it is an easy change but it isn't always as straightforward as the above example.

Ordering and comparing decimal fractions

In order to compare decimal fractions to see which is the bigger or smaller, the decimals need to have the same amount of digits after the decimal sign. (Remember that adding zeroes to the back of a decimal fraction does not change the value of the fraction)

For example:

a. Which is bigger: 3,21 or 3, 201

Add zeroes to the end of the shortest fraction so that both (or all if there are more than two) fractions have the same number of digits after the decimal sign.

3, 210 3, 201

Now you can compare thousandths (3 decimal places) with thousandths. It should be easy to see that 210 is larger than 201.

$\therefore\ 3,21 > 3,201$

b. Arrange the following in descending order:

0,28 ; $\frac{1}{4}$; 0,3

First, we need to change the common fraction into a decimal.

$\frac{1}{4} = 1 \div 4$

$$\begin{array}{r|l} 4 & 1,00 \\ \hline & 0,25 \end{array}$$

Now make sure that all the decimals have the same amount of digits after the decimal sign.

0, 28; 0, 25; 0, 30

Descending → going down → need to start with the biggest

0, 30 0, 28 0, 25

$\therefore\ 0,3;\quad 0,28;\quad \frac{1}{4}$

Rounding

1. When rounding a decimal, we are always most interested in the digit to the right of where we want to round to (so if we are rounding to 2 decimal places, the digit in the 3rd place is of importance).
2. If that digit is 5 or higher, we will round the digit in correct position UP. If that digit is 4 or smaller, we will leave the digit as it is.
3. For example:

0,31425
Rounded to 2 decimal places: 0,31
(4 is in the 3rd – thousandth – position so we leave the digit in the 2nd position as it is)

Rounded to 4 decimal places: 0,3143 (5 is in the 5th position so we round the digit in the 4th position '2' up to 3)

Addition and Subtraction

1. The simplest way to add or subtract decimals is in columns. The decimal signs must be exactly underneath each other so the place values match. (Tens underneath Tens, hundredths underneath hundredths and so on).
2. If need be, zeroes can be paced at the end to make the decimal fractions the same 'length'.
3. For example:

a. 3,2 + 1,45

3, 20
+ 1, 45
4, 65

b. 4,291 + 5,64 (carrying over required)

4, 291
+ 5, 640
9, 931

c. 5, 12 – 2, 452 (borrowing is required)

5, 120
-2, 452
2, 668

Multiplication of decimals

1. Multiplying by powers of 10

This is very easy to do – for every 10 you are multiplying by, the digits move one place to the left.
(Try and imagine the decimal being fixed or locked into position)
Remember to check that your number has become larger
(as it should when you are multiplying)

For example:

3,21 x 10 = 32,1

4,192 x 100 = 419, 2

2. Multiplication of decimals by integers

The method of multiplying in columns as was done in primary school is used here. While you are in the process of multiplying, decimal signs can be ignored. Once the multiplication is complete, count the total number of digits after the decimal sign in the decimal fraction – this is how many places there should be in the answer. Insert the decimal to make this correct.

For example:
4,62 x 3

$$\begin{array}{r} 462 \\ \underline{\times \quad 3} \\ \underline{1386} \end{array} \qquad \therefore\ 4,62 \times 3 = 13,86$$

3. Multiplication of decimal fractions by decimal fractions

The method of multiplying in columns as was done in primary school is used here as well.
While you are in the process of multiplying, you can ignore any decimal signs.
Once the multiplication is complete, count the total number of digits after the decimal sign in both decimal fractions – this is how many places there should be in the answer.
Insert the decimal to make this correct.

For example:
2, 43 x 0, 6

```
    243
X     6
   1458
```

As there are 3 digits after the decimal signs in total (2 in 2,43 and 1 in 0,6), there needs to be 3 in the final answer.

$$\therefore\ 2{,}43 \times 0{,}6 = 1{,}458$$

Division of decimal fractions

1. Dividing decimal fractions by powers of 10

   This is very easy to do – for every 10 you are dividing by, the digits move one place to the right.(Try and imagine the decimal being fixed or locked into position). Remember to check that your number has become smaller (as it should when you are dividing)

For example:
34, 21 ÷ 10 = 3, 421
34, 21 ÷ 100 = 0, 3421
34, 21 ÷ 1000 = 0, 03421

2. Dividing decimals by integers

   Dividing using short or long division, keeping the decimal in the same position for the answer (quotient) as it is in the question. Add zeroes to the end of the decimal if necessary until there is no remainder.

For example:
48, 24 ÷ 3

```
3 |48,24
   16,08
```

$\therefore$ 48,24 ÷ 3 = 16,08

3. Dividing decimal fractions by decimal fractions

   In order to do this, we need to multiply both decimals in the question by the same number of 10's until we are dividing by an integer, then we can follow the steps above.

   For example:

   0,62 ÷ 0,2

   (In order to make sure we are dividing by an integer, we need to multiply by 10. Both the divisor and the dividend must be multiplied by 10)

   = 6,2 ÷ 2
   = 3,1

Note: If learners are unsure why this 'rule' works, use an example such as 10 ÷ 5. Ask what the answer to that would be (2). Then multiply both the dividend and the divisor by 10. Now the question becomes 100 ÷ 50. Ask what the answer is now. Note that it is still 2. If both numbers are multiplied by the same number it still gives the same answer as the original.

Squares, Cubes, Square roots and Cube roots

1. Squares and cubes

   If you square a decimal fraction, the answer will have twice as many decimal places as the original decimal fraction.
   If you cube a decimal fraction, the answer will have three times as many decimal places as the original decimal fraction.

   Note: It would be a good idea if you allowed learners to discover this for themselves. You could give them a number of questions to do using a calculator then asking if they could find a pattern.

   For example:

   $(0,2)^2 = 0,04 \qquad (2^2 = 4)$

   $(0,2)^3 = 0,008 \qquad (2^3 = 8)$

   So, a short way to square or cube a decimal fraction is to:

   - Ignore the decimal
   - Square/cube the number you can see other than zeroes
   - Place the decimal sign in the answer according to the twice as many (squaring) or three times as many (cubing) rule.

For example:

a) $(0{,}03)^2$

Square the 3 $(3^2 = 9)$
Number of decimal places required in answer: 4
(twice as many as the original 2)

$\therefore\ (0.03)^2 = 0{,}0009$

b) $(0{,}12)^3$

$12^3 = 1728$ and answer should have 6 decimal places
(three times as much as 2 decimal places in question)

$\therefore\ (0{,}12)^3 = 0{,}001728$

You could also change the decimal fractions into common fractions, then square or cube them.

2. Square roots and Cube roots

   The previous rule of the answer having twice as many or three times as many is now reversed.

   If you square root a decimal fraction it will have half as many decimal places. If you cube root a decimal fraction it will have a third as many decimal places.

For example:

$\sqrt{0.25}$ $(\sqrt{25} = 5)$

$= 0{,}5$ need one decimal place – half as much as the original two decimal places)

$\sqrt[3]{0{,}000001}$ $(\sqrt[3]{1} = 1$
$= 0{,}01$

and we need 2 decimal places – a third of the original 6)

Solving problems

As decimals are used extensively in everyday life it is important that learners solve problems showing this.

For example:
Calculate the total amount that Jacob would pay for a television if he had to pay a deposit of R250, then monthly instalments of R152,45 per month for two years.

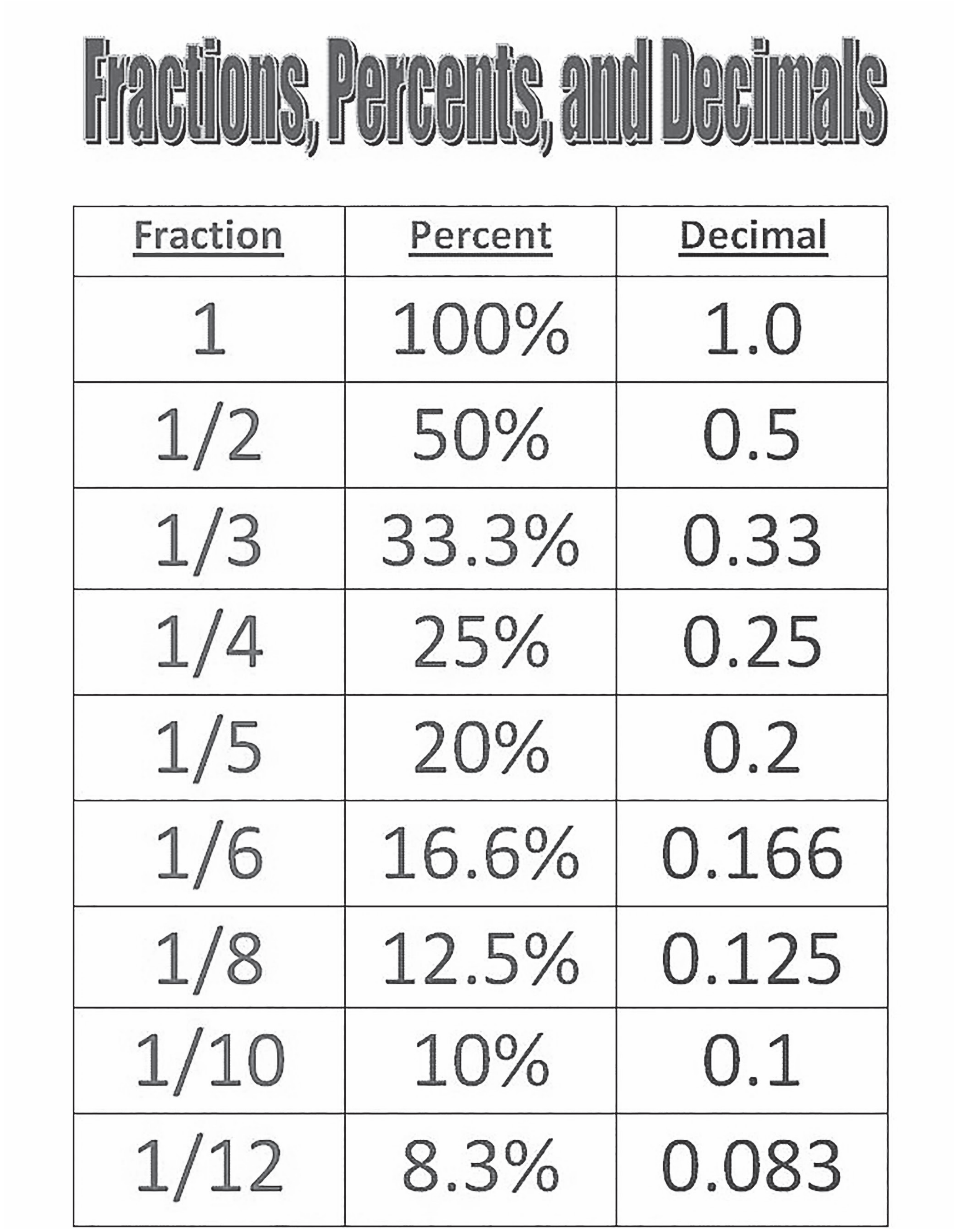
Solution:
R250 + (24 x R152,45)
= R250 + R3658,80
= R3908,80

Teaching Tip:
Ensure learners are exposed to questions involving all four operations.
The above example links back to the finance section done in Term 1, so this would be an ideal time to remind them of some of the skills covered then.
Links to other topics or previous years is always an important aspect of teaching.

Equivalent forms

The resource given at the end of the previous topic, common fractions, can still be used while doing decimal fractions. Learners need to recognise the most commonly used fractions and their decimal equivalent. A further resource is included in the resource section.

TOPIC 2: RESOURCES



Fractions, Percents, and Decimals

Fraction	Percent	Decimal
1	100%	1.0
1/2	50%	0.5
1/3	33.3%	0.33
1/4	25%	0.25
1/5	20%	0.2
1/6	16.6%	0.166
1/8	12.5%	0.125
1/10	10%	0.1
1/12	8.3%	0.083

TOPIC 3: THEOREM OF PYTHAGORAS

INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area, 'Measurement', and counts for 10% in the final exam.
- The unit covers the relationship between the lengths of the sides of a right-angled triangle which in turn develops an understanding of the Theorem of Pythagoras.
- When introducing the Theorem of Pythagoras, learners should be encouraged to investigate it for themselves by measuring the lengths of all three sides of a right-angled triangle and discussing with their peers whether they could find a rule or not.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE / GRADE 7	GRADE 8	GRADE 9 / FET PHASE
LOOKING BACK	**CURRENT**	**LOOKING FORWARD**
• Not covered in the previous grade or phase	• Determining whether a triangle is right-angled using the theorem • Finding length of missing sides in a right-angled triangle	• Using the theorem to solve problems • Used extensively in trigonometry

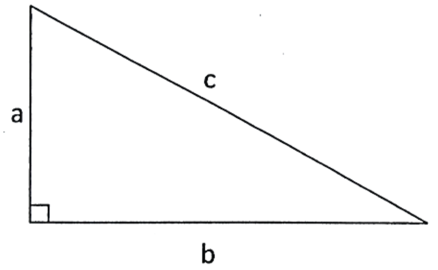
GLOSSARY OF TERMS

Term	Explanation / Diagram
Theorem of Pythagoras	In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides
Triangle	A 3 sided closed figure
Hypotenuse	The longest side in a right angled triangle. It is opposite the right angle
Square	To multiply a number by itself. For example: 5 x 5 = 25 (25 is a perfect square)
Square root	Finding what number was multiplied by itself to get a square number For example: $\sqrt{25}$ = 5 (5 is the square root of 25)
Sum of	Addition of
Surd	A root sign For example: $\sqrt{10}$

SUMMARY OF KEY CONCEPTS

The Theorem of Pythagoras is used in right-angled triangles ONLY.
It is used to find the length of an unknown side.
It can also be used to check if a triangle is right-angled if the lengths of all 3 sides have been given.

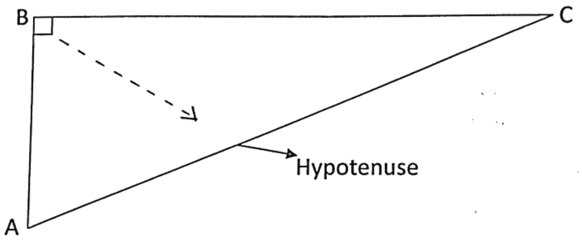
The Theorem of Pythagoras states:
In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



$$c^2 = a^2 + b^2$$

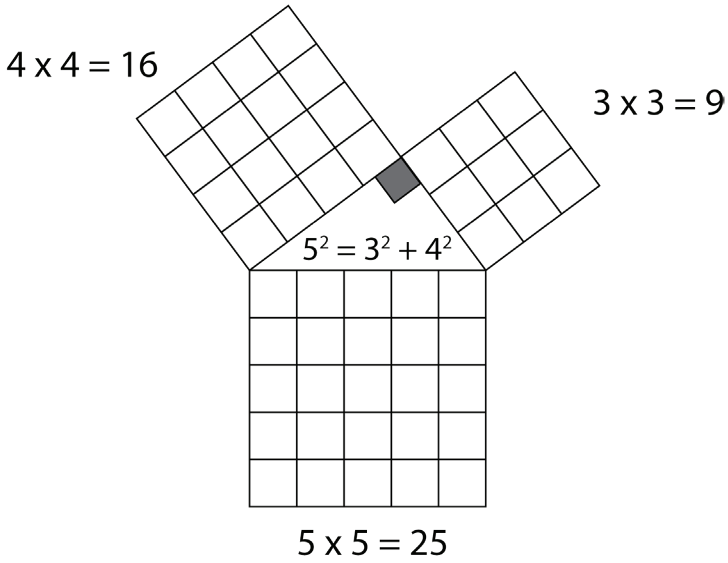
Make sure learners are clear on where the hypotenuse is.

Teaching Tip:
Tell learners that it is always the longest side in a right-angled triangle and it is always opposite the right angle. (The right angle is like an arrow that 'points' to it for you)



Teaching Tip:
As mentioned earlier, learners should spend some time investigating this theorem themselves. The following diagram should be used during the course of the introducing of this topic.
It should help to demonstrate to learners why we say the square on the hypotenuse' and 'the square on the other two sides'. Count the squares with the learners to show why the square on the hypotenuse really does equal the sum of the squares on the other two sides.

The most common Pythagorean triple has been used in this instance (3, 4, 5). It is worth pointing out to learners that they must not expect the numbers to always be consecutive and that in fact, this is the only one that is. The resource at the end of the topic shows some of the more commonly used Pythagorean triples.



Determining whether a triangle is right-angled when given the lengths of the three sides

It is recommended that some time is spent having learners look at different types of triangles here, including right-angled, obtuse-angled and acute-angled and comparing the lengths of the three sides in order to investigate if there is a relationship between the areas on the three sides.
From a more algebraic point of view, the following steps can be followed to determine if a triangle is right-angled or not:

1. Square the lengths of the two shortest sides.
2. Add these two answers together
3. Square the largest (from the longest side).
4. Check if your addition sum is equal to the square of the longest side.

For example:
Is the following triangle a right-angled triangle? Sides: 6cm, 8cm and 10cm

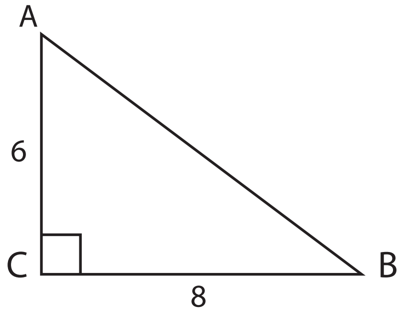
Step 1: $6^2 = 36$ $8^2 = 64$ $10^2 = 100$
Step 2: 36 + 64 = 100

Since the sum of the squares of the two smaller sides equals the square of the longest side, this is a right-angled triangle.

Finding the length of a missing side in a right-angled triangle

Teaching Tip:
Point out to learners that as this is part of Euclidean Geometry, they still need to be aware of the careful setting out of a solution. A statement and reason column ensures good habits that will hold them in good stead when dealing with geometry in the FET phase.

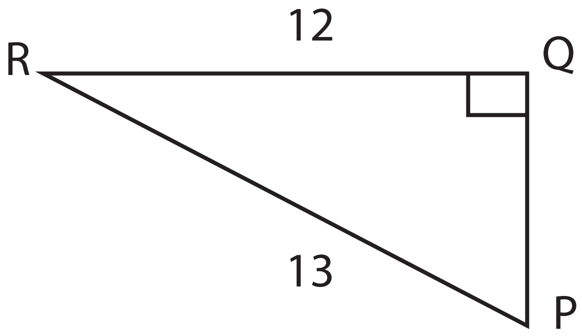
1. Finding the hypotenuse (longest side)



STATEMENT	REASON
$AB^2 = AC^2 + BC^2$	Pythagoras
$AB^2 = 6^2 + 8^2$	
$AB^2 = 36 + 64$	
$AB^2 = 100$	
$\sqrt{AB^2} = \sqrt{100}$	
$AB = 10$	

For example:

2. Finding a short side



STATEMENT	REASON
$PQ^2 + 12^2 = 13^2$	Pythagoras
$PQ^2 + 144 = 169$	
$PQ^2 = 25$	
$\sqrt{PQ^2} = \sqrt{25}$	
$PQ = 5$	

Sometimes the number that needs to be square rooted is not a perfect square.

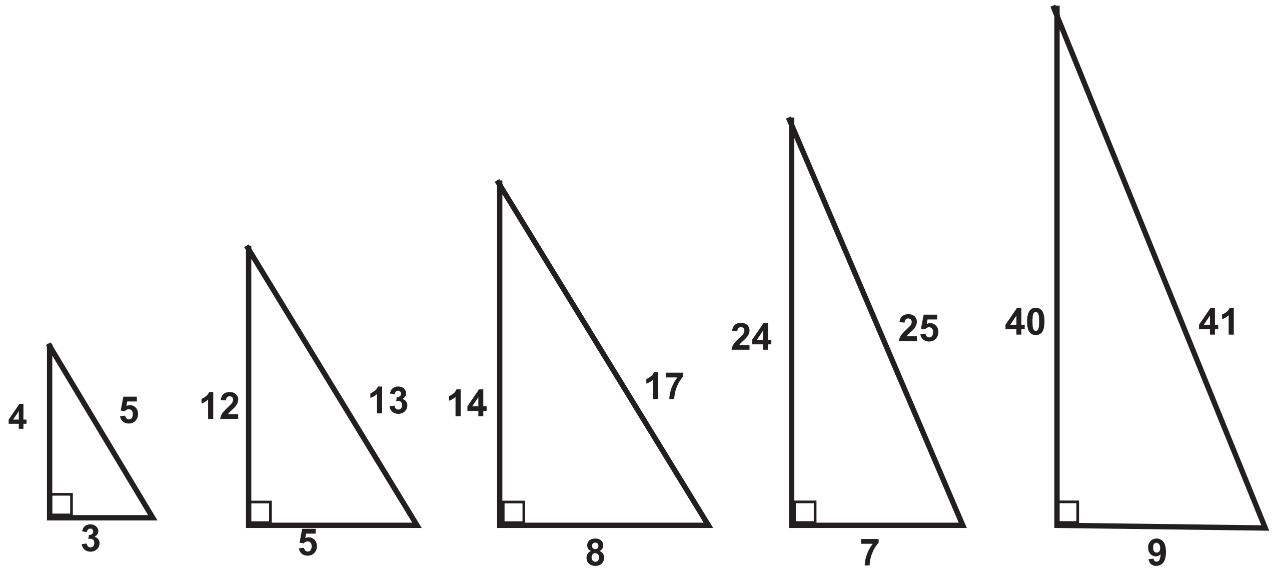
For example:

$$PQ^2 = 10$$

$$\therefore PQ = \sqrt{10}$$

Remember that it is acceptable to leave an answer in 'surd' form (with a root sign) unless the instruction was to round your answer to a certain amount of decimal places – then a calculator may be used to do so.

TOPIC 3 RESOURCES



TOPIC 4: AREA AND PERIMETER OF 2D SHAPES

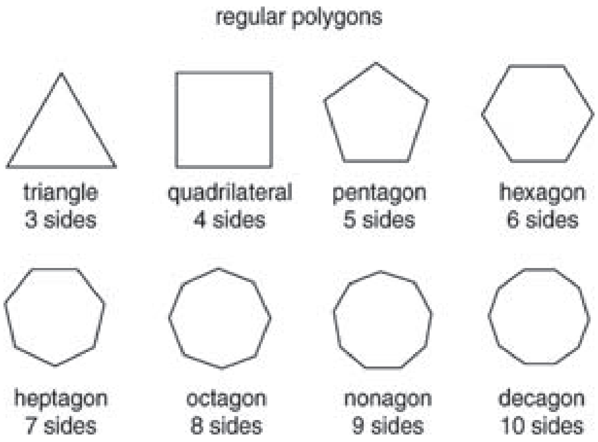
INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area, 'Measurement', and counts for 10 % in the final exam.
- The unit covers the use of appropriate formulae to find area and perimeter of polygons and circles.
- When dealing with triangles, time needs to be spent discussing how the base and the height must meet at right angles and that the height could be measured outside an obtuse-angled triangle.
- Time also needs to be spent discussing the relationship between the circumference and the diameter (Pi / π)

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/ GRADE 7	GRADE 8	GRADE 9 / FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Perimeter of regular and irregular polygons • Perimeter and area of squares, rectangles and triangles • Solve problems involving perimeter and area • Use and convert between appropriate SI units, including: $mm^2 \leftrightarrow cm^2$ and $cm^2 \leftrightarrow m^2$	• Perimeter and area of squares, rectangles, triangles and circles • Areas of irregular polygons • Use and describe the relationship between the: (i) radius, diameter and circumference of a circle (ii) radius and area of a circle • Solve problems, involving perimeter and area of polygons and circles • Use and describe the meaning of the irrational number Pi (π) in calculations involving circles • Use and convert between appropriate SI units, including: $mm^2 \leftrightarrow cm^2 \leftrightarrow m^2 \leftrightarrow km^2$	• Solve problems and calculate perimeter and area of polygons and circles • Investigate how doubling any or all of the dimensions of a 2D figure affects its perimeter and its area

GLOSSARY OF TERMS

Term	Explanation / Diagram
Square	A four sided polygon with all four sides equal in length and all four angles are right angles
Rectangle	A four sided polygon with both pairs of opposite sides equal in length and all four angles are right angles
Triangle	A three sided polygon
Circle	A 2-dimensional shape made by drawing a curve that is always the same distance from a centre.
Polygon	A closed 2D shape in which all the sides are made up of line segments. A polygon is given a name depending on the number of sides it has. A circle is not a polygon as although it is a closed 2D shape it is not made up of line segments 
Area	The space taken up by a two dimensional polygon.
Perimeter	The sum of the length of the outside of the shape. The distance around the shape.
Diameter	The distance across a circle through its centre.
Radius	The distance from the centre of a circle to the circumference.
Circumference	The perimeter of a circle. The distance around the side of a circle.
Pi (π)	The ratio of the circumference divided by the diameter for ANY circle no matter what its size is.

SUMMARY OF KEY CONCEPTS

Perimeter

Perimeter is a distance and therefore a linear measurement. The unit of measurement used would depend on the size of the shape. For example, a swimming pool's perimeter would be measured in metres (m) but a piece of paper's perimeter would be measured in centimetres (cm) or millimetres (mm).

Area

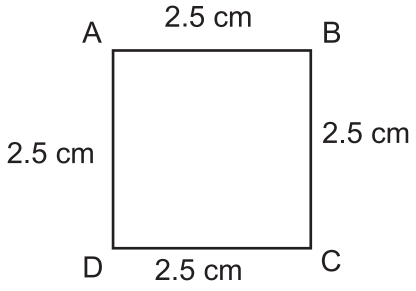
Area deals with 2-Dimensonal shapes and therefore the measurement is always 'squared'. When dealing with area, we are calculating the amount of space a 2D shape takes up.

Teaching Tip:
The use of manipulatives is an important part of teaching this section. Learners need to be measuring actual objects themselves (this would form the concrete phase of their learning) then finding the perimeter and area using their own measurements. Once they have spent time doing this, moving onto the representational phase where sketches would be made of what has been measured, as well as filling in the measurements and labelling accordingly, needs to be encouraged. Finally, a move to the abstract phase, using formulae and substitution would need to be done in order to complete the learning process. Examples of what manipulatives can be used will be given in the teaching tip at the end of each shape listed below.

Squares

1. To find the perimeter of a square, all 4 sides are added together.

   For example:



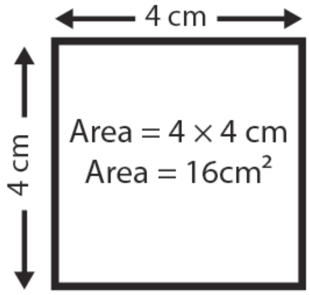
Perimeter $= 2{,}5\text{cm} + 2{,}5\text{cm} + 2{,}5\text{cm} + 2{,}5\text{cm}$

$= 4\ (\ 2{,}5 \text{ cm})$

$= 10 \text{ cm}$

2. To find the area of a square the length is multiplied by the breadth. As these are the same measurements it could also be said that the length is squared.

For example:

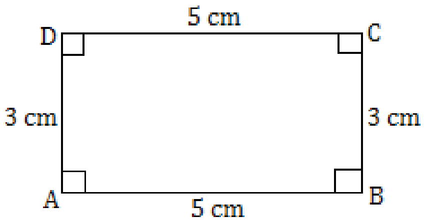


Teaching Tip:
Ask learners to bring in a 2D shape in the form of a square. This could be cut from a cereal (or other food) box. Alternately, perhaps they find a piece of material at home in the form of a square or a place mat. Besides having an actual physical item of their own, also encourage the learners to look around the classroom and see what they can see that is a square (perhaps a window or poster on the wall or even their desk or the bottom of a chair). The shapes brought in could be used by one learner then passed on to another. Group work would also be ideal at this stage where discussion should be encouraged. One lesson should be spent allowing learners to measure and discuss their findings.

Rectangles

1. To find the perimeter of a rectangle, all 4 sides are added together.

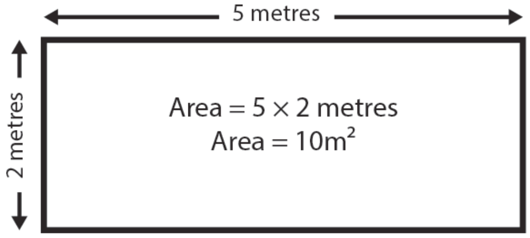
For example:



Perimeter = 5cm + 5cm + 3cm + 3cm
= 2 (5cm) + 2 (3cm)
= 10 cm + 6cm
= 16cm

2. To find the area of a rectangle the length is multiplied by the breadth.

For example:

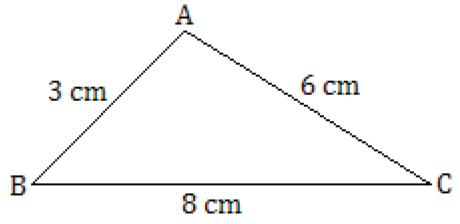


Teaching Tip:
All the information from the teaching tip on the previous page needs to be taken into account again. Rectangles are much easier to locate: doors, windows, books, envelopes, cell phones, money or the paper they are writing on are just a few examples.

Triangles

1. To find the perimeter of a triangle, all 3 sides are added together.

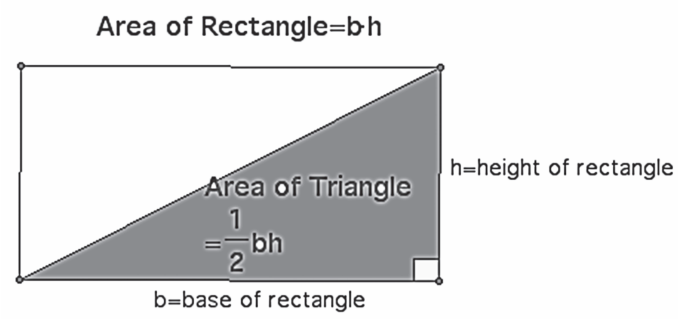
   For example:

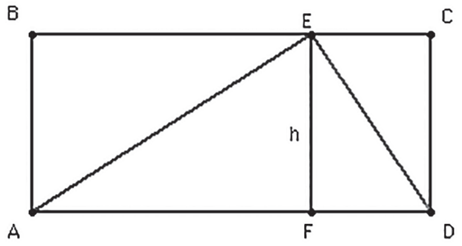


Perimeter = 3cm + 6cm + 8cm
= 17cm

2. To find the area of a triangle, the base is multiplied by the perpendicular height then multiplied by half.

   This formula comes from the use of area of a rectangle. The following 2 diagrams show this:

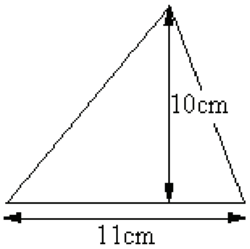




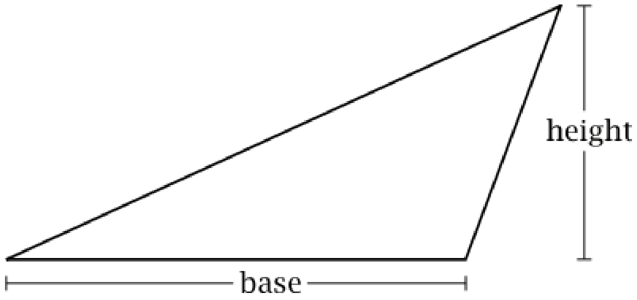
Note in this diagram that AD would be the length of the rectangle as well as the base of the triangle and 'h' would be the breadth/width of the rectangle as well as the height of the triangle. Multiplying the base and height would give the area of the rectangle. Halving this answer would give the area of the triangle.($\triangle$ECD is the same size as $\triangle$DEF and $\triangle$ABE is the same size as $\triangle$AEF)

For example:

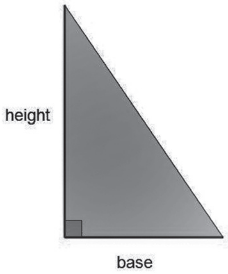
$$AREA = \frac{1}{2} base \times perpendicular\ height$$
$$= \frac{1}{2} \times 11 \times 10 = 55cm^2$$



3. In an obtuse-angled triangle, the height is measured outside the triangle:



4. In a right-angled triangle, the base and height are the two sides that form the right angle:



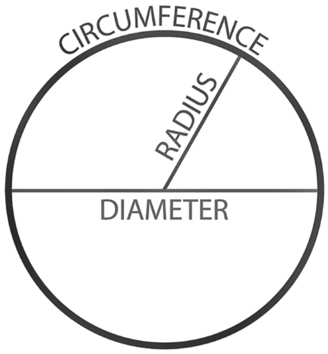
The height and base are interchangeable which means the labels could be switched around.

Teaching Tip:
All the information from the teaching tip on squares needs to be taken into account again.
Triangles may be a little more difficult to find. Due to this if you as the teacher have access to the internet and printing facilities it would be a good idea to find pictures of pyramids and road signs. Some images are available at the end of this section on the resource page. It would be useful at this stage to reinforce the idea that if the shape is not flat (like a sandwich) then to find area or perimeter we would only be dealing with the top (or bottom) triangle. Tell learners that the sandwich (or any other non 2D shape that is being discussed) is actually a 3D shape and that these will be dealt with in the next section.

Teaching Tip:
The theorem of Pythagoras can often be used with a question on area or perimeter of a triangle. Learners may need to find the length of a missing side before a question on either perimeter or area can be answered.

Circles

1. There are four main parts of a circle that need to be learnt in Grade 8, namely: the centre, radius, diameter and circumference.



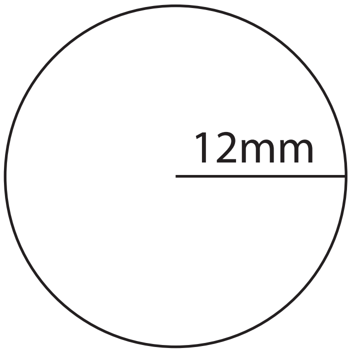
2. Earlier on in the year irrational numbers were covered. When dealing with the circle one of these numbers is of great importance: π

   The approximation of π is 3,142…..
   On a scientific calculator the π button could be used to make calculations more accurate. $\frac{22}{7}$ or 3,14 can be used as approximations of π.
3. No matter what size a circle is, the circumference divided by the diameter will ALWAYS give the same answer: π (Pi).
4. Since , $\pi = \frac{C}{d}$ we can use this to find the formula for calculating the circumference (perimeter) of a circle:

   By multiplying both sides by ‘d’, the formula becomes: $\pi d = C$

   (but remember that a diameter is 2 times the length of a radius so we could also use: $C = 2\pi r$)

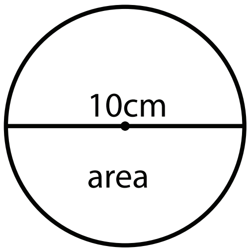
For example: Find the circumference of a circle with a radius of 12mm.

		OR:
	$C = \pi d$	$C = 2\pi r$
	$C = \pi(24mm)$	$C = 2\pi(12mm)$
	$C = 75,4mm$	$C = 75,4mm$

5. To find the area of a circle, π is also used. Area of a circle = πr^2

   NOTE: r^2 is NOT equal to 2r and can therefore NOT be replaced with diameter.
   For example:

Find the area of this circle



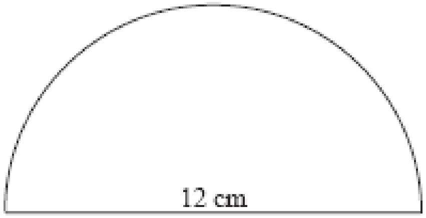
Area = π x radius x radius

Area
= π x 5 x 5
= $78,53cm^2$

Notice that although diameter was given in this example, it couldn't be used directly. First it had to be halved and converted to the radius before it could be used in the area formula.

6. In Grade 8 learners need to be able to use their knowledge of whole shapes and be able to answer questions using this knowledge.

   For example: Find the area and perimeter of the following shape:



   Area is a little more straightforward than perimeter as the above shape is a half of a circle (semi-circle). The formula is therefore:

$$
\begin{aligned}
Area &= \frac{\pi r^2}{2} \\
&= \frac{\pi(6cm)^2}{2} \\
&= \frac{\pi(36cm^2)}{2} \\
&= \frac{113,097cm^2}{2} \\
&= 56,55cm^2
\end{aligned}
$$

   To find the curved part of the perimeter, using the formula for circumference and halving it will be useful. However, the straight line forming the diameter and closing the shape off also needs to be considered so this will need to be added to the curved part.

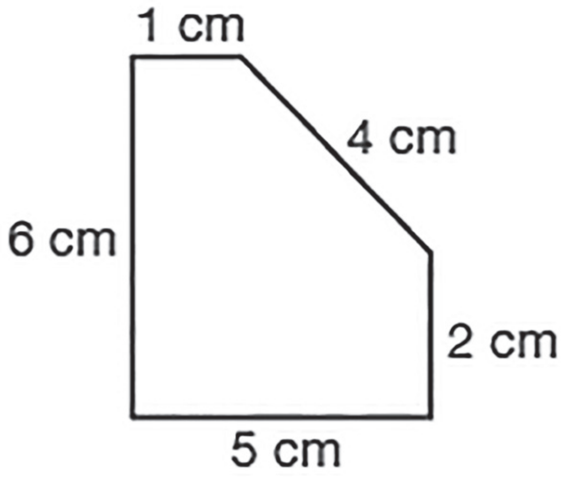
$$
\begin{aligned}
Perimeter &= \frac{\pi d}{2} + d \\
&= \frac{\pi(12cm)}{2} + 12cm \\
&= \frac{37,7cm}{2} + 12cm \\
&= 18,85cm + 12cm \\
&= 30,85cm
\end{aligned}
$$

Teaching Tip:
All the information from the teaching tip on squares needs to be taken into account again.
Examples of circles could be: coins, a biscuit, the face of a watch, the top (or bottom) of a cold drink can or a ring. A few pictures of larger objects can be found on the resource page. If any interest is shown, it may be a worthwhile exercise to discuss the difference between a circle, a cylinder and a sphere at this stage.

Polygons made up of more than one shape

1. When dealing with more unusual shapes, they need to broken up into one or more of the more familiar shapes in order to find the area or the perimeter of them.

   For example:



Note that this shape could be broken up into a rectangle at the bottom (5cm by 2cm) and a rectangle on the top left (1cm by 4cm because the 2cm from the previously mentioned rectangle has been taken away from the 6cm) and a triangle with a base of 4cm (the 1cm has been taken away from the top measurement) and height of 4cm to match the 2nd rectangle. This is not the only possible way to break this irregular shape up. If learners see another way it needs to be encouraged.
In this case, perimeter is quite simple as it just requires adding up all the measurements that can be seen. It is possible in another question that the Theorem of Pythagoras may need to be used.

$$Area = (5cm)(2cm) + (1cm)(4cm) + \frac{1}{2}(4cm)(4cm)$$

$$= 10cm^2 + 4cm^2 + 8cm^2$$
$$= 22cm^2$$

$$\text{Perimeter} = 1\text{cm} + 6\text{cm} + 5\text{cm} + 2\text{cm} + 4\text{cm}$$
$$= 18\text{cm}$$

Topic 4 Area and Perimeter of 2D Shapes

Conversions of measurements

1. Linear measurement

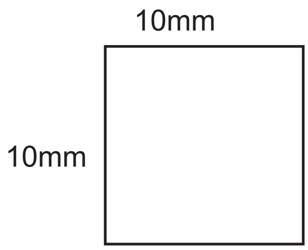
Consider the length of a line:

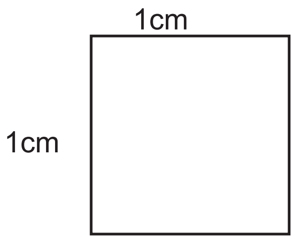
1m = 100cm and 1cm = 10mm and 1km = 1000m

2. Area measurements

When converting area measurements it isn't as straightforward as multiplying or dividing by 10, 100 or 1000 as it is for linear measurements.

Consider this square:





Area = 10mm x 10mm
= 100mm^2

Area = 1cm x 1cm
= 1cm^2

Since 1cm = 10mm, these squares are the same size, so therefore

$$100\text{mm}^2 = 1\text{cm}^2$$

Normally, we would think of the conversion 1cm = 10mm, but since we are dealing with area we need to remember that

$1\text{cm}^2 = 10\text{mm} \times 10\text{mm} = 100\text{mm}^2$

Summary:

CONVERSION OF AREAS SUMMARY
$1cm^2 = 100mm^2$ (10 x 10)
$1m^2 = 10000cm^2$ (100 x 100)
$1km^2 = 1000000m^2$ (1000 x 1000)

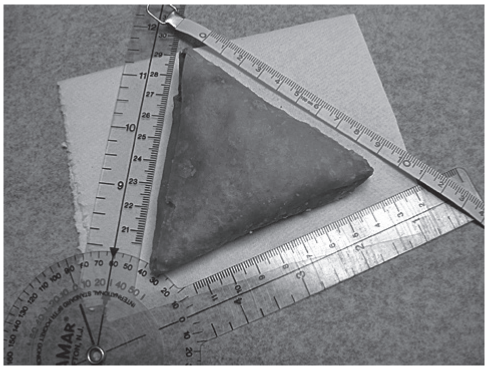
SUMMARY OF ALL 4 MAIN SHAPES:

SHAPE	AREA FORMULA	PERIMETER
Square	$l \times l = l^2$	$4l$
Rectangle	$l \times b$	$2l + 2b$
Triangle	$\frac{1}{2}b \times \perp \text{height}$	$s1 + s2 + s3$
Circle	πr^2	πd or $2\pi r$

TOPIC 4 RESOURCES

TRIANGLES:

NOTE: With this sign it would need to be the inner triangle being discussed due to the rounded edges on the outer one.



CIRCLES:

TOPIC 5: SURFACE AREA AND VOLUME OF 3D OBJECTS

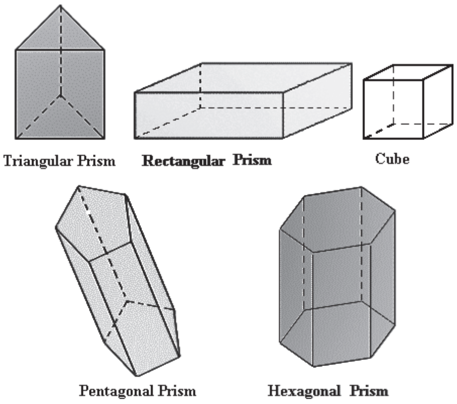
INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area, ‘Measurement’, and counts for 10 % in the final exam.
- The unit covers the use of appropriate formulae to find surface area, volume and capacity of prisms.
- This section is often easier to understand if taught with manipulatives. If the learners can learn through visual representations they will understand better. Use boxes (cubes and cuboid/rectangular prism), cooldrink cans (cylinder) and try and find a piece of wood in the shape of a triangular prism or even a Toblerone box (chocolate) to use in the lessons. Ask the children to bring in 3D objects.
- Although the cylinder is not covered in Grade 8 it would still be worth showing or discussing one in the introductory stages of this topic and relate this to what makes it a right prism (the fact that its base is perpendicular to its height). Similarly it may be helpful to discuss 3D shapes that are not right prisms such as pyramids and cones. Pictures of all of these are included on the resource page at the end of the section.
- The purpose of teaching surface area is mostly to find how much material would be needed to make a 3D shape (useful in deciding what the best container would be for packaging a type of food). This would include knowing how much space is used on the surfaces of all the shapes making up a 3D object (useful if those areas need painting for instance).
- The purpose of teaching volume would be either to find how one or many objects could fit into a certain space or how much liquid a 3D object could hold (capacity).

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Calculate the surface area, volume and capacity of cubes and rectangular prisms • Describe the interrelationship between surface area and volume • Solve problems involving surface area, volume and capacity • Use and convert between appropriate SI units, including: • $mm^2 \leftrightarrow cm^2$ • $cm^2 \leftrightarrow m^2$ • $mm^3 \leftrightarrow cm^3$ • $cm^3 \leftrightarrow m^3$ • Use equivalence between units when solving problems: • $1\,cm^3 \leftrightarrow 1\,ml$ and • $1\,m^3 \leftrightarrow 1\,kl$	• Calculate the surface area, volume and capacity of cubes, rectangular prisms and triangular prisms • Describe the interrelationship between surface area and volume • Solve problems, with or without a calculator, involving surface area, volume and capacity • Use and convert between appropriate SI units, including: $mm^2 \leftrightarrow cm^2 \leftrightarrow m^2 \leftrightarrow km^2$ $mm^3 \leftrightarrow cm^3 \leftrightarrow m^3$ $ml\,(cm^3) \leftrightarrow l \leftrightarrow kl$	• Solve problems using appropriate formulae and conversions between SI units • Calculate the surface area, volume and capacity of cubes, rectangular prisms, triangular prisms and cylinders • Investigate how doubling any or all the dimensions of right prisms and cylinders affects their volume

GLOSSARY OF TERMS

Term	Explanation / Diagram
2D	2-dimensional
3D	3-dimensional
Polygon	A 2D shape in which all the sides are made up of line segments. A polygon is given a name depending on the number of sides it has. Example: A 5 sided polygon is called a pentagon
Solid	An object that occupies space (3-dimensional)
Polyhedron	A solid in which all the surfaces (faces) are flat.
Prism	A solid with parallel equal bases. The bases are both polygons.
Right Prism	A prism which has the sides at right angles to the base. 
Face	A flat surface of a prism
Edge	Where the faces of a prism meet
Vertex	Where the edges of a prism meet (the corner)
Cube	A solid with six equal square faces
Cuboid/Rectangular Prism	A solid with six rectangular faces
Triangular prism	A solid with two equal triangular faces (one is the base) and three rectangular faces
Cylinder	A solid with two equal circular faces (one is the base) and one rectangle (curved)
Net	A 2D shape, that when folded forms a 3D shape. For example:  This is a net of a triangular prism.
Surface Area	The area taken up by the net of a 3D solid. The sum of the area of all the faces.
Volume	The space taken up by a 3D solid. To find volume, the area of the base is multiplied by the perpendicular height. This only works for right prisms.
Capacity	The amount of liquid a 3D shape can hold. It is directly linked to volume.

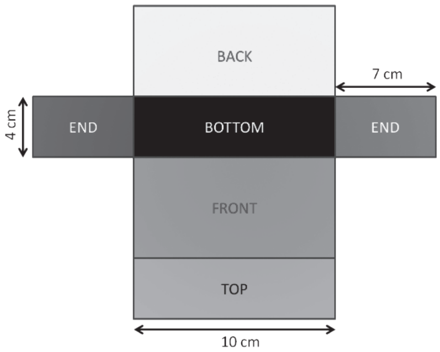
SUMMARY OF KEY CONCEPTS

Surface Area

1. The term ‘surface area’ is linked to 3-dimensional objects only. (When dealing with 2-dimensional shapes the term ‘area’ is used)
2. The answer will always be in measurement squared. For example, cm^2
3. To find the surface area of a 3D object is to find the total area taken up by the net of the 3D shape
   (what the 3D shape looks like in its flattened form)

For example: A box looks like this in its 3D form

But it will look like this in its flattened out 2D form (this is the net of a rectangular prism)



It is best not to try and learn the formulae for surface area of solid shapes but to rather ensure you know what the net of the shape looks like and know how to find the area of the 2D shapes that make up the net (square, rectangle and triangle). The surface area of the 3D shape would then be those added together.

SHAPE	AREA FORMULA
Square	$l \times l = l^2$
Rectangle	$l \times b$
Triangle	$\frac{1}{2}b \times \perp$ height
Circle	πr^2

Consider the box again. It is made up of 6 rectangles. To find the surface area you will need to find the area of each of these rectangles, then add all of them together.

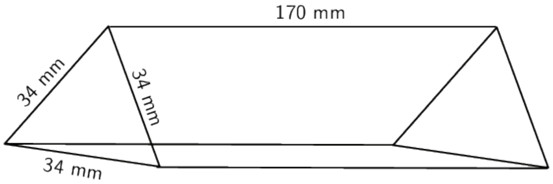
The length is 10cm, the breadth is 4cm and the height is 7cm. However, notice that the front and back are the same size, as are the top and bottom and the two sides.

∴ The Surface Area of the rectangular prism can be found by:

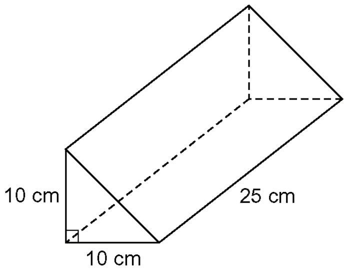
$$
\begin{aligned}
& 2(l)(b)+2(l)(h)+2(b)(h) \\
&= 2(10cm)(4cm)\ +\ 2(10cm)(7cm)\ +\ 2(4cm)(7cm) \\
&= 80cm^2\ +\ 140cm^2\ +\ 56cm^2 \\
&= 276cm^2
\end{aligned}
$$

4. When finding the surface area of a triangular prism, it must be noted that this shape is made up of 3 rectangles and two triangles. The two triangles will always be the same size as it is a right prism.

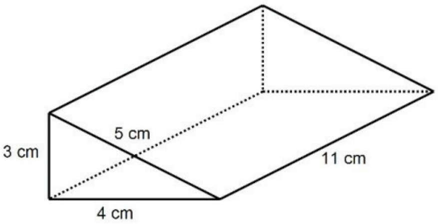
   The 3 rectangles' sizes depends on the kind of triangle forming the base and top of the triangular prism.
   If it is an equilateral triangle, the 3 rectangles will all be the same size.
   If it is an isosceles triangle, 2 of the rectangles will be the same size and one will be different.
   If it is a scalene triangle, all 3 rectangles will differ in size.
   However, ALL 3 rectangles will share one dimension – the length which matches with the height of the prism.
   The following three diagrams demonstrate this:



In this case, all 3 rectangles have dimensions of 170mm by 34mm

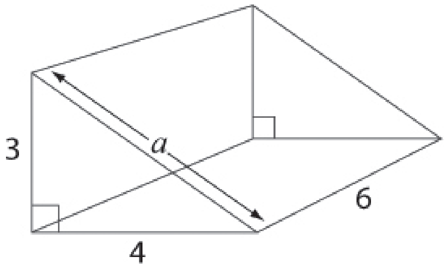


In this case, all three rectangles have one dimension they share – 25cm.
However, two of the rectangles have a second dimension of 10cm and the third one would require using the theorem of Pythagoras to find its other dimension (14,14cm).



In this case, all three rectangles have one dimension they share – 11cm. However, the second dimension of each of the 3 rectangles is 3cm, 4cm and 5cm respectively.

5. A worked example of finding surface area of a triangular prism:



Teaching Tip:
Encourage learners to draw a net of the prism if necessary to help them see more clearly what shapes make up the prism and what all the dimensions are. It should be clear that one of the dimensions is missing in order to find the area of one of the rectangles (the large on one the top right of the figure).

Learners need to notice that they will need to use the theorem of Pythagoras before completing the task of finding the surface area of the whole shape.

Solution:

$First\ find`a'$

$a^2 = 3^2 + 4^2$ Pythagoras

$a^2 = 9 + 16$

$a^2 = 25$

$a = 5$

$$\begin{aligned}\text{Surface Area} &= 2 \text{ triangles} + 3 \text{ rectangles}\\ &= 2\left(\frac{1}{2}(4\text{cm}) \times (3\text{cm})\right) + (6\text{cm})(4\text{cm}) + (6\text{cm})(3\text{cm}) + (6\text{cm})(5\text{cm})\\ &= 2\,(6\text{cm}^2) + 24\text{cm}^2 + 18\text{cm}^2 + 30\text{cm}^2\\ &= 12\text{cm}^2 + 24\text{cm}^2 + 18\text{cm}^2 + 30\text{cm}^2\\ &= 84\text{cm}^2\end{aligned}$$

Teaching Tip:
Make it a habit to continually ask learners what it is they have actually found when they have answered a surface area question. Encourage them to tell you that they now know how much cardboard they would need to make this 3D shape or they now know how much surface space is around this 3D shape if they were required to paint it.

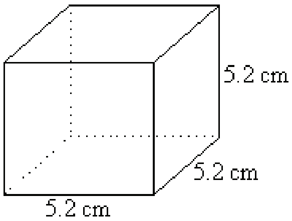
Volume

1. The term 'volume' is linked to 3-dimensional objects only.
2. The answer will always be in measurement cubed. For example, cm^3
3. To find the volume of any right prism, the basic formula is:
   Area of base x perpendicular height
   Notice again that you are required to know how to find the area of the basic shapes (square, rectangle and triangle).

VOLUME OF:	AREA OF BASE x HEIGHT
Cube	$(l \times l) \times ht$ $= l \times l \times l = l^3$
Rectangular prism (cuboid)	$(l \times b) \times h = lbh$
Triangular prism	$(\frac{1}{2} b \times h) \times H$ Note: The first 'h' represents the height of the triangle which is required in order to find the area of the base. The 2nd 'H' represents the height of the prism.

For example:

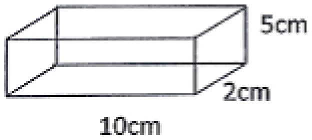
a)



To find the volume of a cube, the area of the base is multiplied by the height. As these measurements are all equal it is the same as cubing the measurement.

$$\begin{aligned} \text{Volume} &= (l \times b) \times h \\ &= l^3 \\ &= (5{,}2\text{cm})^3 \\ &= 140{,}61\text{cm}^3 \end{aligned}$$

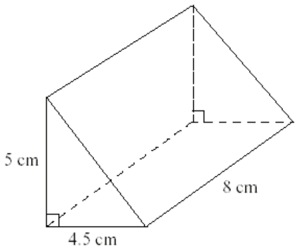
b)



To find the volume of this rectangular prism (or cuboid), we need to find the area of the base (a rectangle) and multiply it by the height. These three dimensions is what gives us the ‘cubed’ in the answer.

$$\begin{aligned}\text{Volume} &= (\text{l}\times\text{b})\times\text{h}\\ &= (10\text{cm}\times 2\text{cm})\times 5\text{cm}\\ &= \ 100\text{cm}^3\end{aligned}$$

c)



To find the volume of this triangular prism, we need to find the area of the base (a triangle) and multiply it by the height. These three dimensions is what gives us the ‘cubed’ in the answer.

Teaching Tip:
Height is not always clear to the learners. Constantly remind them that height of a prism refers to the distance from the shape that ‘gives it its name’ to same shape on the opposite end. So for a triangular prism, the height is the distance from the triangle to the triangle. For a rectangular prism, the height is the distance from the rectangle at one end to the rectangle of the same size at the other end. Height does not always mean from bottom to top.

$$\begin{aligned}\text{Volume} &= (\tfrac{1}{2}\text{b}\times\text{h})\times\text{H}\\ &= (\tfrac{1}{2}\ (4.5\text{ cm})(5\text{ cm}))\times 8\text{ cm}\\ &= 11{,}25\text{ cm}^2\times 8\text{ cm}\\ &= \ 90\text{ cm}^3\end{aligned}$$

(This step is not essential – it is shown in order to remember that the area of the base needs to be multiplied by the height)

Capacity

1. Capacity is how much liquid a 3D shape (solid) can hold. It is directly linked to volume.
2. The following three conversions should be learnt:

$$1\text{cm}^3 = 1\text{ ml}$$
$$1000\text{cm}^3 = 1000\text{ml} = 1\text{l}$$
$$1\text{m}^3 = 1000\text{l} = 1\text{kl}$$

3. For example:

A teaspoon holds 5millilitres. This means its size is 5cm^3
A carton of fruit juice holds 1 litre (1 000 millilitres). This means its size is 1000cm^3.

4. A large fish tank has the following dimensions: 110cm, 45cm and 60cm.

Find how many litres of water the tank can hold.
First we need to find the volume:

$$\begin{aligned}\text{Volume} &= (\text{l}\times\text{b})\times\text{h}\\ &= (110\text{cm} \times 45\text{cm}) \times 60\text{cm}\\ &= 297\ 000\text{cm}^3\end{aligned}$$

This needs linking back to capacity: $297\ 000\ \text{cm}^3 = 297\ 000\text{ml} = 297\text{l}$

Topic 5 Surface Area and Volume of 3D Objects

Conversions

Although part of this has already been covered in the notes on Area and perimeter of 2D shapes it is worth looking at again.

1. Linear measurement

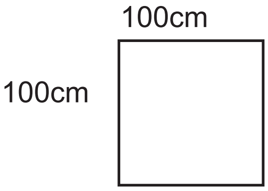
   Consider the length of a line:

   1m = 100cm and 1cm = 10mm and 1km = 1000m

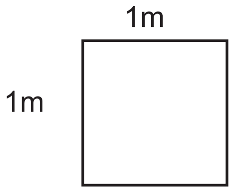
2. Area measurements

   When converting between units of area it isn't as straightforward as multiplying or dividing by 10, 100 or 1000 as it is for linear measurements.

   Consider this square:



$$\begin{aligned}\text{Area} &= 100\text{cm} \times 100\text{cm}\\ &= 10000\text{cm}^2\end{aligned}$$



$$\begin{aligned}\text{Area} &= 1\text{m} \times 1\text{m}\\ &= 1\text{m}^2\end{aligned}$$

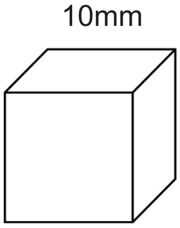
Since $1\text{m} = 100\text{cm}$, these squares are the same size, so therefore

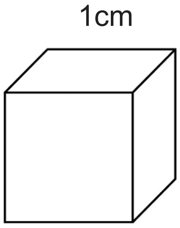
$10000\text{cm}^2 = 1\text{m}^2$

Normally, we would think of the conversion $1\text{m} = 100\text{cm}$, but since we are dealing with area we need to remember that

$1\text{m}^2 = 100\text{cm} \times 100\text{cm} = 10000\text{cm}^2$

3. Similarly for volume:





$$\begin{aligned}\text{Volume} &= l^3\\ &= (10\text{mm})(10\text{mm})(10\text{mm})\\ &= 1000\text{mm}^3\end{aligned}$$

$$\begin{aligned}\text{Volume} &= l^3\\ &= (1\text{cm})(1\text{cm})(1\text{cm})\\ &= 1\text{cm}^3\end{aligned}$$

As the cubes are the same size $(10\text{mm} = 1\text{cm})$, their volumes must be the same as well

$1000\text{mm}^3 = 1\text{cm}^3$

Normally we would think of the conversion $1\text{cm} = 10\text{mm}$, but since we are dealing with volume we need to remember that

$1\text{cm}^3 = 10\text{mm} \times 10\text{mm} \times 10\text{mm} = 1000\text{mm}^3$

Here is a summary of the more common conversions required:

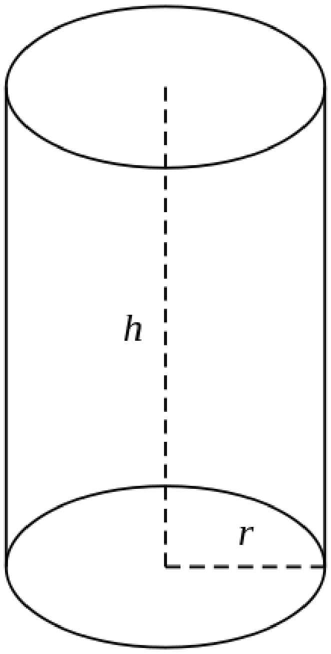
AREA	VOLUME
$1\text{cm}^2 = 100\text{mm}^2$ (10×10)	$1\text{cm}^3 = 1000\text{mm}^3$ $(10 \times 10 \times 10)$
$1\text{m}^2 = 10000\text{cm}^2$ (100×100)	$1\text{m}^3 = 1000000\text{cm}^3$ $(100 \times 100 \times 100)$
$1\text{km}^2 = 1000000\text{m}^2$ (1000×1000)	$1\text{km}^3 = 1000000000\text{m}^3$ $(1000 \times 1000 \times 1000)$

TOPIC 5 RESOURCES

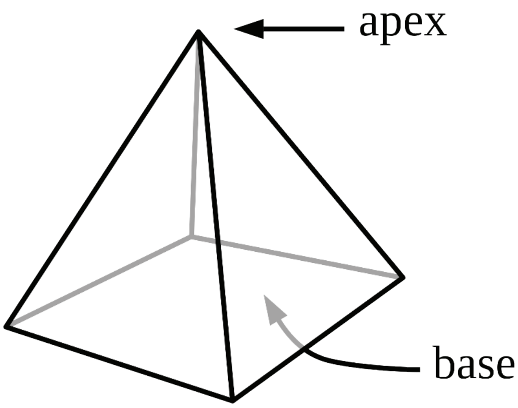
Triangular Prism:

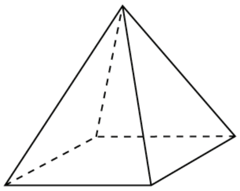
For discussion purposes only: (as mentioned in the introduction)

1. Cylinders (right prisms)

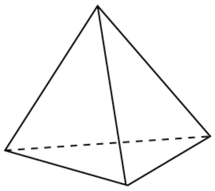


2. Pyramids:

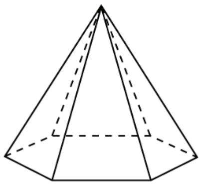




square-based pyramid

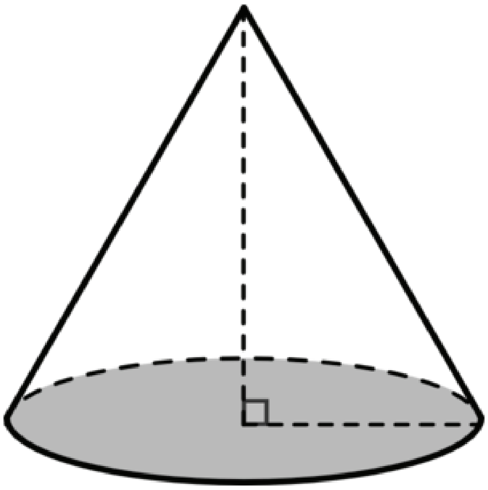


triangular-based pyramid



hexagonal-based pyramid

3. Cones:



TOPIC 6: DATA HANDLING

INTRODUCTION

- This unit runs for 10,5 hours, which includes: Collect, organize and summarize data (4 hours); Represent data (3 hours) and Interpret, analyse and report data (3,5 hours)
- It counts for 10 % in the final exam.
- The unit covers all aspects of data handling as laid out above
- It is important to note that learners need to be exposed to a variety of contexts that deal with social and environmental issues. Learners also need to practice collecting, organising, representing and analysing data. Time needs to be spent discussing and showing learners the differences between the different types of graphs and when one may be more useful than the other.
- If you have access to computers, the learners could use them to draw some of the graphs in Excel.
- The purpose of teaching data handling is to provide learners with the knowledge of how data is collected and the ways in which it can be represented.
- Surveys, graphs and charts are often used by the media to inform, persuade and at times, mislead the audience. Learners need to be made aware of this.

SEQUENTIAL TEACHING TABLE

INTERMEDIATE PHASE/GRADE 7	GRADE 8	GRADE 9/ FET PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Collect data using appropriate sources • Pose questions relating to issues in own environment • Distinguish between samples and populations • Design and use simple questionnaires to answer questions: with yes/no type responses and with multiple choice responses • Organize and record data • Group data into intervals • Summarize and distinguish between ungrouped numerical data by determining mean, median and mode • Determine the range of the data • Critically read and interpret data • Critically analyse data • Summarize data in short paragraphs	• Collect data • Pose questions relating to social, economic, and environmental issues • Select appropriate sources for the collection of data • Distinguish between samples and populations • Design and use simple questionnaires to answer questions with multiple choice responses • Organize and record data • Group data into intervals • Summarize and distinguish between ungrouped numerical data by determining measures of central tendency and measures of dispersion • Summarize data in short paragraphs that include the role of extremes in the data	• Collect data • Pose questions relating to social, economic, and environmental issues • Select and justify appropriate sources for the collection of data • Distinguish between samples and populations • Select and justify appropriate methods for collecting data • Organize numerical data in different ways in order to summarize by determining measures of central tendency, measures of dispersion, including extremes and outliers • Organize data according to more than one criteria • Critically read and interpret data • Critically compare two sets of data related to the same issue • Critically analyse data • Summarize data in short paragraphs that include the role of extremes and outliers in the data

GLOSSARY OF TERMS

Term	Explanation / Diagram
Data	Facts or information collected from people or objects
Population	The entire group of people or objects that data is being collected from
Sample	A smaller part of the population if the population is too large
Random	How to choose a smaller sample of the population in an attempt to not be biased
Questionnaire	A set of printed questions with a choice of answers used in the data collection process
Survey	The collecting of data from a group of people
Discrete data	Data that can only take certain values. For example, the number of learners in a class (there can't be half a learner)
Continuous data	Data that can take on any value within a certain range. For example, the heights of a group of learners (heights could be measured in decimals)
Tally	A way of keeping count by drawing marks. Every fifth mark is drawn across the previous four (to form a gate-like diagram) so you can easily see groups of five.
Frequency tables	A table that lists a set of scores and their frequency. Often used with tallies. It summarises the totals and shows how often something has occurred.
Stem-and-leaf tables	A way to summarise data by splitting each value (eg 26) into a stem (2) and a leaf (6)
Mean	The average of a set of numbers. Calculated by adding all the values then dividing by how many numbers there are.
Median	The middle number in a sorted list of numbers. To find the median, place all numbers in order from smallest to biggest and find the middle number.
Mode	The number that appears the most often in a set of data. There can be two modes. There could also be no mode in a set of data.
Range	The difference between the highest and lowest value in a set of data.
Bar Graphs	A graph drawn using rectangular bars to show how large each value is. The bars could be horizontal or vertical
Double bar graphs	A double bar graph displays information using two bars beside each other comparing two sets of data linked to the same thing. Example: A school's results over two years would form the double bar and a different school's results could be in a separate double bar next to it.
Histograms	A graph representing data that is grouped into ranges and each bar represents data that follows on from the previous bar. Example, one bar could represent how many learners got a mark from 40-49 and the bar immediately next to it would represent 50-59.
Pie chart	A circular chart divided into sectors. Each sector represents an amount of something relative to its size.
Broken line graph	A graph that uses points connected by lines to show how something changes in value as time goes by or as something else happens.

SUMMARY OF KEY CONCEPTS

Data handling is an important section of mathematics as it used in research. Tertiary education makes extensive use of research.

It involves the following steps:
- finding a research question
- collecting data (through questionnaires and interviews etc)
- organising and summarising the data
- representing the data
- interpreting and analysing the data

Collecting Data

1. Data can be collected in a number of ways. A questionnaire is often used. In order to make the analysing of the data easier after having people fill out a questionnaire, it needs to be carefully thought out. It is best to give multiple choice answers instead of open ended questions.

For example:
What Grade are you in? (Place a cross next to your grade)

Gr 8		Gr 9		Gr 10		Gr 11		Gr 12	

2. A difficult question to analyse would be: What do you think about the new school tuckshop?
3. There would be a large variety of answers making it more difficult to organise and analyse. In order to collect data, the population needs to be considered. If a person wanted to do a survey amongst teenagers for example, it will be impossible to survey all the teenagers in the country or even a school. Therefore a sample of your population (teenagers) would be needed. A random sample is the better way to choose a sample so that each person in the population has an equal opportunity to be part of the survey. This could be done by taking class lists and picking every fifth learner. It would be a biased sample if a person just chose their friends.

Teaching Tip:
Take some time to discuss the difference between a population and a sample with the learners. Discuss a few situations and ask learners who the population is and how they would go about choosing a sample of the population. A discussion of the word biased would also be appropriate at this stage.

Topic 6 Data Handling

Organizing and Summarizing Data

1. Once the data has been collected, it needs to be organised and recorded.
2. Tally and frequency tables could be used at this stage or if a computer is available, all the data could be put into an excel spreadsheet (excel can also create graphs for you from the data).

   For example:
   If you had done a survey on learner's favourite colours, a tally table with the frequency recorded may look like this:

Yellow	||||	4
Red	𝍸	5
Blue	𝍸 |	6
Green	|	1
Pink	||||	4

3. Stem-and-leaf plots could also be used to summarise a set of data.

   If your data represented learner's results in mathematics it could look like this:

<table>
<tr><th>Step 1</th><th>Step 2</th><th>Step 3</th></tr>
<tr><td>Make the stem by writing the tens digits from least to greatest</td><td>Make the leaves by writing each ones digit in to the right of its ten digit</td><td>draw a line to separate the stems and leaves. Add a title and key</td></tr>
<tr><td>4
5
6</td><td>4 8 9
5 4 5 8
6 0 3</td><td>Title<table><tr><th>Stem</th><th>Leaf</th></tr><tr><td>4</td><td>8 9</td></tr><tr><td>5</td><td>4 5 8</td></tr><tr><td>6</td><td>0 3</td></tr></table></td></tr>
</table>

4. In order to summarise the data, it is good to look at the measures of central tendency and the measures of dispersion.
5. <u>Measures of central tendency.</u> A measurement of data that indicates where the middle of the information lies.

   A measure of central tendency is a single value that describes the way in which a group of data cluster around a central value. It is a way to describe the centre of the data set. There are three measures of central tendency: mean, median and mode

For example: The following numbers represent the shoe sizes of some learners in your class

8 6 6 5 7 9 8 6 5 8 9 6 7

To find the:

a. Mean

Add all the values and divide by the number of values

$$\frac{90}{13} = 6{,}92$$

b. Median

Arrange the values from smallest to biggest and find the middle value. (If there is an even amount of data, add the two middle values and divide by two – in other words find the mean of the two middle numbers)

5 5 6 6 6 6 7 7 8 8 8 9 9

The median is 7

c. Mode

Find a number that appears most often. There can be two modes – if this is the case we call the set of data bimodal.

The mode is 6

Mean is the most common form used to find one value to represent a set of data. It is considered a good measure as it takes all the data into account. However, it also has its limitations – if there is an outlier (an extreme value which is much higher or much lower than the the other values) then it isn't always a good measure. It also can't be used if the data is not numerical – for example if your data is a list of colours like in the above example of a stem-and-leaf plot

6. <u>Measures of dispersion.</u> This shows how spread out the data is. There are many measures of dispersion which will be covered in later grades but this year the only one required is 'range'.
   To find the range, subtract the lowest value from the highest value.
   From the shoe sizes set of data:

$$9 - 5 = 4$$

4 is the range

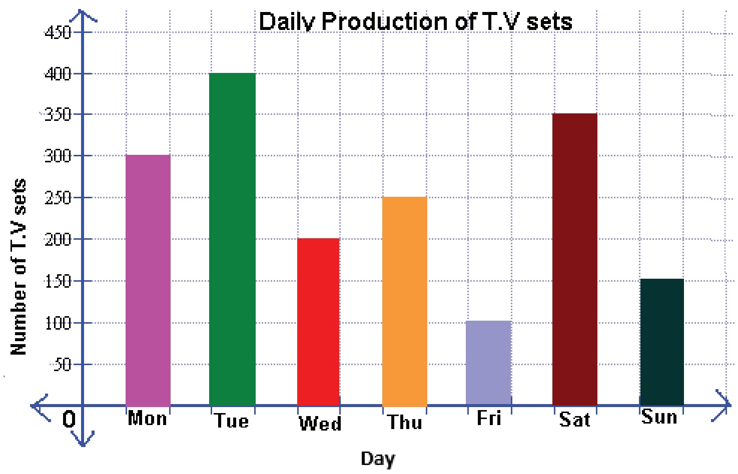
Representing Data

1. Once the data has been organised and summarised it needs to be represented in a visual way that is easy to read and understand.
2. Graphs are the common choice to represent data. The visual representation makes it easier for a person to understand the data collected rather than a long wordy explanation.
3. There are two types of data and each one lends itself to being represented using a different graph. Discrete data has clear separation between the different possible values, while continuous data doesn't. We use bar graphs for displaying discrete data, and histograms for displaying continuous data.
4. Graphs and examples:

a. Bar Graphs

A bar graph is usually used to represent discrete data and is usually in categories.

For example:

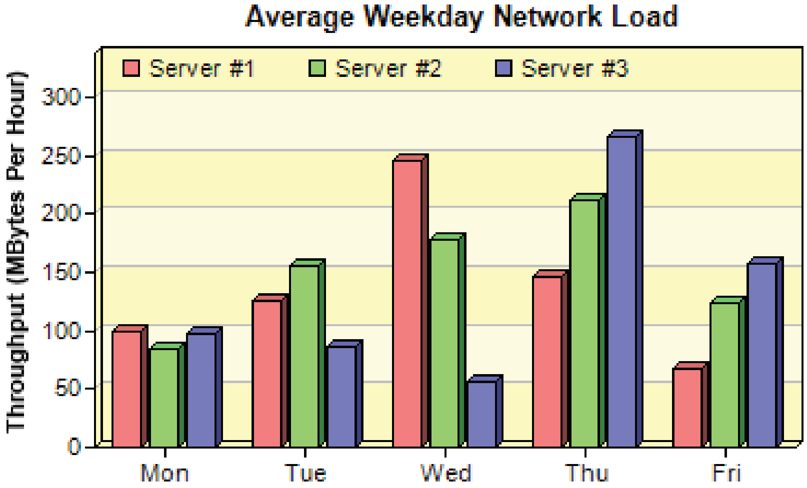


In this example, the number of televisions made is the discrete data and the day of the week is the category

b. Double bar graphs

A bar graph that has a double bar to compare two sets of similar data.

For example:

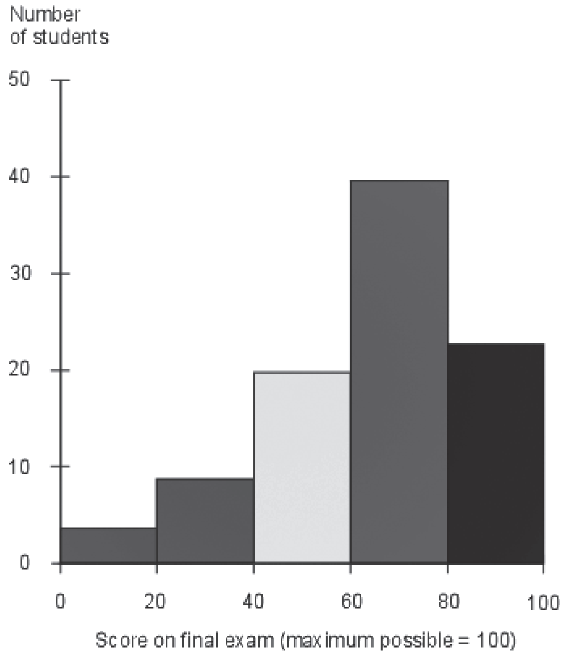


This double bar graph is comparing the network load of three different servers on each day of the week.

c. Histogram

A histogram is used to show grouped data. One bar's data follows directly from the previous one.

For example:

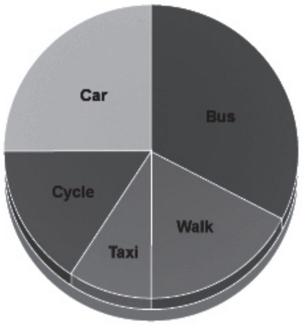


Notice that 20-40 follows on immediately from 0-20 and 40-60 follows on from 20-40 and so on

d. Pie Chart

A pie chart is a circle that is divided into slices where the size of the slice represents the size of the data.

For example:



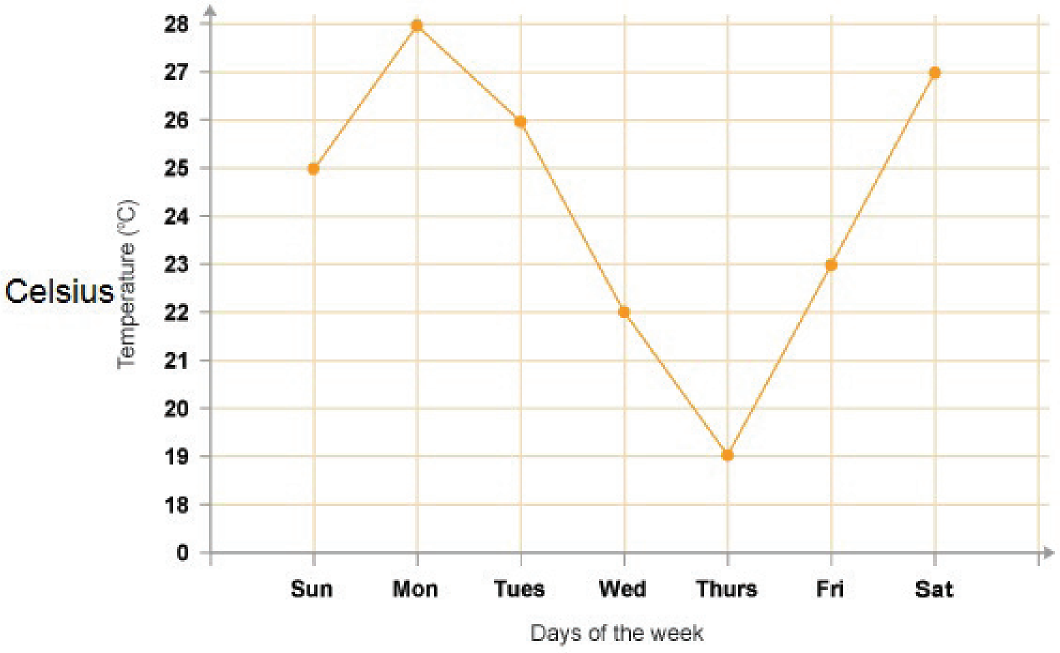
This pie chart represents the transport used by a group of people. It is quick and easy to see that the bus is the most used and that a taxi is the least used.

e. Broken Line Graph

A broken line graph shows change.

For example:

The line graph shows the weather in new york city the first week of June 2011.



Interpreting, Analysing and Reporting Data

1. Interpreting data means to look at the data that has been collected and represented and consider what it all means.
2. It is at this stage that consideration needs to be given to whether the data could be misleading (has it been presented in a way that could give the wrong impression?) or if it has been manipulated (has it been changed in order to mislead?).
3. Changing the scale of the axes on a graph or changing the class intervals on a histogram can lead to data not being represented correctly.
4. When reporting on the results of a research the following information needs to be considered:

An explanation needs to be given discussing:

- the purpose of the research
- the population and how a sample was taken
- the way the data was collected
- what conclusions you have come to

5. It is also useful to try and -
   - make predictions based on the data collected
   - discuss any limitations that may have affected the study.

Notes