Grade 6

MATHEMATICS CONTENT BOOKLET: TARGETED SUPPORT

Term 4

A MESSAGE FROM THE NECT

NATIONAL EDUCATION COLLABORATION TRUST (NECT)

Dear Teachers,
This learning programme and training is provided by the National Education Collaboration Trust (NECT) on behalf of the Department of Basic Education (DBE)! We hope that this programme provides you with additional skills, methodologies and content knowledge that you can use to teach your learners more effectively.

What is NECT?
In 2012 our government launched the National Development Plan (NDP) as a way to eliminate poverty and reduce inequality by the year 2030. Improving education is an important goal in the NDP which states that 90% of learners will pass Maths, Science and languages with at least 50% by 2030. This is a very ambitious goal for the DBE to achieve on its own, so the NECT was established in 2015 to assist in improving education and to help the DBE reach the NDP goals.

The NECT has successfully brought together groups of relevant people so that we can work collaboratively to improve education. These groups include the teacher unions, businesses, religious groups, trusts, foundations and NGOs.

What are the Learning programmes?
One of the programmes that the NECT implements on behalf of the DBE is the 'District Development Programme'. This programme works directly with district officials, principals, teachers, parents and learners; you are all part of this programme!

The programme began in 2015 with a small group of schools called the Fresh Start Schools (FSS). Curriculum learning programmes were developed for Maths, Science and Language teachers in FSS who received training and support on their implementation. The FSS teachers remain part of the programme, and we encourage them to mentor and share their experience with other teachers. The FSS helped the DBE trial the NECT learning programmes so that they could be improved and used by many more teachers. NECT has already begun this embedding process.

Everyone using the learning programmes comes from one of these groups; but you are now brought together in the spirit of collaboration that defines the manner in which the NECT works. Teachers with more experience using the learning programmes will deepen their knowledge and understanding, while some teachers will be experiencing the learning programmes for the first time.

Let's work together constructively in the spirit of collaboration so that we can help South Africa eliminate poverty and improve education!

www.nect.org.za

Contents

INTRODUCTION:
THREE PRINCIPLES OF TEACHING MATHEMATICS

PRINCIPLE 1: TEACHING MATHEMATICS DEVELOPMENTALLY

What is developmental teaching and what are the benefits of such an approach?

- The human mind develops through phases or stages which require learning in a certain way and which influence whether a child is ready to learn something or not.
- If learners are not ready to learn something, it may be due to the fact that they have not reached that level of development yet or they have missed something previously.
- The idea that children's thinking develop from concrete to abstract (Piaget, 1969), was refined (Miller & Mercer, 1993) to include a middle stage, namely the "concrete-representational-abstract" stages. This classification is a handy tool for mathematics teaching. We do not need to force all topics to follow this sequence exactly, but at the primary level it is especially valuable to establish new concepts following this order.
- This classification gives a tool in the hands of the teacher, not only to develop children's mathematical thinking, but also to fall back to a previous phase if the learner has missed something previously.
- The need for concrete experiences and the use of concrete objects in learning, may gradually pass as learners develop past the Foundation Phase. However, the representational and abstract development phases are both very much present in learning mathematics at the Intermediate and Senior Phases.

How can this approach be implemented practically?

The table on page 7 illustrates how a developmental approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the topics, suggestions are made to implement this principle in the classroom:

- Where applicable, we suggest an initial concrete way of teaching and learning a concept and we provide educational resources at the end of the lesson plan or topic to assist teachers in introducing the idea concretely.
- Where applicable, we provide pictures (representational/semi-concrete) and/or diagrams (representational/semi-abstract). It may be placed at the clarification of terminology section, within the topic itself or at the end of the topic as an educational resource.
- In all cases we provide the symbolic (abstract) way of teaching and learning the concept, since this is, developmentally speaking, where we primarily aim to be for learners to master mathematics.

PRINCIPLE 2: TEACHING MATHEMATICS MULTI-MODALLY

What is multi-modal teaching and what are the benefits of such an approach?

- We suggest that teachers present mathematics topics in three forms to provide for all learners' learning styles and preferences. They (a) explain the idea by speaking about a topic, (b) illustrate it by showing pictures or diagrams and finally (c) present the idea by symbolising it in numbers and mathematical symbols.
- Teaching in more than one form (multi-modal teaching) includes hearing the same mathematical idea in spoken words (auditory mode), seeing it in a picture or a diagram (visual mode) and writing it in a mathematical way (symbolic mode).
- Learners differ in the way they learn and understand mathematical ideas. For one learner it is easier to understand through hearing and for the other through seeing. That is why we open both pathways to the symbolic mode – because here they do not have a choice, they all have to reach that mode, be it through hearing or seeing.

How can this approach be implemented practically?

The table on page 8 illustrates how a multi-modal approach to mathematics teaching may be implemented practically, with examples from several content areas.

What does this look like in the booklet?

Throughout the booklets, within the topics at the Senior Phase, we suggest ways to apply this principle in the classroom:

- The verbal explanations under each topic and within each lesson plan, provide the "speak it" or auditory mode.
- The pictures and diagrams give suggestions for the "show it" mode (visual mode).
- The calculations, exercises and assessments under each topic and within each lesson plan, provide the "symbol it" or symbolic mode of representation.

PRINCIPLE 3: SEQUENTIAL TEACHING

What is sequential teaching and what are the benefits of such an approach?

- Learners with weak basic skills in mathematics will find future topics increasingly difficult. A solid foundation is required for a good fundamental understanding.
- In order to build a solid foundation in maths, we teach concepts systematically. If we teach concepts out of that order, it can lead to difficulties in grasping concepts.
- Systematic teaching according to CAPS builds progressive understanding and skills.
- In turn, this builds confidence in the principles of a topic before he/she is expected to apply the knowledge and proceed to a higher level.
- We have to continuously review and reinforce previously learned skills and concepts.
- If learners link new topics to previous knowledge (past), understand the reasons why they learn a topic (present) and know how they will use the knowledge in their lives ahead (future), it can help to motivate them and to remove many barriers to learning.

How can this approach be implemented practically?

If a few learners in your class are not grasping a concept, you as the teacher need to take them aside and teach them the concept again (perhaps at a break or after school).

If the entire class are battling with a concept, it will need to be taught again, however this could cause difficulties in trying to keep on track and complete the curriculum in time.

To finish the year's work within the required time and to also revise topics, we suggest:

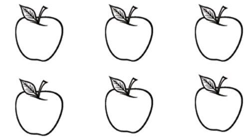
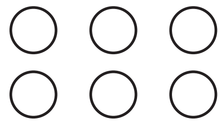
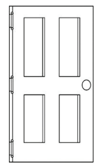
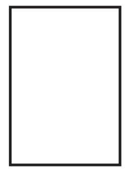
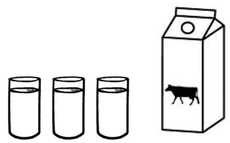
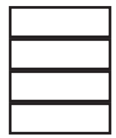
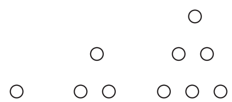
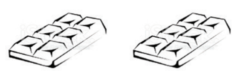
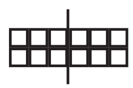
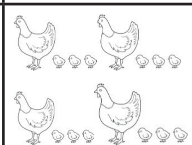
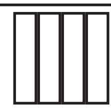
- Using some of the time of topics with a more generous time allocation, to assist learners to form a deeper understanding of a concept, but also to catch up on time missed due to remediating and re-teaching of a previous topic.
- Giving out revision work to learners a week or two prior to the start of a new topic. For example, in Grade 8, before you are teaching Data Handling, you give learners a homework worksheet on basic skills from data handling as covered in Grade 7, to revise the skills that are required for the Grade 8 approach to the topic.

What does this look like in the booklet?

At the beginning of each topic, there are two parts that detail

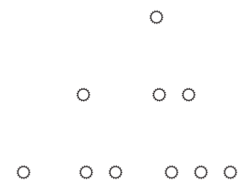
- The SEQUENTIAL TEACHING TABLE lays out the knowledge and skills covered in the previous grade, in the current grade and in the next grade.
- The LOOKING BACK and LOOKING FORWARD summarises the relevant knowledge and skills that were covered in the previous grade or phase and that will be developed in the next grade or phase.

THREE-STEP APPROACH TO MATHEMATICS TEACHING: CONCRETE-REPRESENTATIONAL-ABSTRACT

CONCRETE: IT IS THE REAL THING		REPRESENTATIONAL: IT LOOKS LIKE THE REAL THING		ABSTRACT: IT IS A SYMBOL FOR THE REAL THING	
Mathematical topic	Real or physical For example:	Picture	Diagram	Number (with or without unit)	Calculation or operation, general form, rule, formulae
Counting	Physical objects like apples that can be held and moved			6 apples	$2 \times 3 = 6$ *or* $2+2+2=6$ *or* $\frac{1}{2}$ *of* $6 = 3$ *or* $\frac{2}{3}$ *of* $6 = 4$
Length or distance	The door of the classroom that can be measured physically			80 cm wide 195 cm high	Perimeter: $2L + 2W = 390 + 160$ $= 550cm$ Area: $L \times W = 195 \times 80$ $= 15\,600cm^2$ $= 1.56m^2$
Capacity	A box with milk that can be poured into glasses			1 litre box 250 ml glass	$4 \times 250ml = 1\,000ml$ $= 1$ litre or 1 litre $\div 4 = 0.25$ litre
Patterns	Building blocks			1: 3: 6…	$\frac{n(n+1)}{2}$
Fraction	Chocolate bar			$\frac{6}{12}$	$\frac{6}{12} = \frac{1}{2}$ or $\frac{1}{2}$ of 12 $= 6$
Ratio	Hens and chickens		* *** * *** * *** * ***	4:12	$4 : 12 = 1 : 3$ Of 52 fowls ¼ are hens and ¾ are chickens. ie 13 hens. 39 chickens
Mass	A block of margarine			500g	$500g = 0,5$ kg or calculations like 2 ½ blocks = 1,25kg

Teaching progresses from concrete -> to -> abstract. In case of problems, we fall back <- to diagrams, pictures, physically.

MODES OF PRESENTING MATHEMATICS WHEN WE TEACH AND BUILD UP NEW CONCEPTS

Examples	SPEAK IT - explain	SHOW IT - embody	SYMBOL IT - enable
	• To introduce terminology • To support auditory learning • To link mathematics to real life	• To help storing and retrieving ideas • To help visual learning • To condense information to one image	• To promote mathematical thinking • To convert situations to mathematics • To enable calculations
IP: Geometric patterns	"If shapes grow or shrink in the same way each time, it forms a geometric pattern or sequence. We can find the rule of change and describe it in words. If there is a property in the shapes that we can count, each term of the sequence has a number value" "You will be asked to draw the next term of the pattern, or to say how a certain term of the pattern would look. You may also be given a number value and you may be asked, which term of the pattern has this value?**"**	 T1 T2 T3 T4 Question: (a) Draw the next term in this pattern. (b) Describe this pattern. What is the value of the 9th term of this pattern (T9)? **Which term has a value of 120?** To draw up to the 9th term of the sequence and to find out which term has a value of 120, is slow. One is now almost forced to deal with this problem in a symbolic way.	Say out loud: 1; 3; 6… 1; 3; 6; 10… 1; 3; 6;10;15 Inspect the number values of terms: T1: 1 = 1 T2: 3 = 1+2 T3: 6 = 1+2+3 T4: 10 = 1+2+3+4 T9: 45 = 1+2+3+4+5+6+7+8+9 General rule: The value of term n is the sum of n number of consecutive numbers, starting at 1.
SP: Grouping the terms of an algebraic expression	"We can simplify an algebraic expression by grouping like terms together. We therefore have to know how to spot like terms. Let us say we have to sort fruit in a number of baskets and explain the variables or the unknowns in terms of fruits. Try to imagine the following pictures in your mind:"	Although not in a real picture, we can paint a mind picture to help us understand the principle of classification: • Basket with green apples (a) • Basket with green pears (b) • Basket with green apples and green pears (ab) • Basket with yellow apples (a^2) • Basket with yellow apples and green pears (a^2b) Or in diagram form a □ b ○ ab □○ a^2 ■ a^2b ■○	Group and simplify the following expression: $4b - a^2 + 3a^2b - 2ab - 3a + 4b + 5a - a - 2ab + 2a^2b + a^2b$ $= -3a + 5a - a + 4b + 4b - 2ab - 2ab - a^2 + 3a^2b + 2a^2b + a^2b$ $= a + 8b - 4ab - a^2 + 6a^2b$

TOPIC 1: WHOLE NUMBERS

INTRODUCTION

- This unit runs for 1 hour.
- It is part of the Content Area, 'Numbers, Operations and Relationships' which counts for 50% in the final exam.
- The unit covers revision of the content taught in detail in Term 2 for calculations with numbers up to 9 digits.

SEQUENTIAL TEACHING TABLE

GRADE 5 / INTERMEDIATE PHASE	GRADE 6 / INTERMEDIATE PHASE	GRADE 7 / SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Order, compare and represent numbers to at least 6-digit numbers • Represent odd and even numbers to at least 1 000. • Recognize the place value of digits in whole numbers to at least 6 digit numbers. • Round off to the nearest 5, 10, 100 and 1 000	• Order, compare and represent numbers to at least 9-digit numbers • Represent prime numbers to at least 100 • Recognize the place value of digits in whole numbers to at least 9-digit numbers • Round off to the nearest 5, 10, 100 and 1 000	• The general topic is not covered in the Senior Phase. • These operations are performed in Algebraic context later in the senior and FET phases.

GLOSSARY OF TERMS

Term	Explanation / Diagram
Whole Numbers	The numbers in the set {0, 1, 2, 3, ...} are called whole numbers. Whole numbers are counting numbers including zero.
Place Value	The value of the digit depends on its position in the number. In 65 314 728, the 2 is in the 'tens' position, so it has a value of 20.
Rounding Off	Numbers are either rounded up or down to the nearest multiple of 10, 100 or 1 000.
Digit	A single character used in a numbering system. In the decimal system the digits are 0 to 9.
Inverse Operations	Addition can be checked by subtraction. Multiplication can be checked by division
Order Of Operation	The acronym BODMAS stands for the order in which multiple operations in the same sum are done: Brackets (parts of a calculation inside brackets) always come first; Orders (numbers involving powers or square roots) are done second; Division and/or Multiplication is done third (if there are more than one division or multiplication required, work from left to right); Addition and/or Subtraction is done last (if there are more than one addition or subtraction required, work from left to right)
Estimation	The best guess arrived at after considering all the information given in a problem. To estimate is to find an answer that is very close to the exact answer

SUMMARY OF KEY CONCEPTS

Counting in millions, ten millions and hundred millions

1. There are many ways to practice counting and it is important to have learners count in the given interval such as millions without always starting with the first multiple of a given number or with zero.
2. Counting can be done orally at the start of a lesson.

Examples:
Counting in millions:
1 000 000, 2 000 000, 3 000 000…
1 340 000, 2 340 000, 3 340 000…

Counting in ten millions:
10 000 000, 20 000 000, 30 000 000…
12 300 000, 22 300 000, 32 300 000…

Counting in hundred millions:
100 000 000, 200 000 000, 300 000 000…
102 000 000, 202 000 000, 203 000 000…

3. Learners must be able to count forwards and backwards.

Numbers up to one hundred million in words and in numeric form

	HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
Digits	5	6	5	3	1	4	7	2	8
What the digit means in terms of its position	This represents 5 hundred millions	This represents 6 ten millions	This represents 5 millions	This represents 3 hundred thousands	This represents 1 ten thousand	This represents 4 thousands	This represents 7 hundreds	This represents 2 tens	This represents 8 ones or units
Numeric	500 000 000	60 000 000	5 000 000	300 000	10 000	4000	700	20	8
How you would say it	Five hundred million	Sixty million	Five million	Three hundred thousand	Ten thousand	Four thousand	Seven hundred	Twenty	Eight
What the value of each digit is in the number	The digit 5 has a value of 500 000 000	The digit 6 has a value of 60 000 000	The digit 5 has a value of 5 000 000	The digit 3 has a value of 300 000	The digit 1 has a value of 10 000	The digit 4 has a value of 4000	The digit 7 has a value of 700	The digit 2 has value of 20	The digit 8 has a value of 8

1. In Numerals: **565 314 728**
2. In Words:
   Five hundred and sixty five million, three hundred and fourteen thousand, seven hundred and twenty-eight.
3. Expanded Form:
   500 000 000 + 60 000 000 + 5 000 000 + 300 000 + 10 000 + 4000 + 700 + 20 +8
4. Identify the place value of each digit in a number up to hundred millions (9 digits)

565 314 728

The digit 5 is in the **hundred millions** place
The digit 6 is in the **ten millions** place
The digit 5 is in the **millions** place
The digit 3 is in the **hundred thousands** place
The digit 1 is in the **ten thousands** place
The digit 4 is in the **thousands** place
The digit 7 is in the **hundreds** place
The digit 2 is in the **tens** place
The digit 8 is in the **unit /ones** place

Teaching Tip:
Learners often misread or write numbers incorrectly when they have zeros. This can be prevented if educators emphasise the importance of zeros as place holders in the number. Learners must know that if hundred millions are mentioned there will be at least 9 digits in the number.

Compare numbers within hundred millions (9 digits):

1. Learners must compare and order numbers according to their value. Working with place value tables or writing numbers directly below each other is a good strategy that will help learners make accurate comparisons.
2. When comparing numbers with a view of finding the smaller number, we use the < sign to indicate that the number to the left of the sign is smaller than the number to the right of the sign.

Example:
Which number is smaller, 467 237 981 or 467 230 600?
When comparing numbers, look at the value of each digit starting from the left and gradually moving to the right of the number.

HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
4	6	7	2	3	7	9	8	1
4	6	7	2	3	0	6	0	0

Zero (0) thousands is smaller than 7 thousands.
So, 467 230 600 is smaller than 467 237 981.
This is written as 467 230 600 < 467 237 981

3. When comparing numbers with a view of finding the larger number, we use the > sign to indicate that the number to the left of the sign is bigger than the number to the right of the sign.

Example:
Which number is greater, 300 712 935 or 300 712 846?

HM Hundred Million	TM Ten Million	M Million	HT Hundred Thousand	TT Ten Thousand	T Thousand	H Hundred	T Ten	U Units (One)
3	0	0	7	1	2	9	3	5
3	0	0	7	1	2	8	4	6

A table makes it easier to work out which number is bigger/smaller than another. Work from left to right. If they are the same, continue to compare until the values of the digits are not the same.

The values of the digits in the hundreds place are not the same.

9 hundred is **greater** than **8** hundred. So 300 712 **9**35
is **greater** than 300 712 **8**46.

This is written as 300 712 935 > 300 712 846

4. Arrange the numbers from smallest to biggest **(ascending order)**

Example:
324 688, 32 468, 3 246 880, 324 560 004

Look at which number has the **least** digits and this will become the first number. It will help if leaners use a place value table and write numbers underneath each other.

Answer: 32 468, 324 688, 3 246 880, 324 560 004

5. Arrange the numbers from biggest to smallest **(descending order)**

324 688, 32 468, 3 246 880, 324 560 004

Look at which number has the **most** digits. This becomes the first number in the sequence and is then followed by the next biggest number and so on.

Answer: 324 560 004, 3 246 880, 324 688, 32 468

Rounding off

1. Decide which digit is the last one you need to keep. You will know this because it is the digit in the place you are asked to round off to.
2. Leave it the same if the next digit is less than 5.

Example:
Round 74 to the nearest multiple of 10.

We want to keep the 7 in the 10s place.
The next digit is 4 which is less than 5, so no change is needed to 7.
74 gets rounded down to 70. 74 ≈ 70

3. Increase the number by 1 if the next digit is 5 or more.

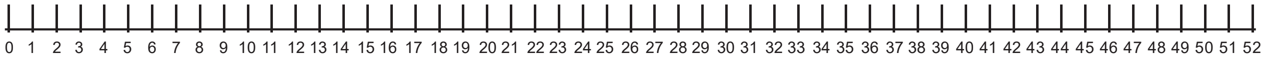
Example:
Round 86 to the nearest multiple of 10.

We want to keep the 8 in the 10s place.
The next digit is 6 which is more than 5, so increase the 8 by 1 to 9.
86 gets rounded up to 90. 86 ≈ 90

4. Learners must be able to round to the nearest 5, 10, 100 or 1000.

Rounding up or down to the nearest multiple of 5

1. From Grade 5 on learners have to round up or down to the nearest multiple of 5, 10, 100, 1 000 and so on. The multiples of 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45 and so on. We can keep on rounding to multiples of 5, even with three- or four- or more digit numbers.
2. A number line can help us to understand this new idea:



When we round down or up to the nearest multiple of 5, the number line helps us to see to which multiple of 5 the given number is closest.

Example:

- 3 is closer to 5 than to 0
- 16 is closer to 15 than to 20
- 34 is closer to 35 than to 30

TOPIC 2: MULTIPLICATION

INTRODUCTION

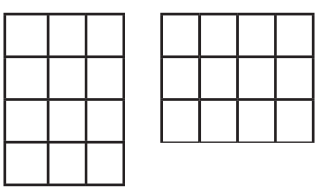
- This unit runs for 5 hours.
- It is part of the Content Area, ‘Numbers, Operations and Relationships’ which counts for 50% in the final exam.
- The unit covers revision of strategies covered in Term 2 where the emphasis is on working with the multiplication of 4-digit by 3-digit numbers.

SEQUENTIAL TEACHING TABLE

GRADE 5 / INTERMEDIATE PHASE	GRADE 6 / INTERMEDIATE PHASE	GRADE7 / SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Multiply 3- by 2-digit numbers
• Estimate the answer to a multiplication calculation
• Use strategies to multiply
  • building up and breaking down numbers
  • use a number line
  • rounding off, compensating
  • doubling and halving
• Know multiples and factors of two digit numbers to 100
• Know the multiplicative property of 1
• Recognise, use commutative property of number
• Recognise and use associative property of number
• Recognise and use distributive property of number
• Solve problems with whole numbers in financial and measurement contexts
• Compare quantities of the same kind (ratio)
• Compare quantities of different kinds (rate)
	• Multiply 4- by 3-digit numbers
	• Estimate the answer to a multiplication calculation
	• Use strategies to multiply, with or without brackets
	  • building up and breaking down numbers
	  • rounding off, compensating
	  • doubling and halving
	  • column method
	• Know multiples and factors of 2-digit and 3-digit numbers and prime factors of numbers to 100
	• Know the multiplicative property of 1
	• Recognise, use commutative property of number
	• Recognise and use associative property of number
	• Recognise and use distributive property of number
	• Solve problems with whole numbers involving multiplication in various contexts
	• Compare quantities of the same kind (ratio)
	• Compare quantities of different kinds (rate)
		• Revise work done in Grade 6
		• Extend the range for multiplication to include the multiplication of
		  • integers
		  • fractions
		  • decimal fractions

GLOSSARY OF TERMS

Term	Explanation / Diagram
Multiples	A number formed by multiplying two other numbers. Example: 28 is the seventh multiple of 4, since 7 x 4 = 28. 28 is also the fourth multiple of 7, since 4 x 7 = 28. The number itself is its own first multiple: 7 is the first multiple of 7 (1 x 7 = 7)
Factors	Whole numbers that divide exactly into another number, or numbers that were multiplied to make that number. For example, 1, 2, 4, 7, 14 and 28 are factors of 28. Prime factors of a composite number are those prime numbers that are all multiplied to reach that number as its product. The prime factors of 28 are 2 and 7, because 2 x 2 x 7 = 28.
Multiplicative Property of One	One multiplied by, or divided into a number does not change that number: one is the identity element for multiplication and division: 14 x 1 = 14; 14 ÷ 1 = 14.
Distributive Property of Multiplication over Addition	If we multiply a number by numbers that are added together, it is the same as multiplying the number by each of the other numbers. Example: Five learners each have three brothers and two sisters. To save time and space, we can write it: 5 times (+) or in numbers = 5(3 + 2) = 15 + 10 = 25 siblings altogether.
Terminology Used in Multiplication Equations or Calculations	9 x 4 = 36 ↓ ↓ ↓ Multiplicand x Multiplier = Product The product is the answer to a multiplication sum.
Inverse Property	Multiplication is the inverse of division. Division is the inverse of multiplication. Example: 4 x 7 = 28 ∴ 28 ÷ 7 = 4 and also 28 ÷ 4 = 7 28 ÷ 7 = 4 ∴ 4 x 7 = 28 and also 7 x 4 = 28
Commutative Law	The order of numbers in addition and multiplication may change and the answer will remain the same. Example: Rectangle a. has three blocks to the side and four down (3 x 4). Rectangle b. has four blocks to the side and three down (4 x 3). Both have 12 blocks altogether because 3 x 4 = 12 and 4 x 3 = 12 
Halving	To divide a number into two equal parts, which is the same as dividing the number by two: when we halve 14, we have two equal parts of seven each.
Doubling	To multiply a number by two, or add the same number to it, so that the answer is twice as many as the number: when we double seven, we have fourteen. A double number is always even.

Term	Explanation / Diagram
Rounding off Symbol	When one number is not exactly equal to, or the same as another number, we use the symbol $\approx$ to indicate that it is approximately, or almost the same as the other when we round off or estimate.
Financial Context	Calculating money is a calculation in a financial context. We calculate it in the currency we use, like Rands and Cents in South Africa.
Ratio	A ratio is a relationship between two numbers indicating how many times the first number contains the second. For example, if a bowl of fruit contains eight oranges and six lemons, then the ratio of oranges to lemons is eight to six (that is, 8:6, which is equivalent to the simplified ratio 4:3).
Rate	Rate is also a ratio. It is used to compare two quantities of things that depend on each other – if one quantity changes, the other is also changing. Price and speed are instances of rate that are familiar in learners' everyday life.

SUMMARY OF KEY CONCEPTS

Multiples, Factors and Factorising

1. A multiple is formed when we multiply two numbers.

   Example:
   28 is the first multiple of 28, since 1 x 28 = 28
   56 is the second multiple of 28, since 2 x 28 = 56
   84 is the fourth multiple of 28, since 3 x 28 = 84

2. This means that a multiple of a number is exactly divisible by that number.

   Example:
   84 is exactly divisible by 28.
   However, it is not only 28 and 3 which 84 can be divided exactly by. 84 can be divided by 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42 and 84.
   This fact makes all of 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42 and 84 factors of 84.

3. A factor is a number that divides exactly into another number. Factor pairs of a number are two numbers that were multiplied to make that number. All the factors of a number are all the numbers that can be exactly divided into that number.

   Example:
   2 x 42 = 84, so a factor pair of 84 is 2 and 42. (All numbers have 1 and themselves as factors too).
   All factors of 84 are:

   1 ; 2 ; 3 ; 4 ; 6 ; 7 ; 12; 14; 21; 28; 42; 84

   - 1 and 84 are a factor pair of 84 because 1 x 84 = 84
   - 2 and 42 are a factor pair of 84 because 2 x 42 = 84
   - 3 and 28 are a factor pair of 84 because 3 x 28 = 84... and so on

4. In Grade 6 learners start factorising numbers into their prime factors, as follows:

   Example:

   a. Find the prime factors of 84

   b. Write 84 as the product of its prime factors.

2	84
2	42
3	21
7	7
	1

   Answer:

   a. 2, 3 and 7 are the prime factors of 84. Note that this is just a list of the prime numbers that are factors of 84.

   b. 84 = 2 x 2 x 3 x 7 or 84 = 2^2 x 3 x 7
   (we can write 2^2 instead)

5. Factorising is easier when we know and apply the rules of divisibility to the number:

Rules of divisibility: A number can be divided exactly

- by 2 if the last digit is an even number: 324 is divisible by 2, because 4 is an even number.
- by 3 if the sum of the digits is a multiple of 3: 324 is divisible by 3, because 3 + 2 + 4 = 9.
- by 4 if the last two digits are a multiple of 4: 324 is divisible by 4, because 24 is a multiple of 4.
- by 5 if the last digit is either 5 or 0: 324 is not divisible by 5, because the last digit is 4.
- by 6 if the number is divisible by 2 and by 3: 354 is divisible by 6, because it is divisible by 2 and by 3.
- by 8 if the last three digits are a multiple of 8: 324 ÷ 2 = 162; 162 ÷ 2 = 81 and 81 cannot be divided by 2 again, so 324 cannot be divided by 8.
- by 9 if the sum of the digits is a multiple of 9: 324 is divisible by 9, because 3 + 2 + 4 = 9.
- by 10 if the last digit is 0: 324 is not divisible by 10, because the last digit is 4.

Multiplication of a 4-digit number by a 3-digit number

1. In Grade 5 learners used various 'break-down' strategies to multiply, which we will now apply to 4-digit numbers multiplied by 3-digit numbers.

   The first two strategies use the breakdown method. However, if we break up the 3-digit number only, it becomes hard to multiply it by a 4-digit number, therefore we break down both numbers. The larger number we expand and write down as the sum of its parts. The 3-digit number we either expand too, or we break it down in its factors if possible.

   Teaching tip: Spend some time to talk to learners about how a number is formed by adding other numbers, or by multiplying other number, as follows:

   Example:
   Multiply 5 235 x 108

   In our example keep in mind that
   108 = 100 + 8 (expanded) and 108 = 12 x 9 (factorised).

   For this strategy, it is important that learners know their multiplication facts, to be able to do short multiplication in the break-down method.

   Option one: both numbers expanded
   5 235 x 108
   = (5 000 + 200 + 30 + 5) x (100 + 8)
   = (5 000x100)+ (5 000x8)+(200x100)+(200x8)+(30x100)+(30x8)+(5x100)+(5x8)
   = 500 000 + 40 000 + 20 000 +1 600 + 3 000 + 240 + 500 + 40
   = 500 000 + 60 000 +4 600 + 740 + 40
   = 565 380

   Option two: 4-digit number expanded and 3-digit number factorised
   5 235 x 108
   = [5 000 + 200 + 30 + 5] x 12 x 9
   = [(5 000 x 12) + (200 x 12) + (30 x 12) + (5 x 12)] x 9
   = (60 000 + 2 400 + 360 + 60) x 9
   = 62 820 x 9
   = [60 000 + 2 000 + 800 + 20] x 9
   = (60 000 x 9) + (2 000 x 9) + (800 x 9) + (20 x 9)
   = 540 000 + 18 000 + 7 200 + 180
   = 565 380

2. The third strategy (**option three**) is only suitable where the 3-digit number ends in 6, 7, 8, 9 or in 60, 70, 80, 90. In this strategy we see the number as the difference between two numbers. This method is also called rounding up and compensating.

Example:

Multiply 5 235 x 108

= (5 000 + 200 + 30 + 5) x (110 – 2)

= [(5 000x110)-(5 000x2)]+[(200x110)-(200x2)]+[(30x110)-(30x2)]+[(5x110)-(5x2)]

= [550 000 – 10 000] + [22 000 – 400]+[3 300 – 60] + [550 – 10]

= 540 000 + 21 600 + 3 240 + 540

= 565 380

3. We can only use doubling and halving in some cases to multiply, because it works only in cases where one of the numbers is a multiple of 8, 16, 32 or 64. The number that can be divided by 8, 16, 32 or 64, is the one that is halved and as that number is halved, the other number is doubled, as follows:

Example: 288 x 2 435

Halving	Doubling
288	2 435
144	4 870
72	9 740
36	19 480
18	38 960
9	77 920
9 x 77 920 = 701 280	

Estimating by Rounding

Learners estimate answers to multiplication sums by rounding to check if the answers are reasonable. Compare the estimations below against the answer that we have calculated above (2 435 x 288 = 701 280) and decide which one gives the best approximate answer:

a. Rounding both numbers to their first digit:
2 435 x 288 ≈ 2 000 x 300 ≈ 600 000

b. Rounding both numbers to the closest multiple of 100:
2 435 x 288 ≈ 2 400 x 300 ≈ 720 000

c. Rounding the 3-digit number to the closest multiple of 100:
2 435 x 288 ≈ 2 435 x 300 ≈ 730 500

Ratio

Ratio is used to compare the sizes of two or more quantities. The key to understanding ratio is to understand that we are comparing the size or magnitude of objects.

Example 1:
In our school there are 840 boys and 910 girls. This is a ratio of 840:910 which can be simplified to 12:13 (we divided the HCF of 70 both sides)

Example 2:
At the SPCA there is one cat for every three dogs. This can be written in the ratio 1:3
Ensure that learners understand that there is not only one cat and three dogs. There are many options possible such as 3 cats and 9 dogs or 10 cats and 30 dogs.
Asking learners for possible options will assist them in understanding the concept.

Rate

Rate is also a ratio, used to compare two quantities that depend on each other – if one quantity changes, the other is also changing. Price and speed are instances of rate.

Example:
A car used 122 litres petrol to travel 1 464 km from Soweto to Cape Town. How many kilometres did the car travel on one litre petrol?
1 464 km/122 litre = 12 km/litre.

Multiplication in vertical columns

Previous strategies have prepared the way for learners to understand this method:

				2		
		3	2	4		
		3	2	4		
		2	4	3	5	(space out numbers properly)
	×		2	8	8	(units directly underneath units, 100s underneath 100s)
	1	9	4	8	0	(x 8)
1	9	4	8	0	0	(x 80 – note the difference between products of 8 and 80)
4	8	7	0	0	0	(x 200 – note the two zeros at the end of the product)
7	0	1	2	8	0	(add all products for final product)

TOPIC 3: COMMON FRACTIONS

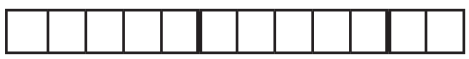
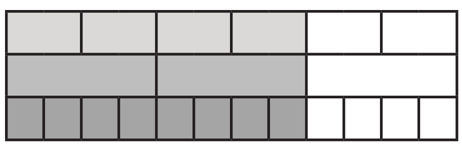
INTRODUCTION

- This unit runs for 5 hours.
- It is part of the Content Area, 'Numbers, Operations and Relationships' which counts for 50% in the final exam.
- The unit covers revision of all the calculations done in Term 2 relating to common fractions, although the greater emphasis this term should be on the comparison of equivalent forms.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Describe and order fractions to at least twelfths • Perform various calculations that involve fractions with the same denominators • Solve problems in contexts that involve grouping and sharing. • Recognise equivalent forms when the denominators are multiples of each other	• Compare and order common fractions including tenths and hundredths • Perform various calculations with fractions where the denominators are multiples of each other • Solve problems in contexts involving fractions • Find percentages of whole numbers • Work with decimal fractions • Recognise and create equivalent forms including decimals and percentages	• Perform a variety of calculations using fractions that may or may not have the same denominator or denominators that are multiples of each other. • Apply all calculation principles relating to fractions including algebraic contexts later in the Senior Phase

GLOSSARY OF TERMS

Term	Explanation / Diagram
Common Fraction	A fraction is a part or parts of a whole that has been shared equally into a number of parts. A fraction can also be a part of a number of things divided into equal groups. We write common fractions with one digit above and one below a fraction line, like $\frac{2}{5}$. 'Common fraction' is one type of fraction. [A 'decimal fraction' is a fraction with an unwritten denominator in powers of 10, indicated by its place value, eg $2.6 = 2\frac{6}{10}$]
Denominator	The digit telling the number of equal parts into which a whole is divided, or the number of equal small groups into which a big group is divided. We write this digit under the fraction line, like in $\frac{2}{5}$ [5 is the denominator].
Numerator	The digit telling how many parts or groups we are dealing with from those into which the whole is divided. That number appears above the fraction line, like $\frac{2}{5}$ [2 is the numerator].
Mixed Number	A mixed number is a way of writing that shows all the parts, like in $\frac{12}{5}$. This is two wholes and fifths, written as $2\frac{2}{5}$ which has a whole number and a fraction. 
Equivalent fractions	Equivalent fractions are fractions that are equal in size or have the same value. The numerator and denominator of one of the equivalent fractions is a multiple of the numerator and denominator of the other. $\frac{4}{6}$ in the first row $=$ $\frac{2}{3}$ in the second row $=$ $\frac{8}{12}$ in the third row. 
Simplify fractions	We can simplify fractions, or write them in their simplest form: • If the numerator is larger than the denominator, we change the fraction to a mixed number: $\frac{23}{6} = 2\frac{5}{6}$ • If the numerator and the denominator can both be divided by the same number, we do that: $\frac{6}{8} = \frac{3}{4}$ because both 6 and 8 can be divided by 2
Lowest Common Multiple (LCM)	The lowest multiple that two or more numbers share. We use this often in addition and subtraction calculations with fractions, to find the LCM of the denominators of the fractions that we are working with. In the example below, the LCM of 4, 6 and 3 is 12. Example: $\frac{3}{4} + \frac{5}{6} + \frac{2}{3} = \frac{9}{12} + \frac{10}{12} + \frac{8}{12} = \frac{27}{12} = 2\frac{3}{12} = 2\frac{1}{4}$
Highest Common Factor (HCF)	The highest factor that two or more numbers share. We use this often in simplifying fractions or getting equivalent fractions. In the example below the highest factor that the numerator (56) and the denominator (84) share, is 28: $\frac{56}{84} = \frac{2}{3}$ [both numerator and denominator were divided by 28]

SUMMARY OF KEY CONCEPTS

Simplification and Equivalent Fractions

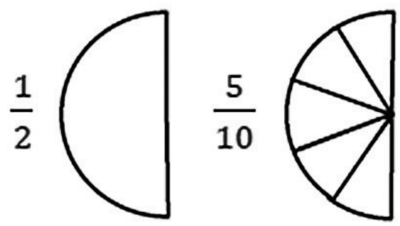
1. Simplification of fractions involves writing the fraction in its simplest form and makes use of the division rules and skills learners have been taught previously.

Example:

$$\frac{10}{15} = \frac{(10 \div 5)}{(15 \div 5)} = \frac{2}{3}$$

This fraction is in its simplest form as the numerator and the denominator do not have any more common factors.

2. Learners must be able to recognise and determine equivalent fractions. Simplification of fractions is an important part of understanding the equivalence of fractions. Equivalent fractions are fractions that have the same value.



To change $\frac{1}{2}$ to $\frac{5}{10}$ we multiply the numerator and the denominator each by 5 like this:

$$\frac{1 \times 5}{2 \times 5} = \frac{5}{10}$$

the value of $\frac{5}{5}$ is 1 so we are not changing the value of the fraction only its appearance

$$\frac{12}{18} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{2}{5} = \frac{4}{10} = \frac{3}{15}$$

3. The numerator and the denominator are multiplied or divided by the same amount as this is the only way to obtain equivalent fractions.

Example:

$$\frac{10}{15} = \frac{(10 \times 4)}{(15 \times 4)} = \frac{40}{60}$$

4. Folding an A4 paper in half, quarters, eighths and sixteenths is a practical way of demonstrating equivalence.
5. Another form of equivalence is introduced in Grade 6, namely the equal forms of common fractions, decimal fractions and percentage.

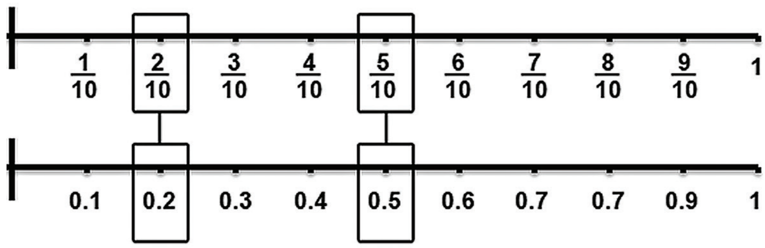
   Complete the table below to show the equivalence of common fractions, decimal fractions and percentage:

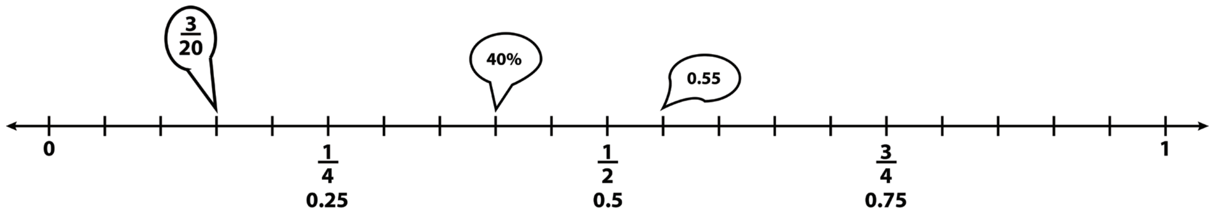
Common fraction	Decimal Fraction	Percentage
Example: $\frac{3}{4}$	0.75	75%
$\frac{3}{5}$		
	0.4	
		35%
$\frac{1}{4}$ of 24 oranges = 6		
		15% of 200 learners

Solution:

Common fraction	Decimal Fraction	Percentage
Example: $\frac{3}{4}$	0.75	75%
$\frac{3}{5}$	0.6	60%
$\frac{4}{10}$	0.4	40%
$\frac{35}{100}$	0.35	35%
$\frac{1}{4}$ of 24 oranges = 6 oranges	0.25 of 24 oranges = 6 oranges	25% of 24 oranges = 6 oranges
$\frac{15}{100}$ or $\frac{3}{20}$ of 200 learners = 30 learners	0.15 of 200 learners = 30 learners	15% of 200 learners = 30 learners

Number lines are essential in forming the concept of the size (magnitude)and position of fractions.





Comparing Fractions

Fractions can be compared in the following way:

1. If any fraction is in the form of a mixed fraction, change it to an improper fraction.
2. Study the denominators of all the fractions and find the LCM.
3. Convert all fractions to their equivalent fractions with the LCM as denominator.
4. Compare the numerators and arrange the fractions in the desired order.

Example:
Arrange the following fractions in ascending order:

$$\frac{5}{6};\frac{1}{2};\frac{2}{3};\frac{7}{15};\frac{9}{10};\frac{3}{5}$$

Solution:
The LCM of 6, 2, 3, 15, 10 and 5 is 30

$$\frac{5\times5}{6\times5};\ \frac{1\times15}{2\times15};\ \frac{2\times10}{3\times19};\ \frac{7\times2}{15\times2};\ \frac{9\times3}{10\times3};\ \frac{3\times6}{5\times6}$$

$$\frac{25}{30};\ \frac{15}{30};\ \frac{20}{30};\ \frac{14}{30};\ \frac{27}{30};\ \frac{18}{30}$$

Having the same denominator, the fractions can be compared and arranged in ascending order according to their numerators.

Therefore, if we arrange in ascending order:

$\frac{14}{30}; \frac{15}{30}; \frac{18}{30}; \frac{20}{30}; \frac{25}{30}; \frac{27}{30}$

But then we need to write the fractions in their original format (simplest form):

$\frac{7}{15}; \frac{1}{2}; \frac{3}{5}; \frac{2}{3}; \frac{5}{6}; \frac{9}{10}$

Four Basic Calculations with Fractions

Look at the following examples from Mathematics materials. Note that addition and subtraction are done in the same manner and multiplication and division are related.

1. Adding and subtracting fractions with denominators that are the same.

$\frac{1}{7} + \frac{3}{7} = \frac{4}{7}$

In this example the denominators are the same and therefore the numerators can simply be added or subtracted.

$\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

2. Adding and subtracting fractions with denominators that differ.

$\frac{7}{15} + \frac{1}{5}$

$\frac{7}{15} + \frac{1 \times 3}{5 \times 3} = \frac{7}{15} + \frac{3}{15} = \frac{10}{15}$

$\frac{7}{8} - \frac{5}{16}$

$\frac{7 \times 2}{8 \times 2} - \frac{5}{16} = \frac{14}{16} - \frac{5}{16} = \frac{9}{16}$

In both the addition and subtraction examples above, the denominators differ and one needs to make them the same. In the addition calculation, 15 is the LCM of 5 and 15 and in the subtraction calculation, 16 is the LCM of 8 and 16. That is why both the numerator 1 and the denominator 5 in the first have to be multiplied by 3 and why both the numerator 7 and the denominator 8 in the second have to be multiplied by 2. Simplify answers if necessary.

3. Multiplying fractions with any denominators

$\frac{2}{5} \times \frac{6}{7} = \frac{2 \times 6}{5 \times 7} = \frac{12}{35}$

$\frac{1}{4} \times \frac{2}{3} = \frac{1 \times 2}{4 \times 3} = \frac{2}{12}$

All numbers must be in fraction form, even whole- and mixed numbers. Now multiply the numerators. Then multiply the denominators. If the answer needs to be simplified, do that.

4. Dividing fractions by fractions or fractions by whole numbers

   Division is the inverse operation of multiplication, so we switch the second number (the numerator becomes the denominator and the denominator becomes the numerator) and proceed to multiply the numbers.

$$\frac{2}{5} \div \frac{2}{3} = \frac{2}{5} \times \frac{3}{2} = \frac{2 \times 3}{5 \times 2} = \frac{6}{10} = \frac{3}{5}$$

take the reciprocal of the divisor

$$\frac{4}{7} \div 2 = \frac{4}{7} \times \frac{1}{2} = \frac{4 \times 1}{7 \times 2} = \frac{4}{14} = \frac{2}{7}$$

Note: Point out to learners that multiplication and division DO NOT require the same denominators. This is a misconception shared by many learners.

TOPIC 4: PROPERTIES OF 3D OBJECTS

INTRODUCTION

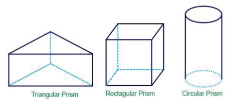
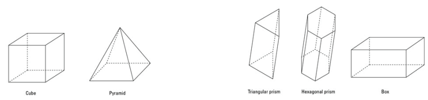
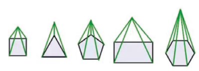
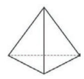
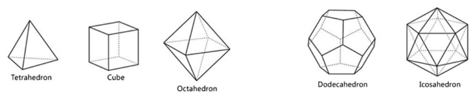
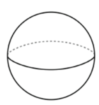
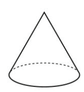
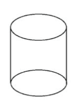
- This unit runs for 5 hours.
- It is part of the Content Area, ‘Space and Shape’ which counts for 15% in the final exam.
- The unit deals with the consolidation and extension of knowledge relating to 3D shapes. In Term 2 learners had to build shapes using nets but now they build skeleton versions of the 3D shapes so that they can compare edges, faces and vertices and how these would determine the classification of the particular shape.
- Although learners have worked on this topic previously it is important that learners are able to not only work with real 3D shapes but also to complete written exercises relating to 3D shapes. Interpreting pictures is more difficult than working with real shapes.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Recognise, visualise and name 3D objects in the environment and geometric settings
• Describe 3D objects focussing on the shape of faces, number of faces and whether they have flat or curved edges.
• Make 3D models from cut out polygons.
• Trace nets of boxes that have been cut open
	• Recognise, visualise and name 3D objects in the environment and geometric settings, focussing on:
	  • Rectangular prisms
	  • Cubes
	  • Tetrahedrons
	  • Pyramids
	• Similarity and differences of pyramids and other tetrahedrons
	• Describe 3D objects focussing on the shape of faces, number of faces, number of vertices and the number of edges.
	• Make 3D models in skeletal form using straws, toothpicks or cocktail sticks and using nets to make the 3D shapes.
		• These skills are applied to geometric understanding and the complete understanding of measurement later in the Senior and FET Phases.

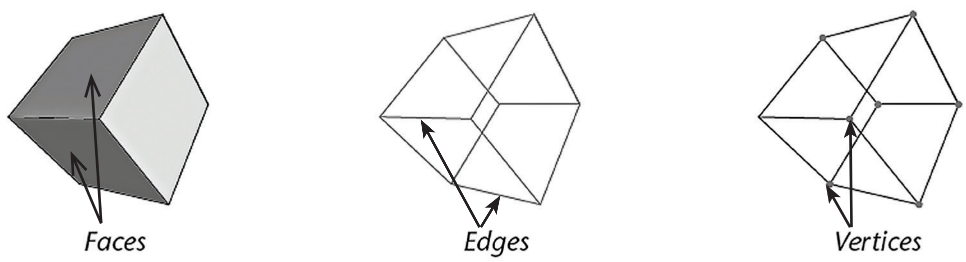
GLOSSARY OF TERMS

Term	Explanation / Diagram
Three-dimensional (3D) Object	3D objects are figures that do not lie in a plane. These objects have length, width (breadth) and height. 3D objects take up space.
Prism	A polyhedron* consisting of two parallel, congruent faces called bases that are joined with rectangular faces. 
Polyhedron	A solid figure that has several polygon faces 
Pyramid	A special type of polyhedron. It has a solid base that is a polygon and the other faces are triangles that meet at an apex.* 
Apex	The highest point of a pyramid.
Tetrahedron	A solid made up of four triangular faces. It can also be considered a pyramid with a triangular base. The regular tetrahedron is also a platonic solid.* 
Platonic Solid	5 Regular polyhedrons that have identical faces and the same number of edges meeting at each vertex.* 
Face	The flat surface or side of a solid shape.
Edge	The intersection of the faces of a 3D shape.
Vertex	The corner of a shape or solid. The place where two edges of a shape meet, or where three or more faces of a solid meet, is a vertex.
Net	A 2D pattern of a 3D shape that can be folded to result in the 3D object.
Sphere	A perfectly round ball. This is a shape that is made of only one curved face of which the end or the beginning cannot be discerned. Every point on the surface of the sphere is the same distance away from the centre of the shape. 
Cone	A cone is a pyramid with a circular base 
Cylinder	A 3D shape with a parallel circular face at each end which is joined by a curved surface. 

SUMMARY OF KEY CONCEPTS

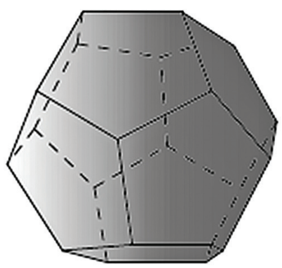
Classifying 3D objects

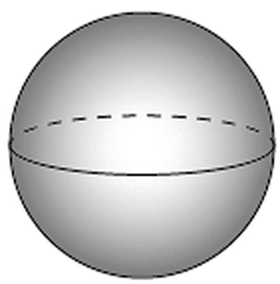
1. The focus of the topic in Term 4 is the classification by means of faces, edges and vertices.



Example:

2. Learners must know that many objects are polyhedra but that there are shapes such as cylinders and spheres that do not fall into this classification because they have curved (not flat) faces.





Example:

Polyhedron

Not a polyhedron

3. Learners must be able to name a given object as well as describe the reason for the name they have chosen for the object.

Teaching tip:

Teachers can use the Glossary of Terms and if possible, copy the pages for each learner as a reference of the appearance and properties of each type of 3D shape. The graphics included in the Glossary are the correct examples of each type of 3D shape and will cut out any confusion. This is also worth a quick class test to ensure that learners have mastered the classifications.

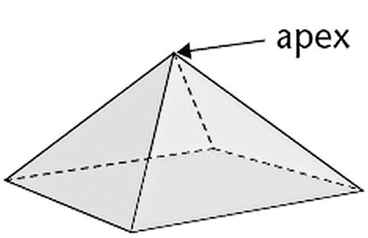
Tetrahedrons and Pyramids

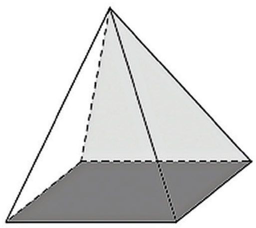
1. Learners must be able to identify and distinguish tetrahedrons and a pyramid.

Teaching Tip:
Many learners find it easier to distinguish between objects if they can touch the actual object. It is a good idea for the teachers to construct a large cardboard model of the objects. Students can touch and discover the differences and this helps them visualise the actual structure of the objects they are learning about.

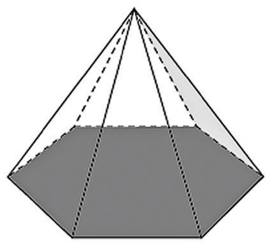
It is vital to discuss the properties of pyramids thoroughly and then discuss the properties of other polyhedra thoroughly as this will ensure a very clear understanding of each individual type of object before learners are expected to tell them apart.

2. Examples of pyramids:



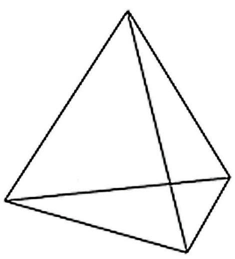


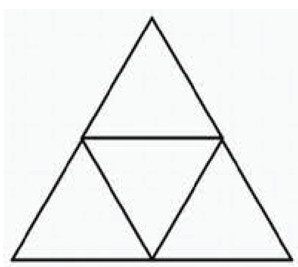
Square-based pyramid
it has 5 faces:
1 square, 4 triangles



hexagonal-based pyramid
it has 7 faces:
1 hexagon, 6 triangles

3. Tetrahedron





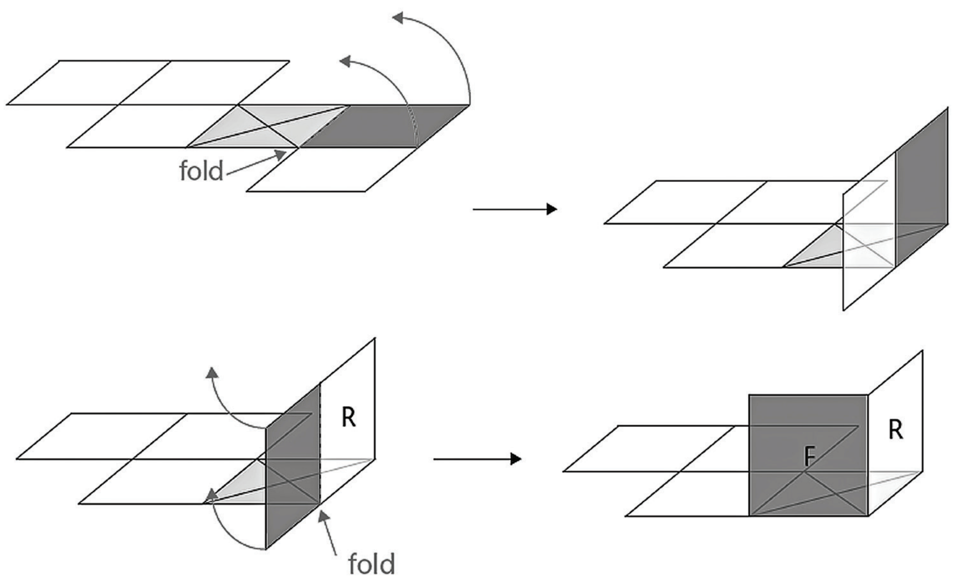
This shape is actually a triangular based pyramid where all faces are the same size equilateral triangles. A tetrahedron is one of the Platonic solids. The picture on the right shows the net from which a tetrahedron is built.

Building 3D models

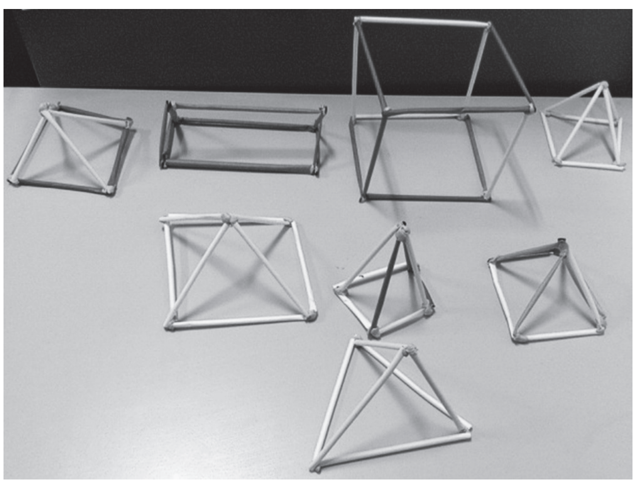
1. Learners are expected to produce models of 3D objects using nets.
2. This also means that learners should be able to identify which nets will deliver the shape they wish to construct.

Teaching Tip:
In Grade 5 learners would have traced nets of a variety of boxes that they have deconstructed. Learners must revisit the requirements for a good net, such as tabs, to ensure their success in constructing these 3D models.

Example:



3. Learners must also be capable of constructions using items such as straws and toothpicks so that they can look at edges and vertices in more detail. These are skeletal form constructions where the emphasis is on the edges and vertices and not on the face of the shape.



TOPIC 5: AREA, PERIMETER AND VOLUME

INTRODUCTION

- This unit runs for 7 hours.
- It is part of the Content Area, 'Measurement' which counts for 15% in the final exam.
- The unit covers perimeter, area and volume of shapes. It also has reference to the history of measurement.
- Learners in Grade 6 are not required to use formulae when calculating perimeter and area but should be measuring using rulers and tape measures. Learners are only expected to start developing an understanding of the calculations that are used for area (only squares and rectangles) and volume (only rectangular prisms).

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Measure perimeter using rulers and tape measures. • Find area using square grids which develops the understanding of square units. • Find volume by filling or packing items into a 3D shape and thus developing an understanding of cubic units.	• Measure perimeter using rulers and tape measures. • Find area of regular and irregular shapes using square grids that allow learners to count squares. • Develop rules for the calculation of area of squares and rectangles. • Find volume by filling or packing items into a 3D shape. • Develop an understanding for the calculation of volume of rectangular prisms using length multiplied by breadth multiplied by height. • Investigate the relationship between perimeter and area of squares and rectangles. • Investigate the relationship between surface area and volume of rectangular prisms.	• Understand that the relationship between area and volume is required for the calculations that are performed relating to 3D objects later in the Senior and FET Phases. • Understand how changes to single or multiple dimensions of a 3D object can influence the area or volume without having to perform complicated calculations but determining them by inference

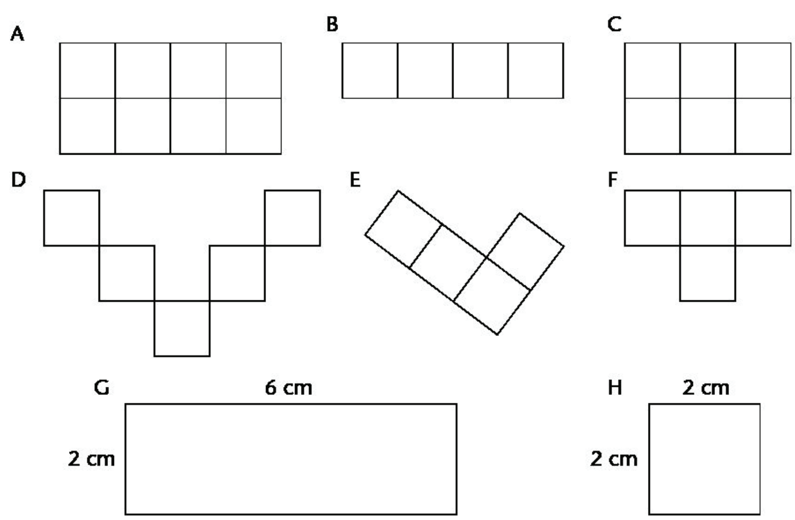
GLOSSARY OF TERMS

Term	Explanation / Diagram
Two-dimensional (2D)	A 2D figure is flat (not solid) so that only two dimensions of it can be measured, namely its length and width. It has no depth and therefore it does not occupy space.
Regular Polygon	A polygon of which the sides are all the same length and of which the angles are all the same size.
Irregular Polygon	A polygon of which the sides are not all the same length and the angles are not all the same size.
Perimeter	The distance around the polygon.
Formula	An expression or equation that is used to express the relationship between certain quantities. Example: The formula for calculating the area of a rectangle is: Area = Length x Width or Area = Length x Breadth, in short: A= L x W or A = L x B
Standard Unit (SI Unit)	A unit of measurement is a definite magnitude of a quantity, defined and adopted by convention or by law, that is used as a standard for measurement of the same kind of quantity, in length measurement being the metre. Any other quantity of that kind can be expressed as a multiple or a fraction of the unit of measurement. For example a kilometre is 1000 x the length of a metre and a centimetre is $\frac{1}{100}$ of a metre.
Area	The surface of a shape or object. It can also be defined as the number of square units that a shape covers.
Composite Shape	An irregular shape that is made up of parts or whole components of other shapes.
Solid Figure	A 3D shape that has length, breadth and height (or depth).
Cube	A 3D figure with 6 identical square faces.
Prism	A 3D shape that has two polygon faces that are parallel and joined by rectangular sides.
Rectangular Prism	A prism made of 6 rectangular faces.
3 dimensional (3D)	These are figures that do not lie in a plane. The figures have length, breadth and height or depth. 3D objects take up space.
Surface Area	The sum of the areas of each of the faces of a 3D shape.
Volume	The volume of an object is the amount of space it takes up. It is measured in cubic units (three-dimensional). Objects with the same volume may have different shapes for example a cube can have the same volume as a cylinder.
Capacity	While volume is the amount of space taken up by an object, capacity is the measure of an object's ability to hold a substance, like a solid, a liquid or a gas.
Net	A 2D pattern that folds to form a 3D shape. It is helpful when calculating surface area as it makes all faces visible so that they are not omitted from the calculation.

SUMMARY OF KEY CONCEPTS

Determining perimeter

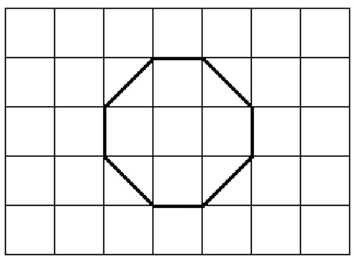
1. Learners must be able to use a variety of measuring instruments like rulers and tape measures accurately.
2. Learners are not expected to use any formulae when finding perimeter at this stage, they are determining perimeter by measurement or by adding the given edge lengths in an exercise.
3. In Grade 6 learners report perimeter and area as follows:
   a. When there is no indication of the unit, they report perimeter in units of length and area in square units. In the example below, A-F will be reported in units and square units. Figure E would then have a perimeter of 10 units, and an area of 4 square units. The way of calculation in this case, is primarily counting.
   b. When the standard unit is indicated, learners report the perimeter in standard units of length (cm) and the area in square standard units (cm^2), as is the case in figure G and H. In G for example, the perimeter is 16 cm and the area is 12 cm^2. The way of calculation in this case was addition for perimeter and multiplication for area.

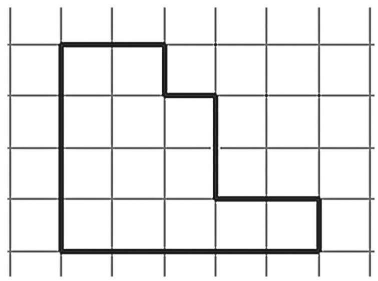


Area of shapes

1. Learners determine the area occupied by a shape that has been drawn on grid paper by counting the squares the shape occupies.
2. These shapes should have straight edges in Grade 6 so that learners can accurately determine the area. Should the shape not occupy the entire block, half blocks can be added to make a whole block.

Examples:



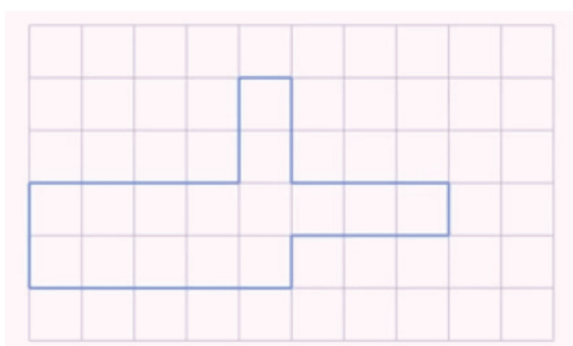


3. Learners estimate the area covered by the shape by taking the following guidelines into account:
   a. Count the whole squares
   b. Combine half squares to make whole squares
   c. Count any parts bigger than a half as a whole square
   d. Ignore the parts that are less than half of the square in size.

4. An estimate is not an accurate calculation. At the same time it is not a wild guess. It is a reasonable judgment.

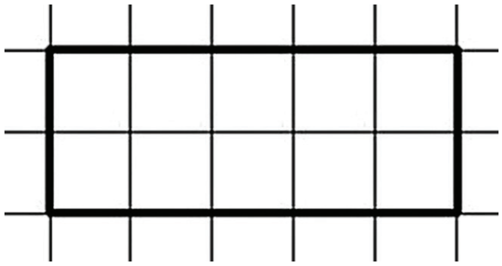
Learners should be encouraged to make up their own shapes and find the perimeter and area with a partner.

An example:



Perimeter: 24 units Area: 15 square units

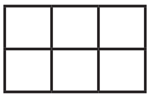
Area of rectangles



1. Learners can determine area by counting the squares on grid paper. Now they start to see the the relationship between the number of rows (breadth) multiplied by the number of blocks per row (length) works out to be the same as the area that they counted before.
2. Exercises that consolidate this skill should allow for counting and using the basic rule given above.
3. The focus should only be on the development of the idea and not on learners using formulae for area.
4. These exercises must be limited to rectangular shapes only.

Investigating the relationship between perimeter and area of rectangles and squares

1. Learners must be able to make some basic conclusions regarding the relationship between area and perimeter by investigating if there is an existing relationship.
2. Make a rough sketch of the rectangle in the first column, then complete the rest of the columns following the example. Calculate (using a calculator) the ratio of perimeter:area, rounded to two decimal places. Ask learners if they can find a pattern

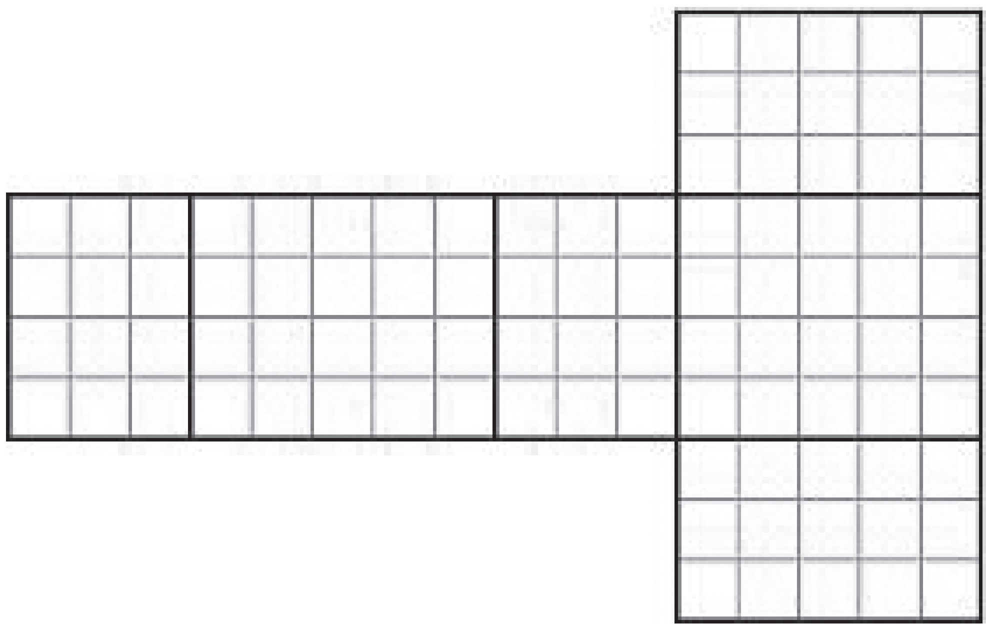
Dimensions of the rectangle	Perimeter of the rectangle	Area of the rectangle	Ratio of perimeter: area
2 cm x 2 cm			
Example: 2 cm x 3 cm 	2 + 3 + 2 + 3 = 10 cm	2 x 3 = 6 cm2	10 ÷ 6 = 1.67
3 cm x 3 cm			
3 cm x 4 cm			
4 cm x 4 cm			
4 cm x 5 cm			
5 cm x 5 cm			
5 cm x 6 cm			
6 cm x 6 cm			
6 cm x 7 cm			

Solution:

Dimensions of the rectangle	Perimeter of the rectangle	Area of the rectangle	Ratio of perimeter: area
2 cm x 2 cm	2 + 2 + 2 + 2 = 8 cm	2 x 2 = 4 cm^2	8 ÷ 4 = 2.00
2 cm x 3 cm	2 + 3 + 2 + 3 = 10 cm	2 x 3 = 6 cm^2	10 ÷ 6 = 1.67
3 cm x 3 cm	3 + 3 + 3 + 3 = 12 cm	3 x 3 = 9 cm^2	12 ÷ 9 = 1.33
3 cm x 4 cm	3 + 4 + 3 + 4 = 14 cm	4 x 4 = 12 cm^2	14 ÷12 = 1.17
4 cm x 4 cm	4 + 4 + 4 + 4 = 16 cm	4 x 4 = 16 cm^2	16 ÷ 16 = 1.00
4 cm x 5 cm	4 + 5 + 4 + 5 = 18 cm	4 x 5 = 20 cm^2	18 ÷ 20 = 0.90
5 cm x 5 cm	5 + 5 + 5 + 5 = 20 cm	5 x 5 = 25 cm^2	20 ÷ 25 = 0.80
5 cm x 6 cm	5 + 6 + 5 + 6 = 22 cm	5 x 6 = 30 cm^2	22 ÷ 30 = 0.73
6 cm x 6 cm	6 + 6 + 6 + 6 = 24 cm	6 x 6 = 36 cm^2	24 ÷ 36 = 0.67
6 cm x 7 cm	6 + 7 + 6 + 7 = 26 cm	6 x 7 = 42 cm^2	26 ÷ 42 = 0.62

Determining surface area

1. Learners must be able to determine the surface area of rectangular prisms.
2. To determine the surface area, they deconstruct the rectangular prism into six rectangles with blocks.
3. Learners would now start to determine the surface area by adding together the areas of each of the rectangles.
4. Make learners aware of the visible and hidden sides of a 3D rectangular prism.
5. Learners are not expected to use formulae yet to calculate surface area. They need to understand the idea of surface area, by handling the physical object first, both as a 3D object, and as its 2D net. Use a net like the one below for this purpose. Let learners cut it out and fold up the 3D prism. After that, they can answer the questions below:
   a. Determine the volume of this rectangular prism. Show all working.
   b. Determine the surface are of this rectangular prism. Show all working.



Solution:

a. The volume of the prism is 3 units x 4 units x 5 unit = 60 units^3

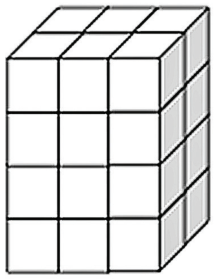
b. The surface area of the prism is 94 units^2

[2 (3 x 4) + 2(4 x 5) + 2 (3 x 5) = 24 + 40 + 30 = 94]

Volume of rectangular prisms

1. Learners must be able to determine the number of blocks that would occupy the volume of a 3D rectangular prism.

   EXAMPLE



This shape has 2 layers of 4 rows of 3 blocks in a row. That makes 12 blocks in a layer. There are 24 blocks that make up the whole shape. So the volume is 24 cubic units.

2. Learners are expected to start developing an understanding as to the relationship between length multiplied by width (breadth) multiplied by height and the number of blocks that occupy the volume of the rectangular prism.

Learners will be expected to use formulae that are developed gradually in the later stages of the Senior and FET Phases to determine volume of a vast number of shapes.

Investigating the relationship between surface area and volume of rectangular prisms

1. Learners must be able to make some basic conclusions regarding the relationship between surface area and volume by investigating if there is an existing relationship.

   Note: This is an advanced exercise and is meant only for learners who have completely mastered the previous parts of the topic. It can be used for enrichment purposes.

2. Do the following calculations of surface area and volume of rectangular prisms. Then calculate through division (using a calculator) the ratio: surface area : volume, rounded to two decimal places. Try to see if there is any pattern that you can make out.

Dimensions of the rectangular prism	Surface area of the rectangular prism	Volume of the rectangular prism	Ratio of surface area : volume
2 cm x 2 cm x 2 cm			
Example: 2 cm x 3 cm x 4 cm	2 x 2 x 3 = 12 cm^2 2 x 3 x 4 = 24 cm^2 2 x 4 x 2 = 16 cm^2 Total SA = 52 cm^2	2 x 3 x 4 = 24 cm^3	52 ÷ 24 = 2.17
3 cm x 3 cm x 3 cm			
3 cm x 4 cm x 5 cm			
4 cm x 4 cm x 4 cm			
4 cm x 5 cm x 6 cm			
5 cm x 5 cm x 5 cm			
5 cm x 6 cm x 7 cm			
6 cm x 6 cm x 6 cm			
6 cm x 7 cm x 8 cm			

Solution

Dimensions of the rectangular prism	Surface area of the rectangular prism	Volume of the rectangular prism	Ratio of surface area : volume
2 cm x 2 cm x 2 cm	Total SA = 24 cm^2	2 x 2 x 2 = 8 cm^3	24 ÷ 8 = 3.00
2 cm x 3 cm x 4 cm	Total SA = 52 cm^2	2 x 3 x 4 = 24 cm^3	52 ÷ 24 = 2.17
3 cm x 3 cm x 3 cm	Total SA = 54 cm^2	3 x 3 x 3 = 27 cm^3	54 ÷ 27 = 2.00
3 cm x 4 cm x 5 cm	Total SA = 94 cm^2	3 x 4 x 5 = 60 cm^3	94 ÷ 60 = 1.57
4 cm x 4 cm x 4 cm	Total SA = 96 cm^2	4 x 4 x 4 = 64 cm^3	96 ÷ 64 = 1.50
4 cm x 5 cm x 6 cm	Total SA = 148 cm^2	4 x 5 x 6 = 120 cm^3	148 ÷ 120 = 1.23
5 cm x 5 cm x 5 cm	Total SA = 150 cm^2	5 x 5 x 5 = 125 cm^3	150 ÷ 125 = 1.20
5 cm x 6 cm x 7 cm	Total SA = 214 cm^2	5 x 6 x 7 = 210 cm^3	214 ÷ 210 = 1.02
6 cm x 6 cm x 6 cm	Total SA = 216 cm^2	6 x 6 x 6 = 216 cm^3	216 ÷ 216 = 1.00
6 cm x 7 cm x 8 cm	Total SA = 292 cm^2	6 x 7 x 8 = 336 cm^3	292 ÷ 336 = 0.87

The History of measurement

1. Learners should learn of some historical methods of measurement and how measurement was recorded.
2. Body parts were used to measure length.

3. In 1791 it was accepted that the standard unit for measuring length would be the metre, which was one ten-millionth of the length of the meridian through Paris from pole to the equator.

TOPIC 6: DIVISION

INTRODUCTION

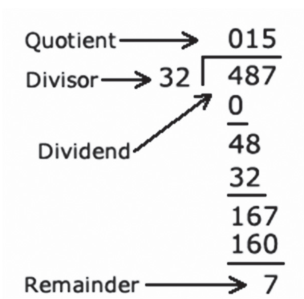
- This unit runs for 7 hours.
- It is part of the Content Area, 'Numbers, Operations and Relationships' which counts for 50% in the final exam.
- This unit is a continuation of the work done in Term 2.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Compare two or more quantities of the same kind (ratio)
• Compare two quantities different kinds (rate)
• Divide at least whole 3-digit by 2-digit numbers
• Use strategies including
  ◦ estimation
  ◦ building up, breaking down
  ◦ rounding off and compensating
  ◦ doubling and halving
  ◦ multiplication and division as inverse operations
• Know multiples and factors of 2-digit numbers to at least 100
• Use properties of whole numbers
• Know multiplicative property of 1
• Know multiplication facts of multiples of 10 and 100
	• Compare two or more quantities of the same kind (ratio)
	• Compare two quantities of different kinds (rate)
	• Divide at least whole 4-digit by 3-digit numbers
	• Use strategies including
	  ◦ estimation
	  ◦ building up, breaking down
	  ◦ rounding off and compensating
	  ◦ doubling and halving
	  ◦ multiplication and division as inverse operations
	• Understand multiples and factors of numbers
	• Use properties of whole numbers
	• Know multiplicative property of 1
	• Know multiplication facts of multiples of 10 and 100
		• All work done in the Intermediate phase will be revised
		• The range of numbers to be divided is extended to include integers and numbers in exponential form
		• Applications of the principles of division are integrated in solving algebraic equations

GLOSSARY OF TERMS

Term	Explanation / Diagram
Division	Sharing out of a quantity into a number of equal portions or groups. Equal sharing, equal groups, rate and ratio all extensions of the same idea. **a.** Equal sharing: Share 35 sweets among 7 children (35 ÷ 7 = 5) **b.** Equal groups: Pack 35 sweets in packets of 5 (35 ÷ 5 = 7) **c.** Rate: Five packets of sweets cost R35, therefore the price per packet is R35 ÷ 5 = R7 (R7/packet) **d.** Ratio:There are 45 girls and 54 boys in Grade 5. This is a ratio of 45:54 or 5:6 if we divide each part by their highest common factor which is 9. The girls form of the grade and the boys form of the grade.
Terms Used in a Division Equation	72 ÷ 6 = 12 72 → dividend, 6 → divisor, 12 → quotient
Multiples	Multiples of a certain number (eg. 5) are the products when we multiply that number by any whole number: 15 is a multiple of 5, because 5 x 3 = 15
Factors	A whole number that divides exactly into another number. Factor pairs are those numbers that were multiplied to make a number. The numbers 2, 14, 7 and 4 are factors of 28; 2 and 14 are a pair, 4 and 7 are a pair.
Prime number	A positive integer that has only two factors, 1 and the number itself. This does not include 1 as it is an identity element for multiplication and division.
Composite Number	Any number that has multiple factors.
Divisible	A number is divisible by another if there is no remainder after division
Remainder	What is left over when you try to share a whole out into a particular number of equal parts, and when the whole is not completely divisible by that number, for example 18 ÷ 4 = 4 with a remainder of 2
Long Division	Long division is the standard algorithm or strategy to divide large numbers. It is a process in which each step of the division is written out in a vertical way and the answer is written on top. 

SUMMARY OF KEY CONCEPTS

Division and multiplication facts

1. Learners can write opposite multiplication and division facts to prove that they understand that multiplication and division are inverse operations.

   Example:
   a. Write two division facts from 223 x 83 = 18 509
      (Solution: 18 509 ÷ 83 = 223; 18 509 ÷ 223 = 83)
   b. Write a multiplication fact from 8 667 ÷ 107 = 81
      (Solution: 107 x 81 = 8 667)

1. In Term 4, Grade 6 learners revise the following multiplication and division facts:

   c. Dividing by 1: When we divide any number by 1, the number stays the same.

   Example:
   14 ÷ 1 = 14: I have 14 sweets. I give it to one child. How many sweets does the child get? The child gets all 14 sweets, because she was the only one.

   d. Dividing by 10 or multiples of 10: When a number ending in a zero is divided by 10 or a multiple of 10, divide the first digits and the zero falls away.

   Example:
   480 ÷ 20 = 24 (48 ÷ 2 = 24, therefore 480 ÷ 20 = 24)

   e. Dividing by 100 or multiples of 100: When we multiply a number by 100 or a multiple of 100, we multiply the first number by the number of 100s and add two zeros to the number. When a number ending in zeros is divided by 100 or a multiple of 100, we divide the first numbers and two zeros fall away.

   Example:
   6 600 ÷ 300 = 22 (66 ÷ 3 = 22, therefore 6 600 ÷ 300 = 22)

f. **Dividing 0:** There is nothing to divide, so the answer is 0.

Example:
$0 \div 14 = 0$

g. **Dividing by 0:** We cannot divide any number by zero – it is undefined.

Note that this concept will become clearer in future years.

Rules of divisibility

A number can be divided exactly

- by 2 if the last digit is an even number: 323 is therefore not divisible by 2
- by 3 if the sum of the digits is a multiple of 3: 324 is divisible by 3, because 3+2+4=9
- by 4 if the last two digits are a multiple of 4: 324 is therefore divisible by 4
- by 5 if the last digit is either 5 or 0: 324 is therefore not divisible by 5
- by 6 if the number is divisible by 2 and by 3: 354 is therefore divisible by 6
- by 8 if it can be divided by 2 three times: 216÷2=108; 108÷2=54; 54÷2=27
- by 9 if the sum of the digits is a multiple of 9: 324 is divisible by 9, because 3+2+4=9
- by 10 if the last digit is 0: 324 is not divisible by 10

Dividing Can Result in Fractions

The remainder resulting from division can be written as a fraction rather than as a remainder. The remainder is the numerator and the dividend is the denominator of the fraction, because a fraction is actually a division calculation.

Example: $2\ 608 \div 48 = 54$ remainder 16 can be written as

$$2608 \div 45 = 54\frac{16}{48} = 54\frac{1}{3}$$

Division Strategies

1. **Estimation:**
   Round both numbers to numbers that can be divided with ease.

   Example: $178 \div 19 \approx 180 \div 20 \approx 9$

2. **Breaking down numbers:**

   - Breaking down the first number and building up the answer

     Example: (This strategy works only where both parts are divisible by the second number)

     $$\begin{aligned} 624 \div 12 &= (600 + 24) \div 12 \\ &= (600 \div 12) + (24 \div 12) \\ &= 50 + 2 \\ &= 52 \end{aligned}$$

   - Breaking down the second number

     Example: (This strategy works only where the first number is divisible by the second)

     $$\begin{aligned} 315 \div 15 &= 315 \div 5 \div 3 \\ &= 63 \div 3 \\ &= 21 \end{aligned}$$

3. **Clue Board:** Use the second number to write down a few multiples of that number.

   Teaching tip:
   The most handy multiples to have in the clue board, are 2, 3, 5, 10 and 20.

   Example: 4 369 ÷ 132:

20 x 132	= 2640	4369 – 2640= 1729
10 x 132	= 1320	1729 – 1320 = 409
3 x 132	= 396	409 – 396 = 13
33 with a remainder of 13		

2 x 132 = 264
3 x 132 = 396
5 x 132 = 660
10 x 132 = 1320
20 x 132 = 2640

   Test the answer: 33 x 132 = 4356; 4356 + 13 = 4369

Topic 6 Division

Equal sharing and grouping problems

h. Equal sharing

In equal sharing, we know the total amount and the number of groups, but we want to find out how many the equal share for each group will be. We divide the total amount by the number of groups, to find the number in each group.

Example:
There are 1 575 children in a certain area. They are grouped in equal groups to receive vaccinations against measles on 15 days. How many children are in each group?

- We know the total number of children that we want to share equally (1575 children)
- We know among how many days (groups) to share the children equally (15 days)
- We want to find out how many children (the equal share) there will be on each day
- As a number sentence we can write 1575 ÷ 15 = $\square$

  (Solution: There will be 105 children on each of the 15 days for vaccination)

i. Grouping

In grouping, we know the total amount and how many the equal share is, but we want to find out how many groups there will be. We divide the total amount by the number in each group, to find the number of groups.

Example:
4320 biscuits need to be packed in boxes containing 96 biscuits each. How many boxes do they need to pack the biscuits?

- We know what the total amount of biscuits is that we want to group (4320 biscuits)
- We know how many biscuits (the equal share) must be in a box (96 biscuits in a box)
- We want to find out how many boxes (groups) they can pack
- As a number sentence, we can write 4320 ÷ $\square$ = 96 or 4320 ÷ 96 = $\square$

  (Solution: 4320 ÷ 45 = 96 or 4320 ÷ 96 = 45)

Rate and ratio problems

In Grade 6 learners must use division to solve rate and ratio problems.

1. **Rate:** In rate two measurements are related to each other, like kilometres and the time to cover that distance in hours. We use the word “per” between the two measurements, and we use the symbol / to mean “per”. Per means to divide.

   Example:
   A truck drives 1 876 km from Lilongwe in Malawi to Soweto in South Africa. At what speed did the truck go if it takes 28 hours to reach Soweto?

   (Solution: 1 876 km ÷ 28 hours = 67 km/hour)

2. **Ratio:** Ratio is the relation between two amounts, like the ratio of boys to girls in a school is 7:8 (said seven to eight). Another way of saying this could be to say, for every 7 boys there are 8 girls.

   Example:
   The learners at school are represented boys : girls in a ratio of 7 : 8. There are 1 485 learners in school. How many are boys and how many are girls?

   (Solution: 1 485 ÷ 15 = 99; 99 x 7 = 693 boys; 99 x 8 = 792 girls)

Long Division or the Standard Algorithm or the Vertical Method of Division

All previous strategies were aimed at making learners use this strategy with understanding. The following two inserts show the method step by step:

			2			15 into 3 doesn't go, so look at the next digit
1	5	3	6	4	0	15 goes into 36 two times, so put a 2 above the 6
	-	3	0			$15 \times 2 = 30$
			6			take that 30 away from the 36 to get your remainder.
						36- 30 = 6

			2	4		
1	5	3	6	4	0	next, carry the 4 down to make 64
	-	3	0			15 goes into 64 four times, so put a 4 above the 4
			6	4		$15 \times 4 = 60$
		-	6	0		take 60 from the 64 to get your remainder
				4		64 - 60 = 4

			2	4	2	
1	5	3	6	4	0	carry down the 0 to make 40
	-	3	0			15 goes into 40 two times, so put 2 above the 0
			6	4		$15 \times 2 = 30$
		-	6	0		
				4	0	take 30 from the 40 to get your remainder
			-	3	0	40 - 30 = 10
				1	0	

		hM	tM	M	hT	tT	T	H	T	U			
		0	0	6	8	1	1	4	0	9	r	1	6
1 4 5		9	8	7	6	5	4	3	2	1			
Step 1	-	8^2	7^3	0									
		1	1	7	6								
Step 2	-	1	1^3	6^9	0								
				1	6	5							
Step 3			-	1	4	5							
				$\cancel{2}^1$	$\cancel{0}^9$	^{14}							
Step 4			-	1	4	5							
					5	9	3						
Step 5				-	5	8	0						
						1	3	$\cancel{2}^1$	^{11}				
Step 6					-	1	3	0	5				
								1	6				

987654321 ÷ 145 = 6811409 r. 16

Teaching Tip:
It saves time and prevents multiplication mistakes to write the multiples of the divisor from 2 to 9 before starting to divide.

TOPIC 7: NUMBER SENTENCES

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area, 'Patterns, Functions and Algebra' which counts for 10% in the final exam.
- The unit is an extension of the work that was covered in Term 1 and is an introduction to algebraic concepts.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD

• Write number sentences to describe a problem situation
• Solve and complete number sentences by inspection and trial and improvement
• Check solution by substitution
	• Write number sentences to describe a problem situation
	• Solve and complete number sentences by inspection and trial and improvement
	• Check solution by substitution
		• Determine, interpret and justify equivalence of different descriptions of the same relationship or rule by number sentences
		• Write number sentences to describe problem situations
		• Analyse and interpret number sentences that describe a given situation
		• Solve and complete number sentences by:
		  • inspection
		  • trial and improvement
		• Determine the numerical value of an expression by substitution.
		• Identify variables and constants in given formulae or equations

GLOSSARY OF TERMS

Term	Explanation / Diagram
Mathematical Problem	A problem that can be written and solved with numbers and with the methods of mathematics. It can be a problem in a real world context or a number problem without context. Example of a problem in context: Thato read 78 pages in 3 hours. How many pages did he read in an hour? Example of a problem without context: How much more is 234 than 123?
Solving by Inspection	A method of solving a number sentence by looking at it carefully and thinking logically what the solution can be without written calculation.
Trial and improvement	A method of solving a number sentence by trying out several methods or possible solutions until you are satisfied with the answer.
Algebraic Expression or Number Sentence (without an = sign)	An algebraic expression is a number sentence that can contain ordinary numbers, an unknown which we write as $\square$ and operators (like add, subtract, multiply, and divide). Example: $\square + 4$ (We do not know what the value in $\square$ is)
Algebraic Equation or Number Sentence (with an = sign)	An algebraic equation is a number sentence with an equal sign, but where one or more of the elements are unknown. The requirement for an equation is that the left hand side must have the same value as the right hand side. Example: $\square + 4 = 7$ (Now find the value in $\square$)
Equivalence	Equivalence is a relation holding between the two parts of an equation where they both have the same value.
Substitution	Substitution is a method to solve a problem or to check if your solution is right. After solving the unknown in an equation, then we substitute that solution in the equation to see if it is the solution that makes the equation true.

SUMMARY OF KEY CONCEPTS

Number sentences and equivalence

1. Number sentences are used to write a problem in mathematical terms so that the problem can be solved.
2. The focus this term is on the equivalence of number sentences and not on the properties of operations.
3. Learners must be given examples that have multiple choice answers to prepare them for this format as it is used in standardised external tests.

EXAMPLE:
Which of the following would have the same value as ___ × 17?

a. ____ + 17
b. ____ - 17
c. 17 ×____
d. 17 + ____

4. A number sentence is used to consolidate the idea of expressing a rule.
5. Where possible learners should use number sentences to solve word problems in a variety of real life contexts.
6. When learners write number sentences they are still encouraged to use symbols to represent the unknown values and should not be using algebraic notation yet.

EXAMPLE:
A certain number multiplied by 2 is equal to 26 minus 4. What is the number?
____ × 2 = 26 – 4

Converting a mathematical problem into a number sentence

1. What we know as a mathematical problem is really a life situation that has been converted into mathematical form to find a solution for the real situation - mathematically.
2. An example is the situation that Thato read 46 pages in 2 hours. The question, the unknown or the problem we want to solve is, how many pages did he read in an hour?
   - This problem, converted into a number sentence would be 46 (pages) ÷ 2 (hours) = $\square$ (pages in one hour).
   - When we start explaining to learners how to set up a number sentence, we first talk through the problem and get some ideas from the learners.
   - We try to discourage them to give the answer immediately, and encourage them to focus on HOW we get to an answer or a solution.
   - The teacher can say the words above (pages, hours and pages in one hour) verbally while writing on the chalk board: $46 \div 2 = \square$.

An algebraic expression

1. An algebraic expression contains an unknown, but is a mathematical sentence which is only a statement, not a question – it does not contain an equal sign (=), which would give enough information to make a solution possible.
2. In the example: $\square + 4$, we do not know what the value in $\square$ is, because we do not have enough information. This expression can therefore not be solved.

An algebraic equation

1. An algebraic equation contains an unknown, and is a mathematical sentence which can be seen as a question. It contains an equal sign (=) and it gives enough information to make a solution possible.
2. In the example: $\square + 4 = 7$, we can find out what the value in $\square$ is, because we have enough information. This expression can therefore be solved. In Grade 6 we solve it either by inspection or by trial and improvement.

Solving by inspection

1. When learners solve a problem by inspection, they do not really calculate, but look at it and “see” what would make sense to make the equation true. A way of teaching them to solve by inspection, is to discuss the number sentence with them by adding a “what?” into the open space or the block.
2. In the example: 15 + $\square$ = 19, the teacher may say: “Fifteen plus what is equal to nineteen?”

Solving by trial and improvement

1. When a problem cannot be solved easily by inspection, learners may solve it by trial and improvement, meaning that they take an initial guess which makes it easy to “see” the answer. They see that the answer is either too big or too small, adapt their first guess to come closer to making the equation true and continue until they find the solution.
2. In the example: 284 – $\square$ = 56, learners may try to subtract 200, which brings them to an easy answer (84); they see that the answer is too high by about 30; they improve the answer by subtracting 30 more, that is they subtract 230; that gives them 54, which is 2 too few; they therefore subtract 2 less, which is 228.

Checking the solution by substitution

1. Through whatever method learners found the solution, teachers bring them into a habit of checking their answers or solutions by substituting them into the unknown space.
2. In the example: $\square$ + 4 = 7, if a learner found 11 as a solution, they substitute the unknown by 11 and say: “11 + 4 = 7”. They immediately see that it does not make sense, and go back to improve or correct the solution. Now they found 3 as the solution and when they substitute 3 into the unknown space, they see 3 + 4 = 7, which they know makes sense and is right.

Problem Types in Grade 6

Learners started writing number sentences mostly with addition and subtraction situations that were given to them in the form of "word problems". Below is a brief summary of addition and subtraction situations followed by some multiplication and division situations:

Note: Teachers may accept alternative ways in which learners put a number sentence, as long as the idea is right and it works out on the desired answer or solution.

1. Addition: Number + number = sum

Unknown number + known number = known sum

Example: Jim had some money. He received R25 more, now he has R68. How much money did he start with?

(We do not know how much money Jim had to start with, so we put an open block to start with. He received R25 more, which means we have to add R25 to the start money. Now he has R68, which means that these two sums together add up to R68.)

Solution: $\square$ + R25 = R68 (OR R25 + $\square$ = R68 OR R68 – R25 = $\square$)

Known number + unknown number = known sum

Example: Jim had R43 and he received some more money, now he has R68. How much money did Jim receive?

(We know that Jim had R43 to start with and that he received more money, so we put a + sign to add more money. We do not know how much more he received, so we put an open block after the + sign. Now he has R68, meaning that the two sums together add up to R68.)

Solution: R43 + $\square$ = R68 (OR $\square$ + R43 = R68 OR R68 – R43 = $\square$)

Known number + known number = unknown sum

Example: Jim had R43 and he received R25 more. How much money does Jim have now?

(We know that Jim had R43 to start with and that he received R25 more, so we put a + sign and add R25. We do not know how much the two sums of money is together, so we put an open block after the = sign.)

Solution: R25 + R43 = □

2. Subtraction: Number – number = difference

Unknown number – known number = known difference

Example: Thabo's dad had a number of sheep. He sold 37 sheep and he was left with 56 sheep. How many sheep did he have to start with?

(We do not know how many sheep Thabo's dad had to start with, so we put an open block to start with. He sold 37 sheep, meaning that we have to subtract 37 from the start number. He is left with 56 sheep, meaning that the difference between these two numbers is 56.)

Solution: □ – 37 = 56 (OR 37 + 56 = □)

Known number – unknown number = known difference

Example: Thabo's father had 93 sheep. He sold some of them and was left with 56 sheep. How many sheep did he sell?

(We know Thabo's dad had 93 sheep to start with and that he sold sheep, meaning we have to subtract something from the start number, so we add a – sign. We do not know how many sheep he sold, so we put an open block. He is left with 56 sheep, meaning that the difference between the two numbers is 56.)

Solution: 93 – □ = 56 (OR □ + R56 = R93 OR R56 + □ = R93
OR R93 – R56 = □)

Known number – unknown number = known difference

Example: Thabo's father had 93 sheep and he sold 37. How many sheep does he have left?

(We know Thabo's dad had 93 sheep to start with, and that he sold 37 sheep, meaning we have to subtract 37 from the start number, so we add a – sign and 37. We do not know how many sheep he was left with, meaning that the difference is unknown. We therefore put an empty block after the = sign)

Solution: 93 – 37 = □ (OR □ + R37 = R93 OR R37 + □ = R93)

3. Multiplication: First number x second number = product

First number (unknown) x second number (known) = product (known)

Example: School B has 216 Grade 4 learners, which is 6 times as many as School A. How many Grade 5 learners does School A have?

Number sentence: □ x 6 = 216

First number (known) x second number (unknown) = product (known)

Example: School A has 36 Grade 5 learners and School B has 216. How many times more learners does School B have than School A?

Number sentence: 36 x □ = 216 (OR □ x 36 = 216)

First number (known) x second number (known) = product (unknown)

Example: School A has 36 Grade 5 learners and School B 6 times more Grade 4 learners. How many Grade 5 learners does school B have?

Number sentence: 36 x 6 = □
(OR □ ÷ 36 = 6 OR □ ÷ 6 = 36 OR 6 x 36 = □)

4. Division: First number ÷ second number = quotient

a. First number (unknown) ÷ second number (known) = quotient (known)

Example: Susan makes up packets of 7 apples from a box of apples and she makes up exactly 13 packets. How many apples were in the box?

Number sentence: $\square \div 7 = 13$ (OR $\square \div 13 = 7$ OR $7 \times 13 = \square$)

b. First number (known) ÷ second number (unknown) = quotient (known)

Example: Su packs 13 bags from a box of 91 apples. How many apples are in each bag?

Number sentence: $91 \div \square = 13$ (OR $91 \div 13 = \square$ OR $13 \times \square = 91$)

c. First number (known) ÷ second number (known) = quotient (unknown)

Example: From a box with 91 apples, Susan makes up packets of 7 apples each. How many packets of apples does she make up?

Number sentence: $91 \div 7 = \square$ (OR $91 \div \square = 7$ OR $7 \times \square = 91$)

Further examples to consolidate:

a. Thili has R350 in and she spends R85. How much money does she have left?

b. Luthando collects marbles. He wants to have 200 marbles in the end and he has 144 up to now. How many more marbles must he get to reach his goal number?

c. There are 55 members in the choir of Sefatlhano Primary. They perform in the competition together with Peme Primary school who has 23 members in their choir. How many choristers are there altogether?

d. Mamile does not know how many books there were on the teacher's table in the morning. She put 15 books extra on the table and now there are 73 books. How many books were there in the morning?

e. There are 37 test papers on the table and Mary-Jane put more test papers on the pile, now there are 64. How many test papers did Mary-Jane add?

Topic 7 Number Sentences

Setting up a number sentence

1. Setting up number sentences from word problems: We use the above problem structures to set up the number sentence from a given situation.
2. Setting up a number sentence from number problems: Learners must also be able to set up number sentences from context free number problems, where only numbers are given, and the problem is not set out in words.

Examples:

a. There is a number that is 9 more than 23. What is that number?

b. A number is 15 less than 38. What is that number?

c. There is a number that is 7 times more than 22. What is that number?

d. If I divide a certain number by 4, the answer is 43. What is that number?

Solutions:

a.	$23 + 9 = \square$	OR	$\square - 9 = 23$	OR	$\square - 23 = 9$
b.	$38 - 15 = \square$	OR	$\square + 15 = 38$		
c.	$22 \times 7 = \square$	OR	$\square \div 7 = 22$		
d.	$\square \div 4 = 43$	OR	$43 \times 4 = \square$		

Solving number sentences and substituting the solution into the number sentence

After setting up the number sentence, learners can solve the problem, or they can solve a number sentence that has already been set up, through inspection or trial and improvement.

After solving the problem, learners can replace the solution into the unknown space of the number sentence to check for correctness of the solution, as has been explained above.

TOPIC 8: TRANSFORMATIONS

INTRODUCTION

- This unit runs for 3 hours.
- It is part of the Content Area, 'Space and Shape (Geometry)' which counts for 15% in the final exam.
- The unit extends the work covered in Term 3 on Transformations where the main focus is on enlargement and reduction of shapes.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Recognise, draw and describe lines of symmetry in 2D shapes • Use transformations to build composite 2D shapes by tracing and by rotating, translating or reflecting 2D shapes • Use transformations to tessellate patterns with 2D shapes • Observe and recognise symmetry and transformations in nature and in the environment • Use reflection, rotation and translation • Describe patterns	• Continue the work and concepts learned in Grade 4 and 5 • Transform 2D shapes through reflection, translation, rotation, enlargement and reduction • Use transformations to describe shapes in the world, in nature and from our cultural heritage • Describe transformations in terms of reflection, rotation, translation, enlargement and reduction	• Recognize, describe and perform translations, reflections and rotations with geometric figures and shapes on squared paper • Identify and draw lines of symmetry in geometric figures • Draw enlargements and reductions of geometric figures on squared paper and compare them in terms of shape and size

GLOSSARY OF TERMS

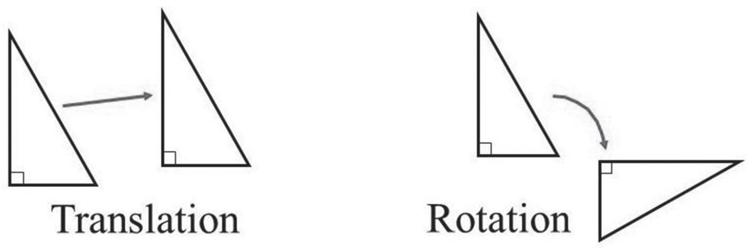
Term	Explanation / Diagram
Transformation	A change in the position, orientation or size of an object or shape
Rigid Transformation	Transformations in which size and shape are preserved (their position, direction and orientation may change though)
Object	The original shape before a transformation has been undergone
Image	The resulting shape after a transformation has been undergone
Enlargement	A transformation where the shape of the object is maintained but the size is increased.
Reduction	A transformation where the shape of the object is maintained but the size is decreased.
Centre Of Enlargement / Reduction	The point from which an enlargement takes place or the point towards which a reduction occurs
Scale Factor	The number of times the image is larger or smaller than the original object
Congruent	Absolutely identical to each other

SUMMARY OF KEY CONCEPTS

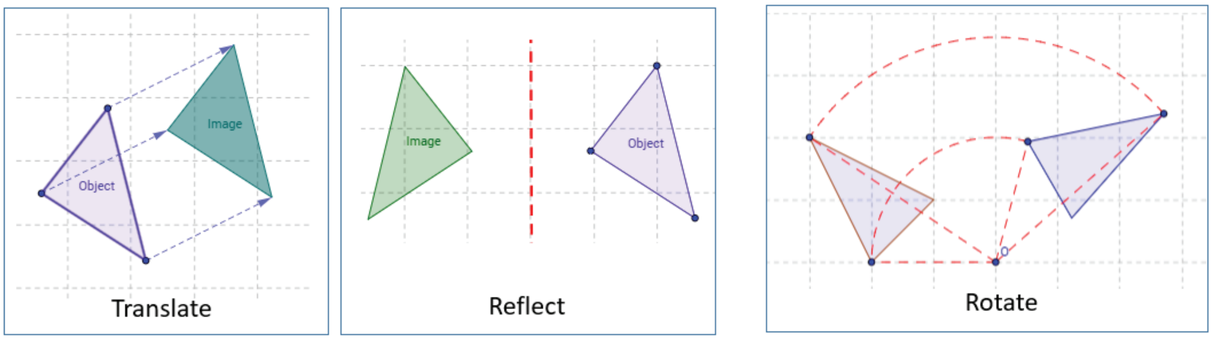
The focus this term is on enlargements and reductions. Below is an overview of what should have already been covered in Transformations.

Revision of Transformations

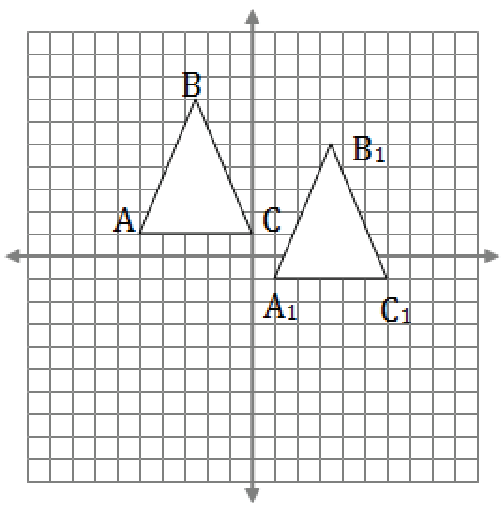
1. To transform means to change. The change can be the position or the size of a shape.
2. Learners may use quad paper to practice transformations.



Example:



Using a grid similar to the one below (which is basically a Cartesian plane without the labels and integers) is a good idea to get learners used to the idea of a Cartesian plane without being formally introduced to it. This way, the blocks are still there to be counted but an understanding of the x- and y-axis system is not required.



3. In the three basic transformations, the size and shape of the original figure remain the same, although its position, direction or orientation is changing.

   a. **Translations**
   A translation is when a point or a shape slides from one position to another. If it is a 2-dimensional shape being translated, the size remains the same.

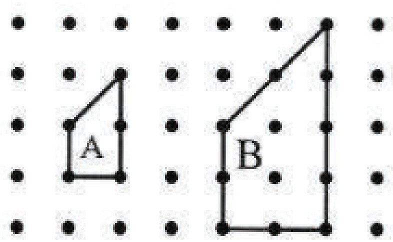
   b. **Reflections**
   A reflection is like a mirror image, which requires a 'mirror line' or a line of reflection. This line can be vertical or horizontal.

   c. **Rotations**
   Rotation means to turn around a centre point. The centre point will always be the origin on the Cartesian plane, where the x–axis and the y–axis intersect. Learners can imagine swinging a compass and the point would be placed at the origin. Shapes can rotate either anti-clockwise or clockwise around the origin.

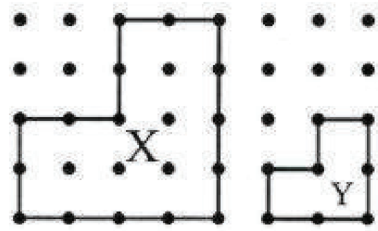
Enlargement and Reduction

1. Enlargements and reductions are different to the previous three transformations in that the shape will now change size.
2. Shapes are enlarged and reduced in such a way that the enlarged or reduced images have similar shapes to the original objects. This is only possible if all the sides of the objects are multiplied or divided by the same number.
3. In the picture below, figure A has been enlarged to image B which is twice as big as A (all sides have been multiplied by 2). Figure X has been reduced to image Y which is half as big as A. All sides have been divided by 2, or we can reason they have been multiplied by $\frac{1}{2}$.

ENLARGEMENT

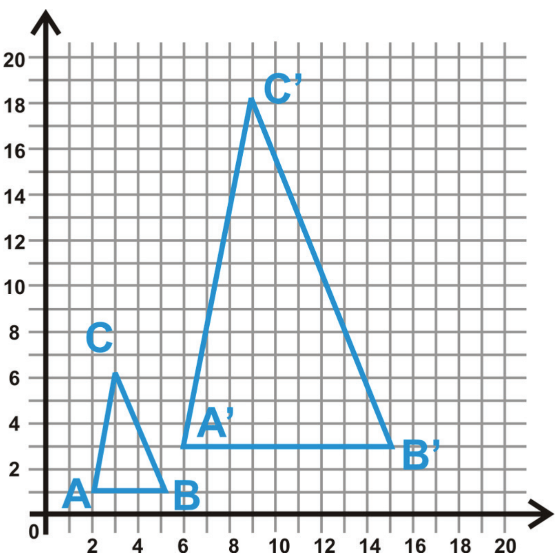


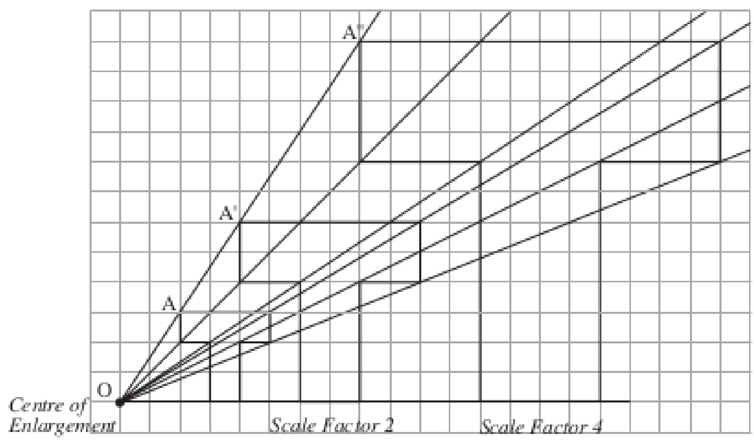
REDUCTION



4. If a figure is multiplied by 1, it remains the same. If it is multiplied by any number more than one, it will be enlarged. If it is multiplied by any number smaller than one, it will be reduced.
5. The number that we multiply the sides of a figure by for enlargement or reduction, is called the scale factor or enlargement factor. Figure A has been enlarged by a scale factor (or enlargement factor) of 2 and figure X has been reduced by a scale factor (or enlargement factor) of $\frac{1}{2}$.
6. Learners draw triangles and rectangles on quad paper, decide on the scale factor, then enlarge or reduce the object. If for example they name the object ∆ABC, the enlarged or reduced image is named ∆A'B'C' to show which is the object and which is the image.
7. To start practising this skill, the two figures (the object and its enlarged or reduced image) can be drawn like in the example above. Later in Grade 8 learners will do enlargements and reductions on a Cartesian plane where all such transformations occur through the origin.

Further examples demonstrating the information above





TOPIC 9: POSITION AND MOVEMENT

INTRODUCTION

- This unit runs for 2 hours.
- It is part of the Content Area, 'Space and Shape (Geometry)' which counts for 15% in the final exam.
- The unit covers the location of objects, drawings and symbols using alpha-numeric referencing.
- Learner skills are extended in Grade 6 to having to give directions to move from one position to another.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Locate position of objects. drawings or symbols using grid referencing in the alpha-numeric context. • Locate a position on a map using alpha-numeric reference. • Follow directions to trace a path between positions on a map.	• Locate position of objects. drawings or symbols using grid referencing in the alpha-numeric context. • Locate a position on a map using alpha-numeric reference. • Give directions to move between positions or places on a map.	• This skill is applied in transformation geometry and analytical geometry later in the senior and FET phases. • The skill of grid referencing is also applied extensively in Geography and Mathematical Literacy

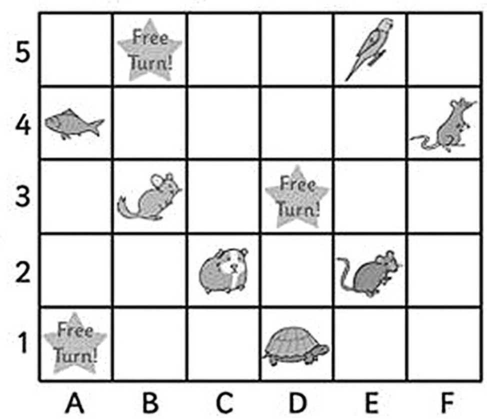
GLOSSARY OF TERMS

Term	Explanation/diagram
Grid	A pattern of blocks or cells running sideways (in rows) and downwards (in columns). The rows are labelled with numbers (1, 2, 3…) and the columns are labelled with letters (A, B, C…)
Grid reference position	A particular cell in a grid, where a column and a row meet. The name of that cell is the label of that column and the label of that row.
Alpha-numeric grid	A grid with letters from the alphabet for the columns and counting numbers for the rows.
Reference	A position on a grid that can be shown or pointed to.
Coordinate	A reference showing the exact position of an object or place.
Scale	Scale is used in plans of houses and for area maps, to draw something in a way that it is a small image of something large.

SUMMARY OF KEY CONCEPTS

Grid Reference of position on a map

1. Finding the position of a specific object or symbol on the map using alpha-numeric reference.



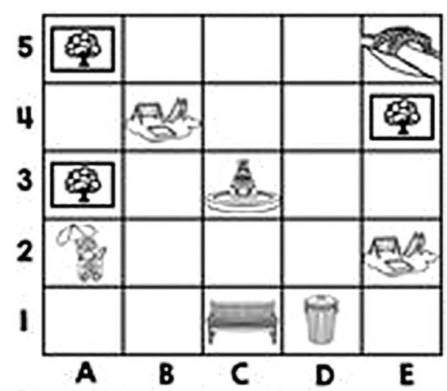
The fish is located in A4 and the turtle in D1.

2. Learners can play games to practice this idea. Games such as battleships or treasure hunts can help learners develop a full understanding of grid referencing.

Describing a change in position on a grid

1. Learners must be able to describe the movement an object makes to change position in the grid.

EXAMPLE:

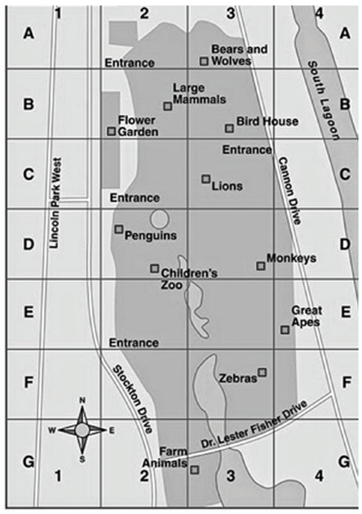


The tree moves upwards 2 blocks from A3 to reach A5, then the tree moves 4 blocks to the right and down 1 block to reach E4.

Working with maps that have grid references

1. Learners must be able to identify places on a map by determining the grid reference and must be able to give direction (compass direction) to reach another location in a different section of the grid.

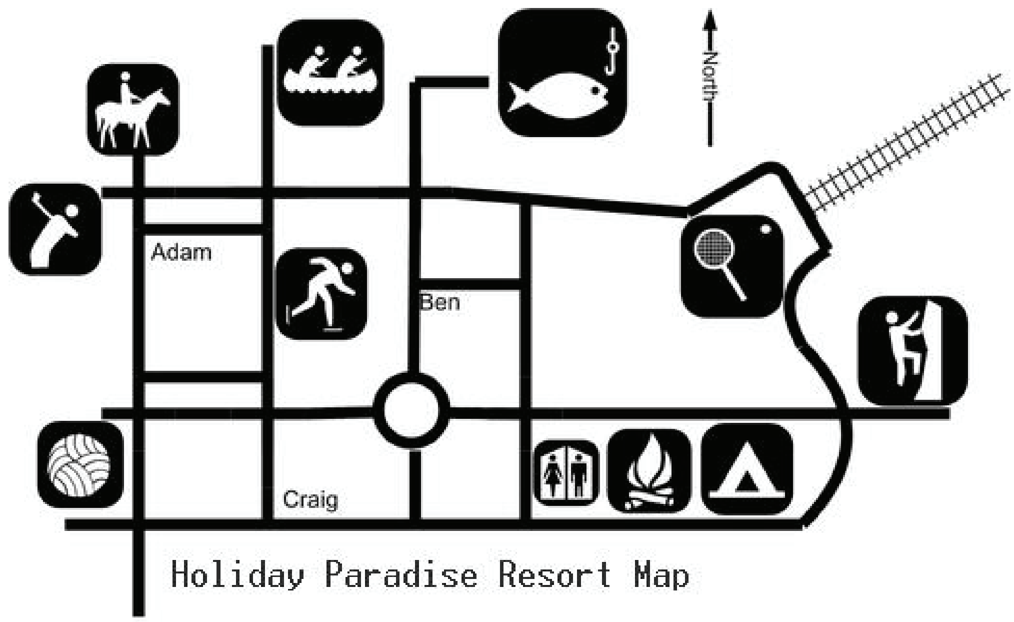
EXAMPLE:



Give the grid reference for the farm animals?
In what direction would you walk from the children's zoo in D2 to the monkeys in D3?

Giving Cardinal Directions (Using North, East, South, West)

Study the resort map below, and answer the questions following.



1. Jabu is fishing and Thabo is playing golf. They decided that the one who is finished first, would walk to the other one. Give both of them clear directions how to reach the other one.

   Jabu, ____
   ____
   ____

   Thabo, ____
   ____
   ____

2. Zinzi is skating and Bibi is playing tennis. They decided that the one who is finished first, would walk to the other one. Give both of them clear directions how to reach the other one.

   Zinzi, ____
   ____
   ____

   Bibi, ____
   ____
   ____

TOPIC 10: PROBABILITY

INTRODUCTION

- This unit runs for 2 hours.
- It is part of the Content Area, 'Data Handling' which counts for 10% in the final exam.
- The unit extends the knowledge of basic chance that was introduced in earlier grades in the Intermediate Phase.

SEQUENTIAL TEACHING TABLE

GRADE 5 INTERMEDIATE PHASE	GRADE 6 INTERMEDIATE PHASE	GRADE 7 SENIOR PHASE
LOOKING BACK	CURRENT	LOOKING FORWARD
• Perform simple repeated experiments • List the possible outcomes of events or experiments • Make tally tables to record actual outcomes • Count and compare the frequency of outcomes	• Perform simple repeated experiments • List the possible outcomes of up to 50 trials • Make tally tables to record actual outcomes • Count, compare frequency of outcomes up to 50 trials of an experiment	• Perform simple experiments where the possible outcomes are equally likely • List the possible outcomes based on the conditions of the activity • Determine the probability of each possible outcome using the definition of probability

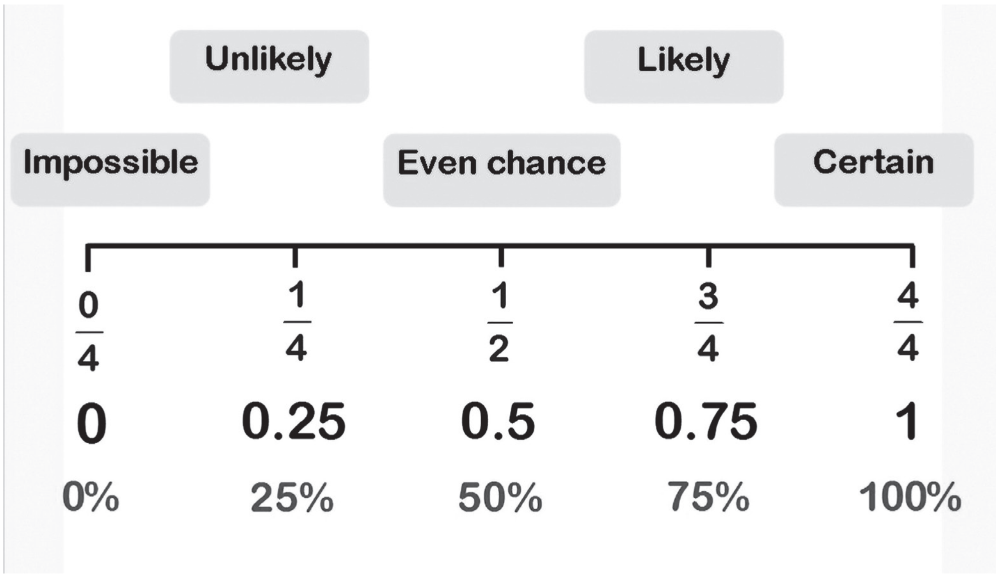
GLOSSARY OF TERMS

Term	Explanation / Diagram
Experiment	Something you do to find out what will happen, like tossing a coin twenty times to see how many times it lands on each side.
Trial	An activity you do in an experiment, like tossing the coin once.
Outcome	The result of a trial, like when I tossed the coin, it landed heads up, therefore the outcome of the trial is "heads up".
Event	A trial together with its outcome are called an event.
Frequency	The number of times that an event occurred.
Probability	The chance that a specific event will occur during an experiment.
Possible outcomes	The number of outcomes that may occur, like rolling a die has six possible outcomes.
Impossible	An outcome that will never happen, like the die can never land on 7 because there is no 7 on the die.
Likely	When there are many chances that something will happen, it is likely, like in winter it is likely that learners will come to school with jerseys.
Unlikely	When there are many chances that something will not happen, it is unlikely, like in summer it is unlikely that learners will come to school with jerseys.

SUMMARY OF KEY CONCEPTS

What is probability?

Probability is the chance that something will happen. It is the likelihood of an event happening. We can draw a line or a scale where we can indicate how likely something is going to happen. Study the scale below and answer the questions following:



Give all answers in the form of a common fraction, a decimal fraction and percentage:

1. What is the probability that Thursday will follow Saturday?
2. What is the probability that Monday will follow Sunday?
3. What is the probability to pick a red ball from a bag with 3 red balls and 3 green balls?
4. What is the probability that it will be very cold in the middle of summer?
5. What is the probability that it will be very cold in the middle of winter?

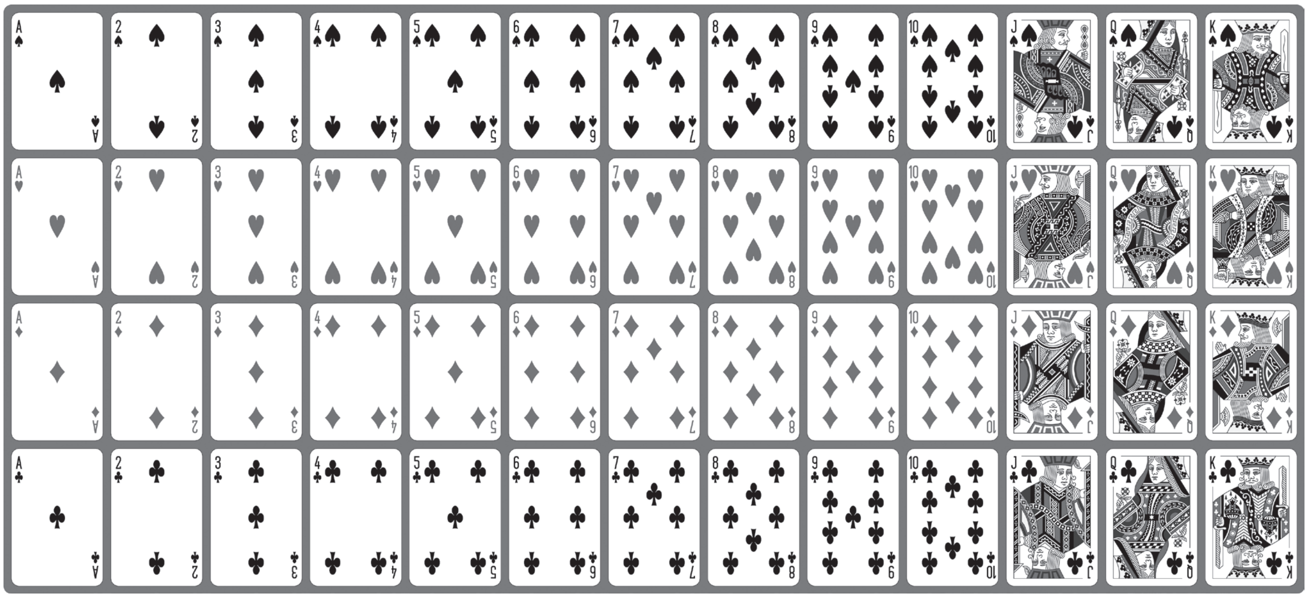
We report all probability on a scale from 0 to 1 or 0% to 100%. In the middle of 0 and 1 is $\frac{1}{2}$; to the unlikely side are all fractions smaller than $\frac{1}{2}$ (not only a quarter) and to the likely side are all fractions larger than $\frac{1}{2}$ (not only three-quarters).

Outcomes

1. An outcome is a possible result of a trial or an experiment.
2. Sometimes there can be more than one possible outcome.

   Examples:
   - When throwing a die, there are six possible outcomes (1, 2, 3, 4, 5 or 6).
   - When a child is born, there are two possible outcomes – a boy or a girl.
   - When a card is chosen from a pack of playing cards, there can be 52 outcomes.
3. Probability questions often refer to the use of playing cards so it is important that learners are familiar with what makes up a pack of cards.

   A regular deck of cards has:
   - 52 cards in total
   - 26 red cards (13 diamonds, 13 hearts) and 26 black cards (13 spades, 13 clubs).
   - Each of the 4 groups (suits) has the cards 2-10, Jack, Queen, King, and Ace.
   - The J, Q, K are called face- (or picture) cards.
   - The Ace and 2 – 10 are called number cards. When the cards are arranged, the Ace acts like a one, but when coming to value, it has the highest value.



Using frequency tables to record outcomes

A frequency table has to have a column for all the possible outcomes, may have a column for the tallies and should have a column for the number of times that that outcome occurred (the frequency), and a row underneath for the total number of trials that were conducted, where all the frequencies are added.

Example:
Jim did an experiment with a spinner with five colours. He did fifty trials and recorded the results in a frequency table.

Experiment with a spinner with five colours		
Possible outcomes	**Tallies**	**Frequency**
Red	~~////~~ ////	9
Green	~~////~~ ~~////~~ //	12
Yellow	~~////~~ ~~////~~ /	11
Blue	~~////~~ ///	8
Brown	~~////~~ ~~////~~	10
Total number of trials that were conducted		**50**

a. How many times did the spinner land on yellow?

b. What was the probability that the spinner would land on green?

c. What was the probability that the spinner would not land on blue?

d. If Dudu did the same experiment, would she also find that the spinner landed on red nine times?

e. What is the ideal number of times that the spinner would land on each colour?