MATHEMATICS

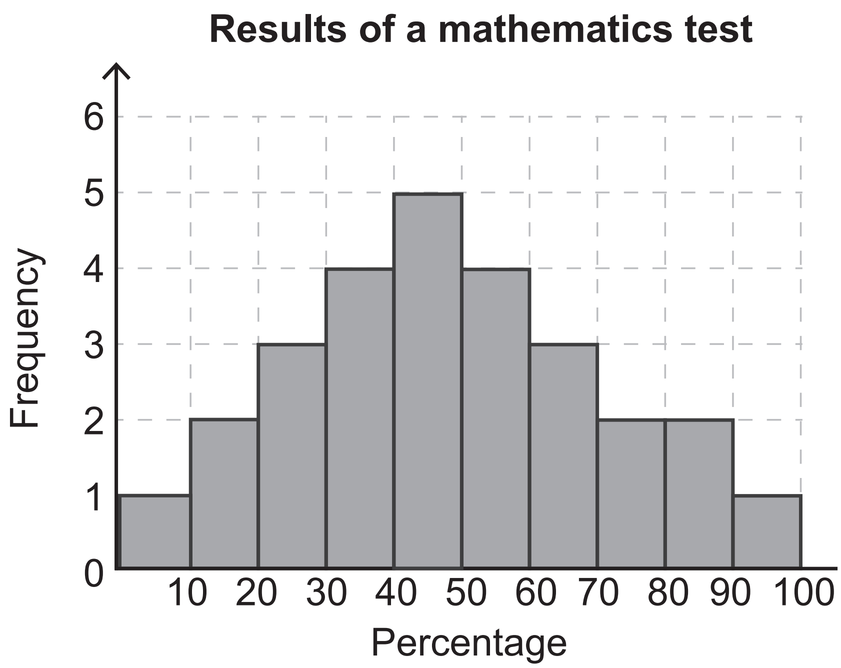
RESOURCE PACK

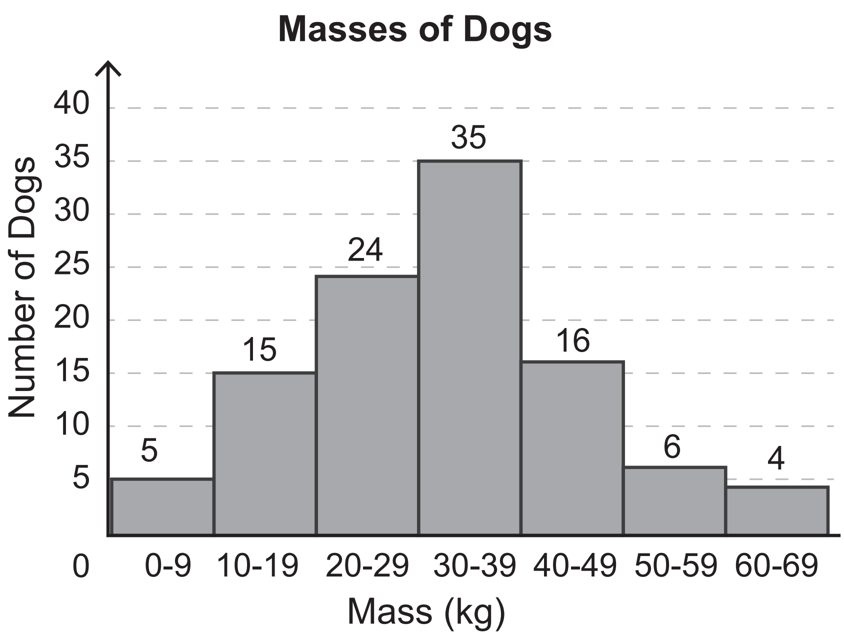
GRADE 11 TERM 4

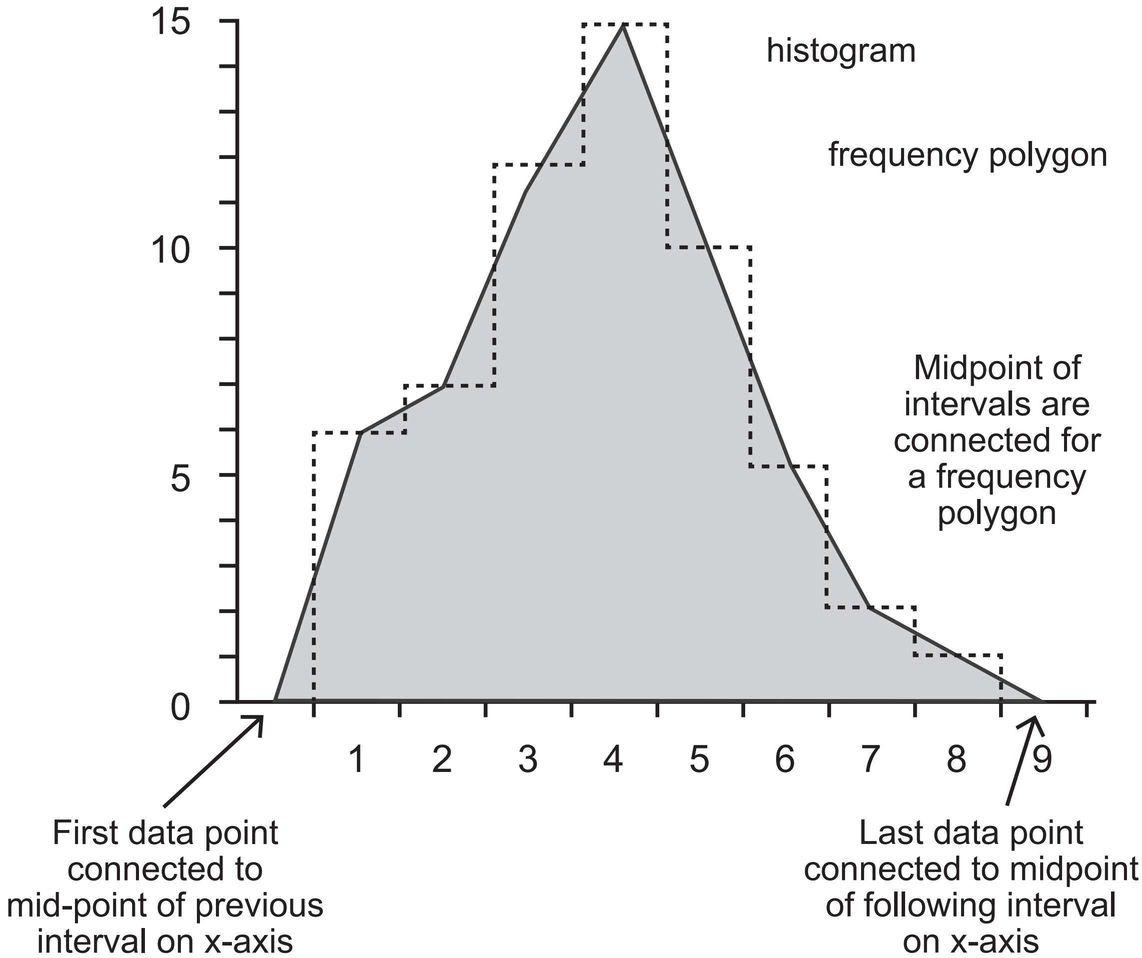
STATISTICS

RESOURCE 1

LESSON 2



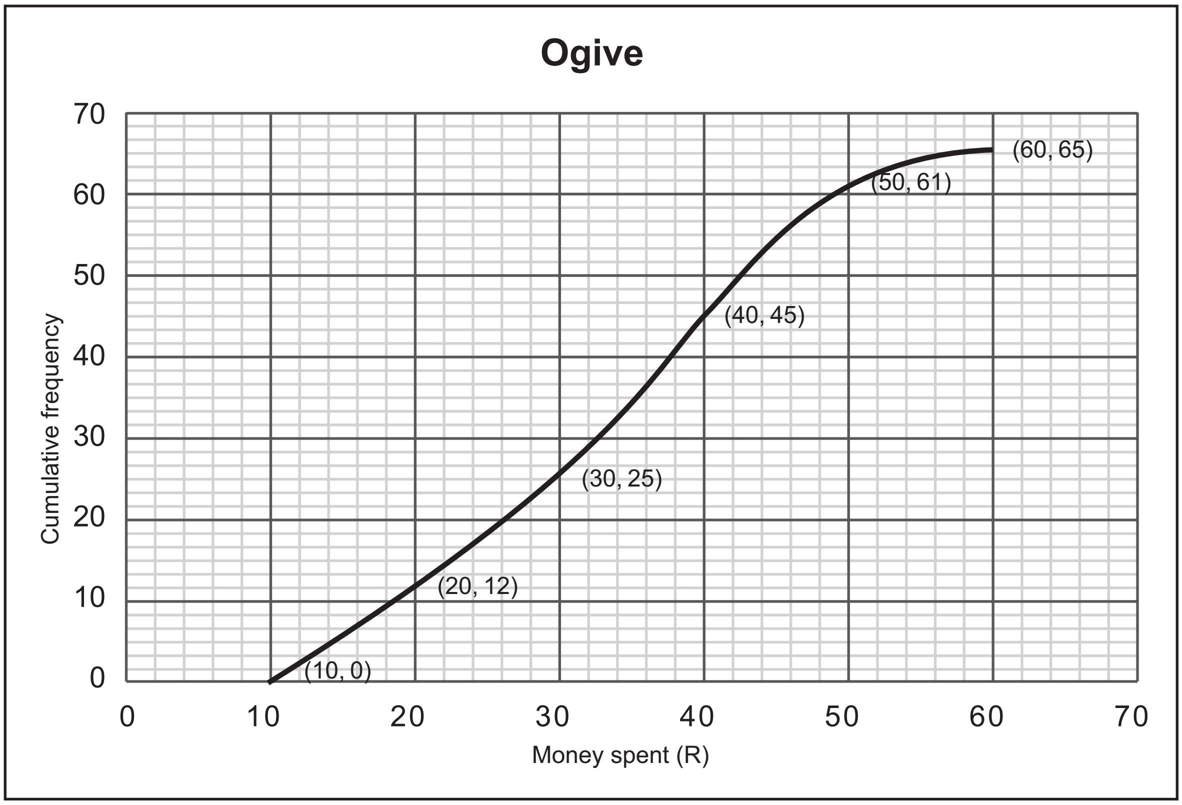



15
10
5
0
histogram
frequency polygon
Midpoint of intervals are connected for a frequency polygon
1 2 3 4 5 6 7 8 9
First data point connected to mid-point of previous interval on x-axis
Last data point connected to midpoint of following interval on x-axis

RESOURCE 2

LESSON 3

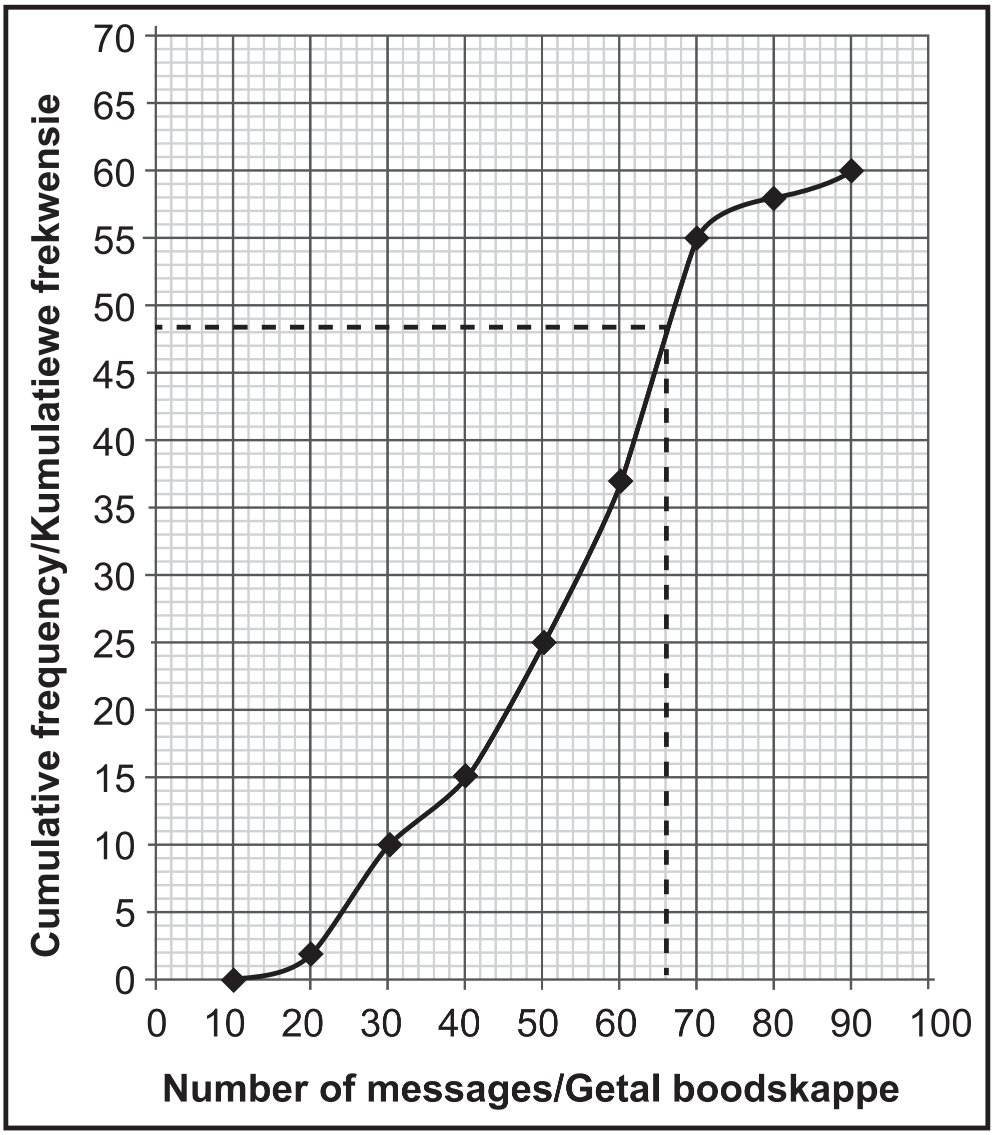
Example 1



Amount of money (in R)	$10 \leq x < 20$	$20 \leq x < 30$	$30 \leq x < 40$	$40 \leq x < 50$	$50 \leq x < 60$
Frequency	a	13	20	b	4

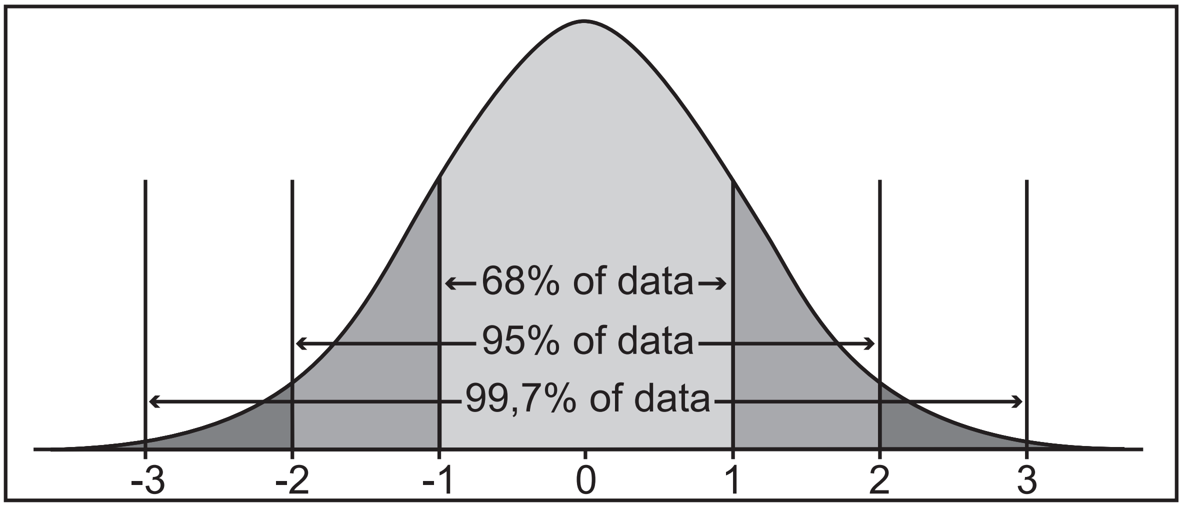
Example 2

NUMBER OF MESSAGES	NUMBER OF DAYS
$10 < x \leq 20$	2
$20 < x \leq 30$	8
$30 < x \leq 40$	5
$40 < x \leq 50$	10
$50 < x \leq 60$	12
$60 < x \leq 70$	18
$70 < x \leq 80$	3
$80 < x \leq 90$	2



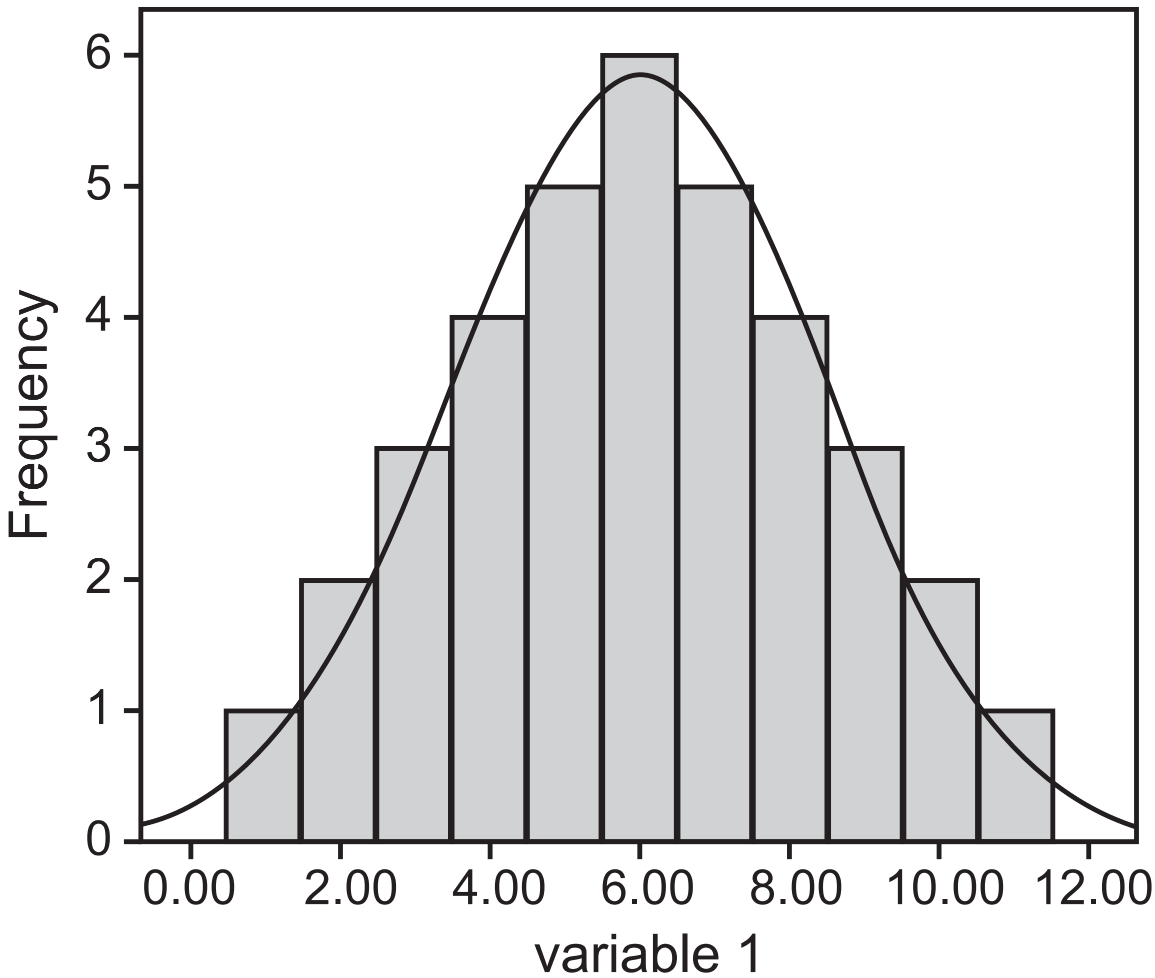
RESOURCE 3

LESSON 4



RESOURCE 4

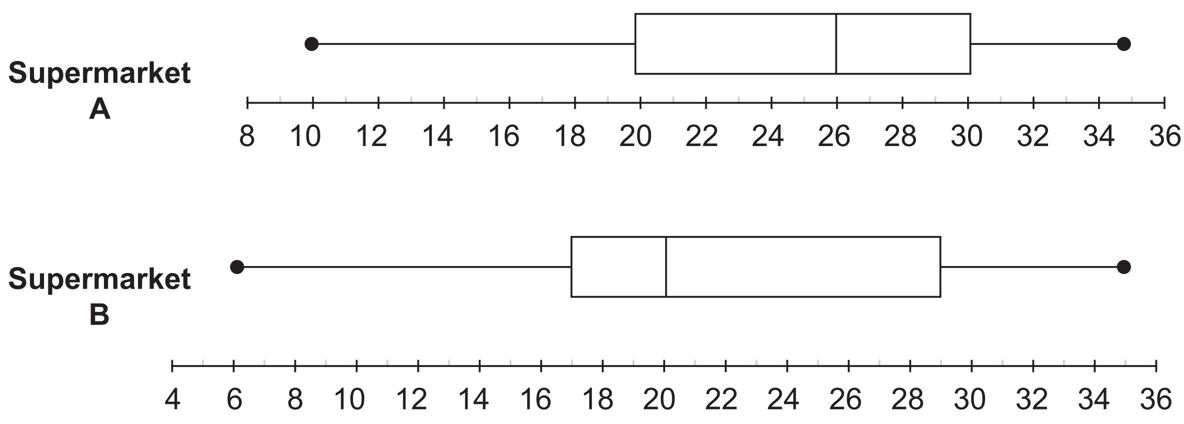
LESSON 5

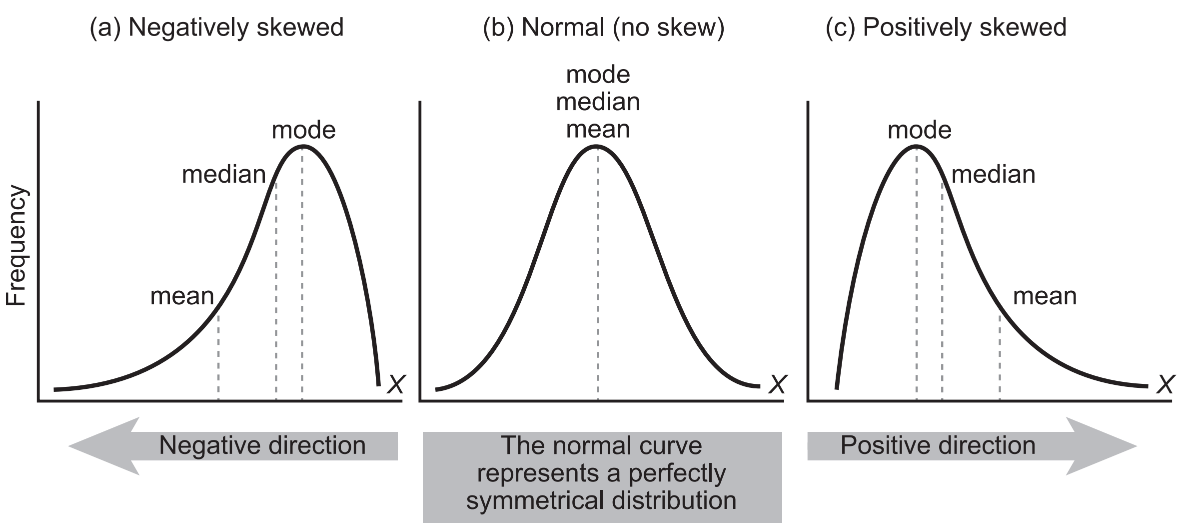


RESOURCE 5

LESSON 5

Example



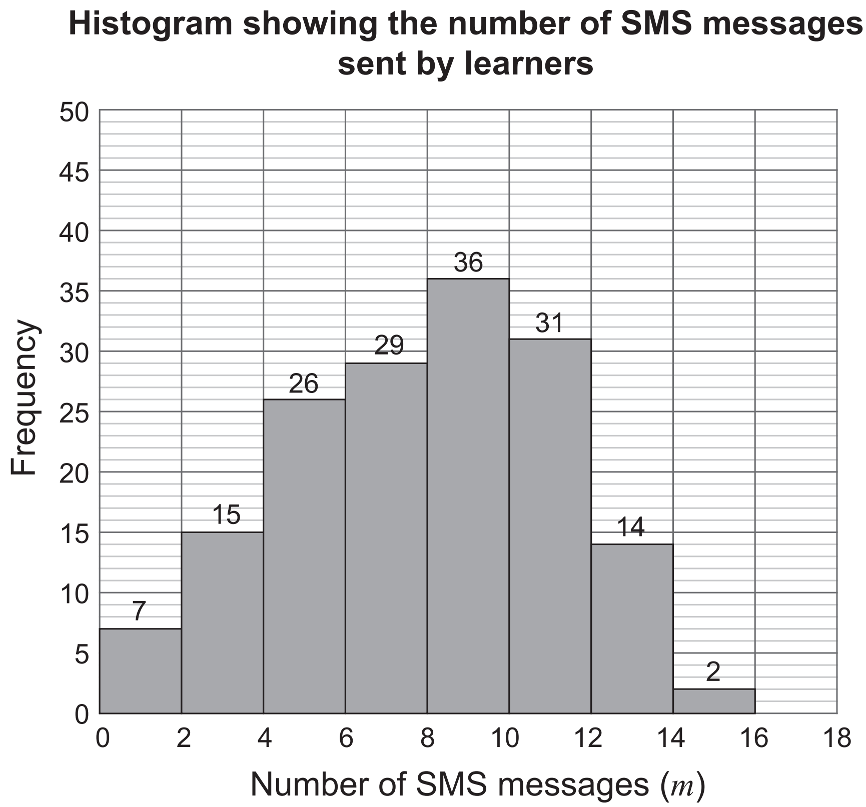


Without Outlier	With Outlier
4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7	4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 7, 300
Mean = 5.45	Mean = 30.00
Median = 5.00	Median = 5.50
Mode = 5.00	Mode = 5.00
Standard Deviation = 1.04	Standard Deviation = 85.03

RESOURCE 6

LESSON 6

Example



CLASS	FREQUENCY	CUMULATIVE FREQUENCY
$0 \leq m < 2$		
$2 \leq m < 4$		
$4 \leq m < 6$		
$6 \leq m < 8$		
$8 \leq m < 10$		
$10 \leq m < 12$		
$12 \leq m < 14$		
$14 \leq m < 16$		

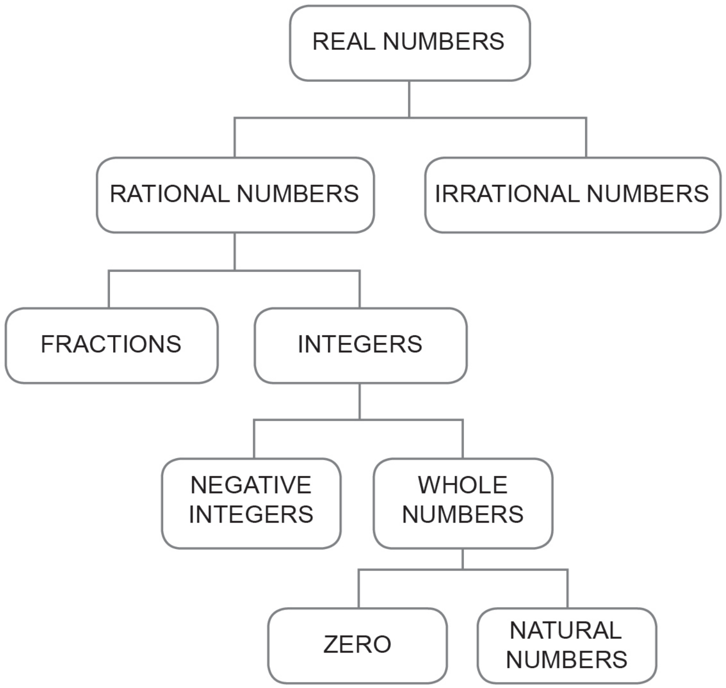
RESOURCE 7

REVISION: Week 1

Summary notes – Paper 1

ALGEBRAIC EXPRESSIONS AND EXPONENTS

The Real Number system



Products

General Rule with brackets: All terms in one bracket must be multiplied by all terms in the other bracket.

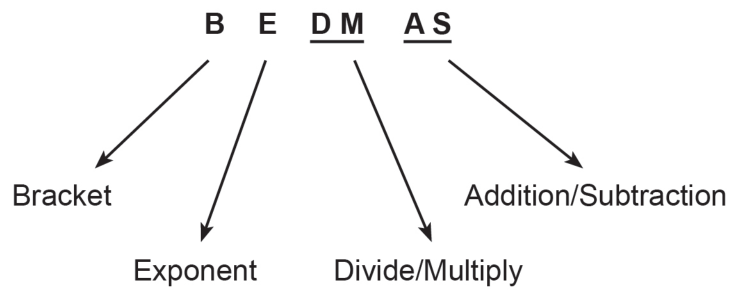
Binomial x Binomial: Use FOIL (first, outer, inner, last)

REMEMBER:

- "names" don't change when adding and subtracting, only the coefficient does
  ($2ab + 4ab = 6ab$ not $6a^2b^2$)
- The sign to the left of a term belongs to it

If there is more than one set of brackets: Work from the inside out.

ORDER OF OPERATIONS is ALWAYS important and needs to be followed.



FACTORS

To factorise an expression is the opposite operation to finding the product.

1. Common factor (including grouping and sign changing)
   - Find HCF to take out
   - Remaining factors go into bracket
   - Original number of terms must equal the number of terms left over in the bracket.
   Example: $3x^2 - 9x^3 = 3x^2(1 - 3x)$

2. Grouping
   - 4 or more terms usually requires grouping
   - Look at the ratios of the coefficients to help decide which terms group together
   - There needs to be a sign between the brackets after grouping. If the sign in front of a bracket is '-' we need to change the sign in the bracket following it.
   - Once grouping has been done, the common factor should be whatever is in the bracket.

   Example, $6p^3 - 4q^3 + 3p^2q - 8pq^2$ (6 & 3 gives the same ratio as 8 & 4)

   $= 3p^2(2p + q) - 4q^2(q + 2p)$ $[2p + q = q + 2p]$

   $= (2p + q)(3p^2 - 4q^2)$

3. Difference of 2 squares
   - Always 2 terms separated by a minus sign
   - Both terms must be perfect squares
   - Remember *not* to multiply out first if brackets are involved

   Example, $(3a+b)^2 - 16$

   $= [(3a + b) + 4][(3a + b) - 4]$

4. Sum and Difference of 2 cubes
   - Always 2 terms separated by a plus or minus sign
   - Both terms must be perfect cubes
   - Factors will always be a binomial and a trinomial
   - Binomial bracket: Cube root each term and keep same sign
   - Trinomial bracket:

   1st term: Square 1st term from binomial bracket

   2nd term: Find product of 2 terms from binomial bracket and change the sign

   3rd term: Square the 2nd term from the binomial bracket

   Example, $27x^3 + 64y^3$

   $= (3x + 4y)(9x^2 + - 12xy + 16y^2)$

5. Trinomials
   - Always 3 terms and factorises into two factors (hence the two brackets)
   - If coefficient of x^2 is not 1:
   - Choose the appropriate signs to match the product of the last term
   - Find factors of the first term and last term
   - Use cross multiplication to find the factors that work

   Examples:

	Working (signs and factors) and solution
$10x^2 + 37x + 7$	Last term positive ∴ need two signs the same 2nd term positive ∴ (+)(+) Factors of 1st term: 1×10 and 2×5 Factors of last term: 1×7 $2x$ ╳ 7 $5x$ 1 $35x$ $2x$ These add up to $37x$ (middle term) Solution: $(5x + 1)(2x + 7)$

$8x^2 - 10x + 3$	Last term positive ∴ need two signs the same 2nd term negative ∴ (–)(–) Factors of term 1: Factors of last term: 1×8 and 2×4 1×3 $\begin{array}{ccc} 4x & & 3 \\ 2x & \times & 1 \\ \hline 6x & & 4x \end{array}$ Because the signs are the same these terms should add up to the middle term – they do. Solution: $(4x - 3)(2x - 1)$
$6x^2 + 5x - 6$	Last term negative ∴ need two different signs Factors of 1st term: Factors of last term: 1×6 and 2×3 1×6 and 2×3 $\begin{array}{ccc} 2x & & 3 \\ 3x & \times & 2 \\ \hline 9x & & 4x \end{array}$ Because the signs are different these terms should make a difference of the middle term – they do. Use the terms to decide which factors go with which signs. To get $+5x$, $+9x -4x$ is required. The sign of $4x$ moves directly above and the sign of $9x$ is placed between the top two factors. Solution: $(2x + 3)(3x - 2)$

$12x^2 - 11 - 15$	Last term negative ∴ need two different signs Factors of 1st term: Factors of last term: 1×12 and 2×6 and 3×4 1×15 and 5×3 $\begin{array}{ccc} 3x & & 5 \\ 4x & \times & 3 \\ \hline 20x & & 9x \end{array}$ Because the signs are different these terms should make a difference of the middle term – they do. Use the terms to decide which factors go with which signs. To get $-11x$, $-20x + 9x$ is required. The sign of $9x$ moves directly above and the sign of $20x$ is placed between the top two factors. Solution: $(2x + 3)(3x - 2)$

<u>REMEMBER</u>: ALWAYS look for a highest common factor first.

Algebraic fractions

1. Multiplication and Division
   - For division, change to multiplication and reciprocate
   - Factorise all numerators and denominators fully
   - Simplify by looking for common factors in any numerator and denominator (remember: you cannot simplify 'next to' an addition or subtraction)

Example, $\frac{(a^2-3a)}{(a^2-4)} \div \frac{(a^2-9)}{(a^2+a-6)}$

$= \frac{(a^2-3a)}{(a^2-4)} \times \frac{(a^2+a-6)}{(a^2-9)}$

$= \frac{a(a-3)}{(a+2)(a-2)} \times \frac{(a+3)(a-2)}{(a+3)(a-3)}$

$= \frac{a}{a+2}$

2. Addition and Subtraction
   - Ensure all denominators are fully factorised
   - Find LCD (lowest common denominator)
   - Change numerators accordingly to ensure equivalent fractions
   - Collect like terms

For example, $\frac{2}{(x^2-1)} + \frac{3}{(x^2+x-2)} - \frac{1}{(x^2+3x+2)}$

$= \frac{2}{(x+1)(x-1)} + \frac{3}{(x-1)(x+2)} - \frac{1}{(x+2)(x+1)}$

$= \frac{2(x+2)+3(x+1)-1(x-1)}{(x+1)(x-1)(x+2)}$

$= \frac{2x+4+3x+3-x+1}{(x+1)(x-1)(x+2)}$

$= \frac{4x+8}{(x+1)(x-1)(x+2)}$

$= \frac{4(x+2)}{(x+1)(x-1)(x+2)}$

$= \frac{4}{(x+1)(x-1)}$

Completing the square

Completing the square is a technique used to express quadratic expressions in the form of:

$$a(x \pm p)^2 + q$$

and is also used to solve for the roots of a quadratic equation.

Steps to completing the square:

1. Take out the coefficient of x^2 if it is not 1
2. Add and immediately subtract (half the coefficient of $x)^2$
3. Factorise (the newly formed perfect square trinomial) and distribute the coefficient.

Example:

Complete the square on the expression: $2x^2 - 10x + 4$

1.		$2(x^2 - 5x + 2)$
	Find ½ the coefficient of x	$\frac{1}{2}(-5) = \frac{-5}{2}$
	and square it	$\frac{25}{4}$
2.	Add and subtract (to keep the expression the same)	$2\left(x^2 - 5x + \mathbf{\frac{25}{4}} - \mathbf{\frac{25}{4}} + 2\right)$
	Note the perfect square *trinomial* you have created	$2\left(\mathbf{x^2 - 5x + \frac{25}{4}} - \frac{25}{4} + 2\right)$
3.	Factorise the perfect square trinomial and collect other 2 like terms	$2\left[\left(x - \mathbf{\frac{5}{2}}\right)^2 - \frac{17}{4}\right]$ **This will always be half the coefficient of x**
	Remove the outer brackets by distributing the coefficient of x^2	$2\left(x - \frac{5}{2}\right)^2 - \frac{17}{2}$

EXPONENTS AND SURDS

Definitions and laws:

Definition/law	Example	Explanation
$x^a \times x^b = x^{a+b}$	$2^3 \times 2^2 \times 2$ $= 2^{3+2+1}$ $= 2^6$	When multiplying like bases keep the bases the same and add the exponents.
$\frac{x^a}{x^b}$ $= x^{a-b}$	$\frac{6x^6}{2x^2} = 3x^4$	When dividing like bases keep the base and subtract the exponent. Divide integers as per usual.
$(x^a)^b = x^{ab}$	$(-2a^2b^3)^4$ $= (-2)^2 \times a^{2\times2} \times b^{3\times2}$ $= 4a^4b^6$	When raising exponents to a power, keep the base and multiply the exponents.
$(xy)^a = x^a y^a$ or $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$	$(a^4 b)^3$ $= a^{12} b^3$ $\left(\frac{a^3}{b}\right)^3 = \frac{a^9}{b^3}$	When more than one base is raised to an exponent, each base is raised to the exponent. When a fraction is raised to an exponent, the numerator and denominator must be raised to that exponent.
$x^0 = 1$	$(x^4 + 4)^0 \times 3^0$ $= 1 \times 1 = 1$	Any base raised to the power of zero is equal to 1. ($x \neq 0$ as 0^0 is undefined)

$x^{-a} = \frac{1}{x^a}$	$3x^{-2} = \frac{3}{x^2}$ and $\frac{3}{x^{-2}} = 3x^2$	A base raised to a negative exponent is equal to its reciprocal raised to the same positive exponent.
$\sqrt[n]{xy} = \sqrt[n]{x} \times \sqrt[n]{y}$	$\sqrt{18} = \sqrt{9} \times \sqrt{2}$ $= 3\sqrt{2}$	When surds are multiplied they can be split apart and rooted individually.
$\sqrt[mn]{x}$ $= \sqrt[m]{\sqrt[n]{x}}$	$\sqrt[6]{64}$ $= \sqrt[3\times 2]{64}$ $= \sqrt[3]{\sqrt{62}}$ $= \sqrt[3]{8} = 2$	When taking the root of a root, it is the same as taking the single root to the product of both roots.

Fractions with exponents

Type 1: When the numerator and/or denominator has more than one term, factorising is required. To find a common factor, Law 1 needs to be used in reverse ($2^{x+1} = 2^x.2^1$). This makes finding the HCF and knowing what remains when it has been taken out much easier.

For example:

$$\frac{3^{1+x} - 5.3^x}{(3^x.6)}$$

$$= \frac{(3^1.3^x - 5.3^x)}{(3^x.6)} \quad \text{Use inverse of Law 1}$$

$$= \frac{3^x(3-5)}{(3^x.6)} \quad \text{Find HCF and factorise}$$

$$= -\frac{2}{6} \quad \text{Simplify}$$

$$= -\frac{1}{3}$$

Type 2: When there is only one term in the numerator and denominator, each base must be written as a product of prime factors then the laws and definitions of exponents are used to simplify.

For example:

$$\frac{3^{n+1}.4^{n-1}}{2^n.6^{n-1}}$$

$$= \frac{3^{n+1}.(2^2)^{n-1}}{2^n.(2.3)^{n-1}} \quad \text{(prime factors)}$$

$$= \frac{3^{n+1}.2^{2n-2}}{2^n.2^{n-1}.3^{n-1}}$$

$$= \frac{3^{n+1}.2^{2n-2}}{2^{2n-1}.3^{n-1}}$$

$$= 3^{n+1-(n-1)}.2^{2n-2(2n-1)}$$

$$= 3^{n+1-n+1}.2^{2n-2-2n+1}$$

$$= 3^2.2^{-1}$$

$$= \frac{9}{2}$$

Simplification of surds

A surd is the root of a number that would result in an irrational number.

For example: $\sqrt{3}$ is a surd as the answer is irrational.
$\sqrt{9}$ has a rational answer (3).

Further examples:

a) $\sqrt{12} - \sqrt{48} + \sqrt{75}$

$= \sqrt{4 \times 3} - \sqrt{16 \times 3} + \sqrt{25 \times 3}$ (break down into the product of a perfect square and another factor

$= 2\sqrt{3} - 4\sqrt{3} + 5\sqrt{3}$ (square root)

$= 3\sqrt{3}$ (simplify)

b) $(\sqrt{5}-2)(\sqrt{5}+2)$ (Difference of 2 squares)
= 5 - 4
= 1

Nature of roots

In the quadratic formula $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, the discriminant (it discriminates between different types of solutions), b^2-4ac is used to find the nature of the roots.

$b^2-4ac>0$ → two real roots
$b^2-4ac=0$ → one real root (the roots are equal)
$b^2-4ac<0$ → non-real roots

EQUATIONS AND INEQUALITIES

1. **Linear equations**
   - Remove brackets (using distributive law) and collect like terms on each side
   - Get all the terms with the variable in them on LHS and all constants on RHS (but remember, whatever is done to one side of the equation must be done to the other side to keep the equation balanced)
   - Collect like terms on each side again and get the variable on its own using division
2. **Equations with fractions**
   - Find LCD. Multiply ALL terms throughout equation by LCD to remove all fractions (no more denominators)
   - There should be NO fractions AT ALL in the next step.
   - Continue the same as for linear equations
3. **Quadratic equations**
   - Recognisable by the "square". You should be expecting two answers.
   - Get ALL terms on LHS so that RHS = 0
   - Factorise the LHS fully
   - Find the 2 possible solutions using the concept that two factors multiplied to equal zero will mean that each one of the factors could possible equal zero.
4. **Simultaneous equations** (Given 2 equations with 2 variables to solve for – usually a quadratic at Grade 11 level)
   - Get ONE of the variables by itself in ONE of the equations
   - Use this information to substitute back into the second equation. You should now have an equation with only one unknown variable in
   - Solve for this variable
   - Substitute the variable found back into the first equation and solve for the second variable.
5. **Exponential equations**
   - Bases must be the same to solve exponential equations – if the bases are the same, the exponents will be the same
   - If bases are not the same, use prime factors to make them the same.
6. **Literal equations**
   - Treat as if it is an ordinary linear equation first (try and ignore the fact that there are many variables and few or no numbers)
   - Focus on the variable you have been asked to solve for.
   - Get all terms with this variable in on one side and all terms without this variable in on the other side.

- If the variable you are solving for is in more than one term (and they're all on one side now), factorise by taking this variable out as a common factor.
- Divide both sides by any other variables 'in the way' and get the variable you're solving for on its own.

7. Equations involving surds

- Isolate the surd
- Square both sides
- Solve for x
- Check your answer

8. Equations with rational exponents

These are exponents with fractions. If an equation is in the form: $x^{\frac{a}{b}} = y$:

- there will be a positive and negative solution if a is even and b odd.
- there will be one solution if a is odd.

Examples:

$x^{\frac{2}{3}} - 16 = 0$

$x^{\frac{2}{3}} = 2^4$ (a is even b is odd $\therefore \pm$ solutions)

$(x^{\frac{2}{3}})^{\frac{3}{2}} = \pm(2^4)^{\frac{3}{2}}$ (both sides raised to $\frac{3}{2}$)

$x = \pm 2^6$

$x = \pm 64$

$3^{\frac{x}{4}} = 27$

$3^{\frac{x}{4}} = 3^3$ (convert 27 to a prime base)

$\frac{x}{4} = 3$ (if the bases are the same the exponents must be equal)

$x = 12$

9. Equations involving factorising

$3^{x+2} + 3^{x+3} - 3^x = 105$	$3^{2x} - 10.3^x + 9 = 0$ $((3^x)^2)$-
$3^x.3^2 + 3^x.3^3 - 3^x = 105$	(use k method)
(inverse of Law 1)	Let $k = 3^x$
$3^x(3^2 + 3^3 - 1) = 105$	$k^2 - 10k + 9 = 0$
(factorise)	$(k-9)(k-1) = 0$ (factorise)
$3^x(35) = 105$	$k = 9$ or $k = 1$
(divide both sides by 35)	$\therefore 3^x = 9$ or $3^x = 1$
$3^x = 3$	$3^x = 3^2$ $3^x = 3^0$
$\therefore x = 1$	(any number raised to the power of zero =1)
	$\therefore$ $x = 2$ or $x = 0$

10. Linear inequalities

- Treat the same as a linear equation
- IF it is required to divide by a negative integer to get the variable alone, the sign (< or >) needs to be changed.
- These solutions may need to be represented on a number line.

11. Quadratic inequalities

Points to remember when solving inequalities:

- If you multiply or divide by a negative number, the sign changes
  (< becomes > etc)
- Because you are solving for a variable, you can NEVER multiply or divide by a variable in an inequality as you don't know whether it is positive, negative or zero.

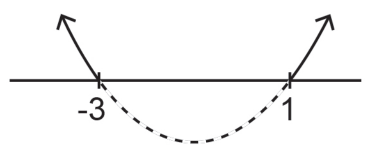
Example

$$x^2 + 2x - 3 \geq 0$$

$$(x + 3)(x - 1) \geq 0$$

- Find the critical values (these are NOT the solutions – merely the values that will assist as they are in a quadratic equation)
- Mark them on a number line (remember that these values represent the x-intercepts of the quadratic function)
- Sketch the function
- Find the part of the function that matches the inequality in the question (in this case greater than or equal to zero. This is the positive part of the function above the x-axis)

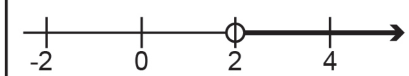
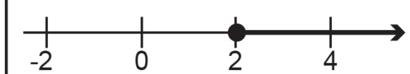
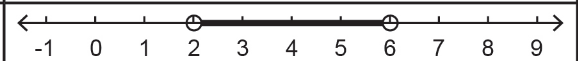
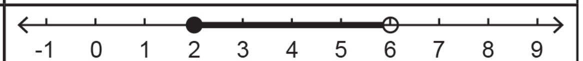
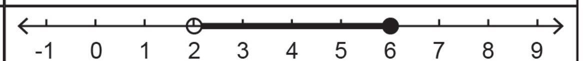
Solution: $x \leq -3$ or $x \geq 1$



Inequalities, Interval Notation and Representation on a number line

Inequality sign	words	Open/closed dot	
>	Greater than	Open	
≥	Greater than or equal to	Closed	
<	Less than	Open	
≤	Less than or equal to	Closed	

Examples:

Inequality	Interval notation	
$x > 2$	$x \in (2 ; \infty)$	
$x \geq 2$	$x \in [2 ; \infty)$	
$2 \leq x \leq 6$	$x \in [2 ; 6]$	
$2 < x < 6$	$x \in (2 ; 6)$	
$2 \leq x < 6$	$x \in [2 ; 6)$	
$2 < x \leq 6$	$x \in (2 ; 6]$	

Interval Notation is used to represent a set of Real Numbers as it is impossible to list them.

NUMBER PATTERNS

Sequence: A set of numbers written in order according to some mathematical rule.

The numbers in a sequence are called terms.

The terms of a sequence are indicated by the symbol T_n

Example, T_2 is the second term of the sequence.

T_n, the n^th term gives the rule for the sequence.

A sequence that goes up or down in equal steps is called an arithmetic sequence.

In an arithmetic sequence, a constant value is either added or subtracted to generate the next term in the sequence.

The difference between any 2 terms in an arithmetic sequence is known as the common difference.

Linear patterns

All these patterns have a common difference between each term. In other words,

$$T_2 - T_1 = T_3 - T_2$$

The general term for a linear pattern can be written as

$$T_n = bn + c$$

This form is like the standard form of the straight-line graph which shows it is a linear pattern. A pattern however would be represented by discrete points and not a continuous line.

To find the general pattern (also known as the n^th term): ● Find the common difference ● Substitute into 'd' in the general format To find the general term use: $T_n = a + (n-1)d$ a = first term d = common difference Example: Find the general term for the pattern 4 7 10 13… Common difference: 3 (7 – 4 = 3 and 10 – 7 = 3)	$T_n = a + (n-1)d$ $T_n = 4 + (n-1)(3)$ $T_n = 4 + 3n - 3$ $T_n = 3n + 1$
Given the position, looking for the term: substitute n with position given and find T_n Example. Find the 20^th term of the above pattern	$T_n = 3n + 1$ $T_{20} = 3(20) + 1 = 61$
Given the term, looking for the position: make an equation and solve for n (substitute in T_n) Example: In which position will the term 151 be in the above pattern?	$T_n = 3n + 1$ $151 = 3n + 1$ $150 = 3n$ $50 = n$ 151 is the 50^th term

Quadratic sequences

In a quadratic sequence the <u>*second difference*</u> is constant.

The first differences of a quadratic sequence form a linear sequence.

8   20   38   62

12   18   24   (first line of difference)

6   6 ⟶ (second difference)

$T_n = an^2 + bn + c$

$a \rightarrow$ half the second difference   ($2a = 2^{nd}$ difference)

$b \rightarrow$ a constant   ($3a + b = 1^{st}$ term of 1^{st} line of difference)

$c \rightarrow$ a constant   ($a + b + c = 1^{st}$ term of original sequence)

$n \rightarrow$ position of the n^{th} term in the sequence

For the above example:

$2a = 6$ $a = 3$	$3a + b = 12$ $3(3) + b = 12$ $9 + b = 12$ $b = 3$	$a + b + c = 8$ $3 + 3 + c = 8$ $6 + c = 8$ $c = 2$
$T_n = 3n^2 + 3n + 2$		

FINANCE AND GROWTH

Simple Interest $A = P(1 + in)$	A = Final amount P = Principal amount i = interest rate n = number of times interest is calculated* * In simple interest it is always annually.
Compound Interest $A = P(1 + i)^n$	
Hire Purchase (buying an item from a shop on credit - you are officially hiring the item until the final payment when you have finally purchased it)	• Always use simple interest formula $A = P(1 + i.n)$ • If insurance is required, it is always on the total purchase price regardless of deposits paid • Deposits are subtracted from purchase price to find amount needed to be 'borrowed'.
Inflation (The increase in an item over the course of time)	• Always use compound interest formula • When working towards a previous time period then you usually have 'A' and are looking for 'P'.
Exchange rates	The rate of one country's money against another country's money. Use ratios to convert between one currency and another.

Effective and nominal Interest (Nominal Interest is what you are quoted from the bank/institution. Effective interest is what you *actually* gain (if it's a savings situation) or *actually* pay (if it's a loan situation) due to the interest being compounded.	To convert between nominal and effective: $i_{eff} + 1 = (1 + \frac{i}{m})^m$ i_{eff} = effective rate i_{nom} = nominal rate m = number of compounding periods/year
Depreciation Assets (cars, machinery etc) reduce in value over time.	Depreciation on a straight-line balance (it will eventually be worth nothing): $A = P(1 - i.n)$ Depreciation on a reducing balance: $A = P(1 - i)^n$

FUNCTIONS

There are two types of functions:

ONE-TO-ONE	MANY-TO-ONE
A single x-value for a particular y-value	More than one x-value for a particular y-value
THERE CAN ONLY BE <u>ONE</u> y-VALUE	

The straight-line graph (Linear function)

Standard form:	$y = ax + q$ Gradient ($a > 0$: / , $a < 0$: \\ ) — Vertical shift (up/down), y-intercept
To draw	Find the x-intercept (make $y = 0$) Find the y-intercept (make $x = 0$)
To find the equation	• Given the y-intercept and another point: Substitute y-intercept for 'q' Substitute other point $(x;y)$ to find 'a' • Given two points Use two points to find gradient ('a') Use any point to substitute and find 'q' Note: check the values of 'a' & 'q' according to what they represent (for example, if you have found that $a < 0$, check that the line has a negative slope)
Domain and Range	Domain (all possible x-values on the function): $x \in R$ Range (all possible y-values on the function): $y \in R$
Other	• If 2 lines are parallel, then $m_1 = m_2$ • If 2 lines are perpendicular, then $m_1 \times m_2 = -1$ • A line perpendicular to the x-axis and parallel to the y-axis (a vertical line): the equation will be in the form $y = c$ • A line perpendicular to the y-axis and parallel to the x-axis (a horizontal line): the equation will be in the form $x = c$

The parabola (Quadratic function)

Standard form:	$y = ax^2 + bx + c$ or $y = a(x - p)^2 + q$ $(p;q) \rightarrow$ turning point a > 0: a < 0: y-intercept 
To draw	Find the x-intercept (make $y = 0$) Find the y-intercept (make $x = 0$) Find the axis of symmetry: $x = \frac{-b}{2a}$ Find the turning point: substitute the value of x from the axis of symmetry into the equation to find the corresponding y-value.

To find the equation	● Given the x-intercepts and another point: $y = a(x - x_1)(x - x_2)$ Substitute x-intercepts for x_1 and x_2 Substitute other point $(x;y)$ to find 'a' ● Given turning point and another point: $y = a(x - p)^2 + q$ Substitute the turning point for p and q Substitute other point $(x;y)$ to find 'a' Note: check the values of 'a' according to what it represents (for example, if you have found that $a < 0$, check that the parabola opens downwards/is upside down)
Domain and Range	Domain (all possible x-values on the function): $x \in R$ Range (all possible y-values on the function): If $a > 0$: $y \in [q\ ;\ \infty)$ If $a < 0$: $y \in (-\infty\ ;q]$
Other	Parabolas can have a minimum or a maximum value. ● If $a > 0$, there is a minimum value The minimum value is $y = q$ ● If $a < 0$, there is a maximum value The maximum value is $y = q$

Examples: Finding equations of parabolas

Finding the equation of a parabola given the TURNING POINT and another point.

Use	Example:
$y = a(x - p)^2 + q$	The turning point of a parabola is (2 ; 6) and it also passes through the point (5 ; -30). Find the equation of the parabola.
Substitute turning point into $(p ; q)$	$y = a(x - 2)^2 + 6$
Substitute other point $(x ; y)$	$-30 = a(5 - 2)^2 + 6$ (1*)
Solve for a	$-30 = a(9) + 6$ $-36 = 9a$ $-4 = a$
Substitute 'a' back into (1*)	$y = -4(x - 2)^2 + 6$
Multiply out and collect like terms ($y = ax^2 + bx + c$)	$y = -4(x^2 - 4x + 4) + 6$ $y = -4x^2 + 16x - 16 + 6$ $y = -4x^2 + 16x - 10$

Finding the equation of a parabola given the x-INTERCEPTS and another point.

Use	Example:
$y = a(x - x_1)(x - x_2)$	A parabola passes through the point (-2 ; 0), (5 ; 0) and (0 ; 5). Find the equation of the graph.
Substitute 2 values of x-intercepts into x_1 and x_2	$y = a(x + 2)(x - 5)$ (1)*
Substitute the other coordinate $(x ; y)$ and solve for a.	$5 = a(0 + 2)(0 - 5)$ $5 = -10a$ $\frac{-1}{2} = a$
Substitute 'a' back into (1)*	$y = \frac{-1}{2}(x^2 - 3x - 10)$
Multiply out and collect like terms ($y = ax^2 + bx + c$)	$y = \frac{-1}{2}x^2 + \frac{3}{2}x + 5$

Minimum and maximum values of quadratic expressions

Once you have completed the square of an expression and have it in the form:

$$a(x - p)^2 + q$$

If $a > 0$, there will be a MINIMUM value
If $a < 0$, there will be a MAXIMUM value

This will be the MAXIMUM or MINIMUM value

Example 1: $-2(x - 1)^2 + 4$

Has a maximum value

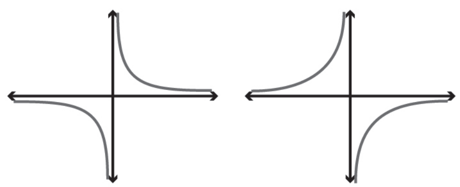
which is

Example 2: $\frac{1}{2}(x - 5)^2 - 5$

Has a minimum value

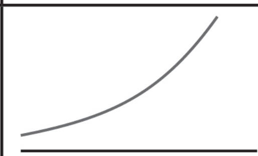
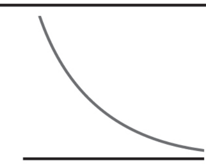
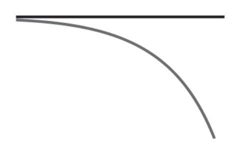
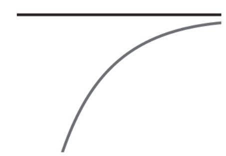
which is

The hyperbola (Hyperbolic function)

Standard form:	$y = \frac{a}{x-p} + q$ $+q$: Vertical shift (up/down); Horizontal asymptote ($y = q$) p: Horizontal shift (left/right); Vertical asymptote ($x = p$) $a > 0$: $a < 0$: 
To draw	Draw in the horizontal asymptote: $y = q$ Draw in the vertical asymptote: $x = p$ Find the x-intercept (make $y = 0$) Find the y-intercept (make $x = 0$)
To find the equation	Given the asymptotes and another point: Substitute the value of the asymptotes for 'p' and 'q' Substitute other point (x ; y) to find 'a' Note: check the values of 'a' according to what it represents (for example, if you have found that $a < 0$, check that the hyperbola is in the correct quadrants)
Domain and Range	Domain (all possible x-values on the function): $x \in R; x \neq p$ Range (all possible y-values on the function): $y \in R; y \neq q$
Other	A hyperbola has two axes of symmetry. ● One has a gradient of '1' and the other has a gradient of '-1' ● They both pass through the point where the asymptotes meet. (p ; q)

The exponential graph (Exponential function)

Standard form:	$y = a.b^{x-p} + q$ $(a \neq 0 ; b \neq 0)$ p: Horizontal shift (left/ right) q: Vertical shift (up/down) Horizontal asymptote ($y = q$)

	b > 1	0 < b < 1
a > 0		
a < 0		

To draw	Decide whether it lies above or below the asymptote. (If $a > 0$, it lies above the asymptote and if $a < 0$ it lies below the asymptote) Decide whether it is increasing or decreasing by considering 'a' and 'b'. Draw in the horizontal asymptote: $y = q$ Find the y-intercept (make $x = 0$) Find the x-intercept (make $y = 0$) Find a few more points if necessary with possible x-values.
To find the equation	The amount of information required is always directly related to the number of variables missing. You are likely to be given the 'format' of the graph. For example, $y = ab^x + q$ (notice there is no 'p'). As you are looking for the values of 3 variables (a, b and q) there must be 3 pieces of information given to you. Given the asymptote and another point: Substitute the value of the asymptote for 'q' Substitute other point (x ; y) to find 'a' or 'b' Simultaneous equations may be necessary.
Domain and Range	Domain (all possible x-values on the function): $x \in R$ Range (all possible y-values on the function): If $a > 0$: $y \in [q\ ;\ \infty)$ If $a < 0$: $y \in (-\infty\ ;\ q]$

General information regarding functions

1. Find the values of x for which:

$f(x) = g(x)$	Make the equations equal and solve for x. If the coordinates are asked for, substitute the x-value(s) into any function and solve for y.
For each of the questions below: ● First find the part of the graph that answers the question (highlight it if possible) ● Find the x-values that correspond to the part of the graph that satisfies the statement.	
$f(x) > g(x)$	Where is the function $f(x)$ greater than (in other words above) the function $g(x)$.
$f(x) < g(x)$	Where is the function $f(x)$ less than (in other words below) the function $g(x)$.
$f(x) \geq g(x)$	Where is the function $f(x)$ greater than (in other words above) or equal to the function $g(x)$.
$f(x) \leq g(x)$	Where is the function $f(x)$ less than (in other words below) or equal to the function $g(x)$.

2. Average Gradient

   This is the average gradient between two points on a curve

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

3. Transformations of functions
   - Reflections

Reflection in the x-axis ($y = 0$)	Rule: $(x ; y) \rightarrow (x ; -y)$ In other words – leave the x-value the same and change the y-value to negative
Reflection in the y-axis ($x = 0$)	Rule: $(x ; y) \rightarrow (-x ; y)$ In other words – leave the y-value the same and change the x-value to negative

PROBABILITY

Probability is the likelihood of something happening or being true. Probability is assigned a value from 0 (impossible) to 1 (certain). The probabilities of the possible outcomes in a sample space must sum up to 1.

Probability of an event occurring and sample space

The probability of Event A occurring is: P(A) = $\frac{n(A)}{n(S)}$

In general, A is the total number of ways a specific event can occur. S is the total number of possible outcomes for the entire event.

Notation

A = {1;2;3;4;5} represents Event (Set) A.

- $n(A)$ → the number of items in set A.
- $P(A)$ → The probability of Event A occurring
- $P(A')$ → The probability of Event A **NOT** occurring. It is also known as the complement of A

- $P(A \text{ or } B)$ = P(A ∪ B)→ The probability of A or B occurring. ∪ is the symbol for **'or'** and is also known as **union**.
- $P(A \text{ and } B)$ = P(A ∩ B)→ The probability of A and B occurring. ∩ is the symbol for **'and'** and is also known as **intersection**.

Inclusive events

Two events that can occur at the same time are inclusive.

$P(A \cap B) \neq 0$ $\quad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Mutually Exclusive events

Two events that are mutually exclusive cannot occur at the same time. There is no intersection.

$P(A \cap B) = 0$ $\quad$ $P(A \text{ or } B) = P(A) + P(B)$

Exhaustive events

Two events A and B are exhaustive if together they cover all the elements of the sample space.

$P(A \text{ or } B) = 1$

Complementary events

Mutually exclusive, exhaustive events are **complementary** events. They are the only two possible outcomes. If one event does not occur, the other event must occur

$P(\text{not } A) = P(A') = 1 - P(A)$

The addition rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If the events are mutually exclusive then: $P(A \cup B) = P(A) + P(B)$, **as** $P(A \text{ and } B) = 0$.

Venn diagrams

Venn diagrams are a graphical way of representing a sample space and its events. If two events can *both* happen at the *same time*, then a Venn diagram is a good way to represent the situation.

Tree diagrams

When there are two or more consecutive events taking place, it is often useful to represent the possible solutions on a tree diagram. Tree diagrams are constructed by showing all possible events. They can be used for dependent or independent events. When dealing with tree diagrams always multiply along the branches (horizontal) and add probabilities moving down branches (vertical) at the end. Write the probability of an event occurring at the top of the branches and the actual event at the end of the branch.

Contingency tables

Contingency tables are statistical tables that show the relationship between two or more variables. They are often used to determine whether events are independent or not.

Dependent and independent events

Two events are independent if the outcome of one event does not affect the probability of another event occurring. If the outcome of one event changes the probability that another event occurs, the events are said to be dependent.

The product rule:

Test for independent events: $P(A \text{ and } B) = P(A) \times P(B)$

Remember: $P(A \text{ and } B)$ can be written as $P(A \cap B)$

RESOURCE 8

REVISION: Paper 1 2017

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 11

MATHEMATICS P1

NOVEMBER 2017

MARKS: 150

TIME: 3 hours

This question paper consists of 7 pages.

IMAT1

EASTERN CAPE

Please turn over

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of NINE questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs et cetera that you have used in determining the answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. Round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(2x-3)(x+7)=0$ (2)

1.1.2 $7x^2+3x-2=0$ (leave your answer correct to TWO decimal places) (3)

1.1.3 $\sqrt{x-1}+3=x$ (6)

1.1.4 $x^2>3(x+6)$ (4)

1.2 Solve for x and y simultaneously:

$2y+x=1$
$x^2+y^2+3xy+y=0$ (6)

1.3 If $f(x)=0$ has roots $x=\dfrac{-5\pm\sqrt{3-12k^2}}{4}$, for which values of k will the roots be equal? (3)
[24]

QUESTION 2

2.1 Simplify fully, WITHOUT using a calculator:

$\dfrac{3^{m+4}-6.3^{m+1}}{7.3^{m+2}}$ (4)

2.2 Solve for x, WITHOUT using a calculator:

2.2.1 $x^{-\frac{3}{4}}=8$ (3)

2.2.2 $4^x-2^x=2$ (4)

2.3 If $x=\dfrac{3-\sqrt{a}}{\sqrt{2}}$ and $y=\dfrac{4+\sqrt{a}}{\sqrt{2}}$, determine the value of $(x+y)^2$ (3)

2.4 Show, WITHOUT using a calculator, that $\sqrt[12]{10}\times\sqrt[6]{640}\times\sqrt[4]{810}\times\sqrt{40}=120$ (4)
[18]

QUESTION 3

3.1 Given the finite linear pattern: 12 ; 17 ; 22 ; … ; 172

3.1.1 Determine a formula for the n^{th} term of the pattern. (2)

3.1.2 Calculate the value of T_{12} . (2)

3.1.3 Determine the number of terms in the pattern. (2)

3.2 Given the first four terms of a linear pattern: 3; x; y ; 30

Calculate the values of x and y . (4)

[10]

QUESTION 4

Given the quadratic pattern: 244 ; 193 ; 148 ; 109 …

4.1 Write down the next term of the pattern. (2)

4.2 Determine a formula for the n^{th} term of the pattern. (4)

4.3 Which term of the pattern will have a value of 508? (4)

4.4 Between which TWO consecutive terms of the quadratic pattern will the first difference be 453? (3)

4.5 Show that all the terms of the quadratic pattern are positive. (4)

[17]

QUESTION 5

Given: $f(x)=\frac{-3}{x+2}+1$ and $g(x)=2^{-x}-4$

5.1 Determine $f(-3)$. (1)

5.2 Determine x if $g(x)=4$. (2)

5.3 Write down the asymptotes of f. (2)

5.4 Write down the range of g . (1)

5.5 Determine the coordinates of the x- and y- intercepts of f . (5)

5.6 Determine the equation of the axis of symmetry of f which has a negative gradient. Leave your answer in the form $y=mx+c$. (2)

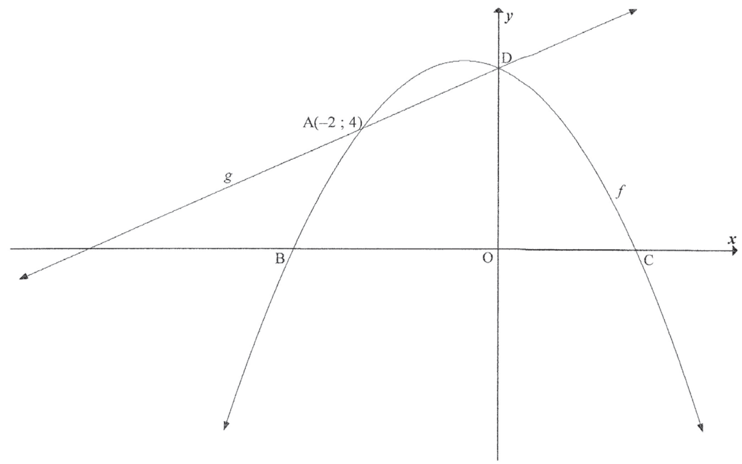
5.7 Sketch the graphs of f and g on the same system of axes. Clearly show ALL intercepts with the axes and any asymptotes. (6)

5.8 If it is given that $f(-1)=g(-1)$, determine the values of x for which $g(x)\geq f(x)$. (2)

[21]

QUESTION 6

The diagram below shows the graphs of $f(x)=-x^2-x+6$ and $g(x)=mx+c$. A$(-2\ ;\ 4)$ is the point of intersection of the graphs.



6.1 Determine the x-intercepts f. (4)

6.2 Write down the equation of the axis of symmetry of f . (2)

6.3 Determine the range of f. (3)

6.4 Write down the equation of g in the form $g(x)=mx+c$. (3)

6.5 Write down the average gradient between points A and D. (1)

6.6 Determine the equation of h, if h is the reflection of f about the x-axis and then translated 3 units to the right. Leave your answer in the form $h(x)=a(x+p)^2+q$. (3)

6.7 Write down the values of x for which $f(x)>0$. (2)

6.8 If $f(p)=f(r)=4$, calculate the value of $p-r$ if $r<0$. (4)

[22]

QUESTION 7

7.1 A company bought machinery costing R80 000. Using the reducing balance method, the machinery had a book value of R20 000 after 5 years.

Calculate the rate of depreciation. (3)

7.2 Calculate the effective interest rate if interest is compounded at 5% p.a., compounded quarterly. (3)

7.3 Sipho invested R30 000 for 6 years. The investment earned interest at 12% p.a., compounded monthly for the first two years. Thereafter the interest rate changed to 10,8% p.a., compounded semi-annually for the rest of the period.

Calculate the value of the investment at the end of 6 years. (No other transactions were made on the account.) (4)

7.4 Mary deposited R25 000 into a savings account with an interest rate of 18% p.a., compounded monthly. Mary withdrew R8 000 from the account 2 years after depositing the initial amount. She deposited another R4 000 into this account 3½ years after the initial deposit. What amount will Mary have 5 years after making the initial deposit in this account? (6)
[16]

QUESTION 8

8.1 A bag contains 3 blue marbles and 2 red marbles. A marble is taken from the bag, the colour is recorded and the marble is put aside. A second marble is taken from the bag, the colour is recorded and then put aside.

8.1.1 Draw a tree diagram to represent the information above. Show the probabilities associated with EACH branch, as well as the possible outcomes. (3)

8.1.2 Determine the probability of first taking a red marble and then taking a blue marble, in that order. (2)

8.2 A and B are two events. The probability that event A will occur is 0,4 and the probability that event B will occur is 0,3. The probability that either event A or event B will occur is 0,58.

8.2.1 Are events A and B mutually exclusive?
Justify your answer with appropriate calculations. (3)

8.2.2 Are events A and B independent?
Justify your answer with appropriate calculations. (3)
[11]

QUESTION 9

A survey was done among 80 learners on their favourite sport.
The results are shown below.

- 52 learners like rugby (R)
- 42 learners like volleyball (V)
- 5 learners like chess (C) only
- 14 learners like rugby and volleyball but not chess
- 12 learners like rugby and chess but not volleyball
- 15 learners like volleyball and chess but not rugby
- x learners like all 3 types of sport
- 3 learners did not like any sport

9.1 Draw a Venn diagram to represent the information above. (5)

9.2 Show that $x = 8$. (2)

9.3 How many learners like only rugby? (1)

9.4 Calculate the probability that a learner, chosen randomly, likes at least TWO different types of sport. (3)
[11]

TOTAL: 150

RESOURCE 9

REVISION: Week 2

Summary notes – Paper 2

ANALYTICAL GEOMETRY

All three formulae require 2 points: $(x_1 ; y_1)$ and $(x_2 ; y_2)$

Gradient	$m = \frac{y_2 - y_1}{x_2 - x_1}$
Distance	$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Midpoint	$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

Useful information:

1. Collinear points: points that lie on a straight line. To prove three points (A, B & C) collinear, prove $mAB = mBC = mAC$ (only two pairs required)
2. 2 lines are parallel if their gradients are equal.
3. 2 lines are perpendicular if the product of their gradients equals -1.
4. To find the y-intercept of any graph, let $x = 0$
   To find the x-intercept of any graph, let $y = 0$
5. To show that two lines bisect each other – the midpoints of each line must be equal.
6. To show that a point lies on a graph: substituting the point should make LHS = RHS
7. To find where two graphs intersect, get both into standard form ($y = \ldots$), solve simultaneously.
8. Properties of quadrilaterals (often needed):
   - Diagonals of rhombus bisect each other at 90º
   - Diagonals of a rectangle are equal in length.
9. To prove a quadrilateral is a parallelogram, prove one of the following:
   - diagonals bisect (same mid-point)
   - both pairs of opposite sides parallel (equal gradients)
   - both pairs of opposite sides equal (equal lengths)
   - one pair of opposite sides parallel and equal (equal lengths & equal gradients)

Finding the equation of a straight line

Examples:

Determine the equation of a straight which:

a) is parallel to the line $=-3x+4$; passing through the point A (4;7).

$m = -3 \quad \rightarrow$ line is ||

$y - y_1 = m(x - x_1)$

$y - 7 = -3(x - 4)$

$y = -3x + 12 + 7$

$y = -3x + 19$

b) is perpendicular to the line $= \frac{-2}{3}x + 2$; with a y-intercept of -3.

$m = \frac{3}{2} \quad \rightarrow$ line is $\perp$ to $y = \frac{-2}{3}x + 2$

Sub $c = -3$

$\therefore y = \frac{3}{2}x - 3$

c) is parallel to the x-axis and passes through the point (-4;3).

$y = 3$ (A line parallel to the x-axis is a horizontal line)

d) is parallel to the y-axis and passes through the point (-4;3).

$x = -4$ (A line parallel to the y-axis is a vertical line)

e) passes through the points (-2;4) and (3;-6)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-6-4}{3-(-2)} = \frac{-10}{5} = -2$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -2(x - (-2))$$

$$y = -2x + 4 + 4$$

$$y = -2x + 8$$

Angle of inclination

Angle of inclination is often shown as θ.

The gradient of a line (m) is equal to the tangent of the angle of inclination (θ).

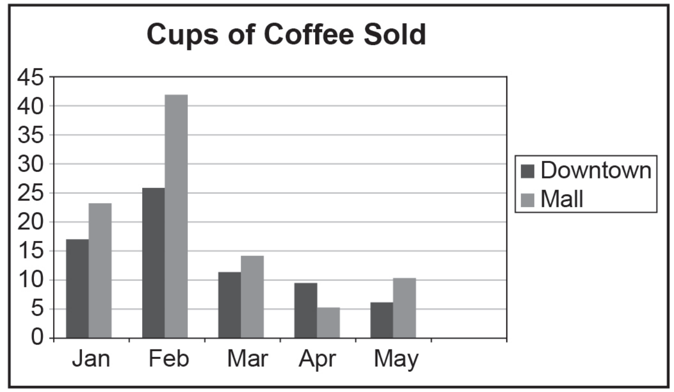
$\tan \theta = m$ where $\theta \in (0°;180°)$

STATISTICS

Ungrouped data

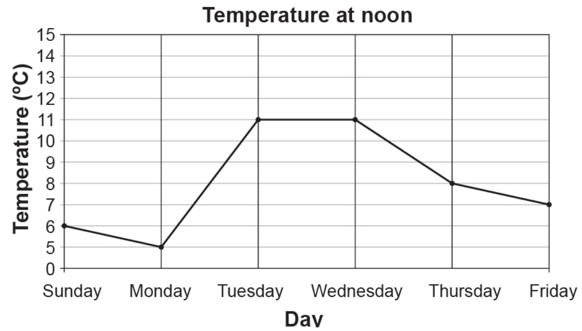
Representing ungrouped data graphically:

Bar Graph



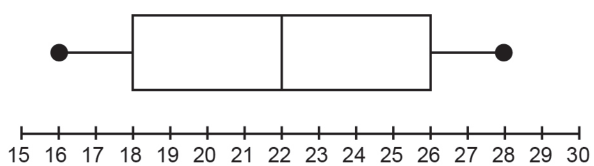
Bar graphs display discrete data.

Broken line graphs



Broken line graphs represent changes in data over time. In such graphs, convention *dictates the independent variable is represented horizontally on the* x*-axis and the dependant variable is represented vertically on the* y*-axis.*

Box-and-whisker plot

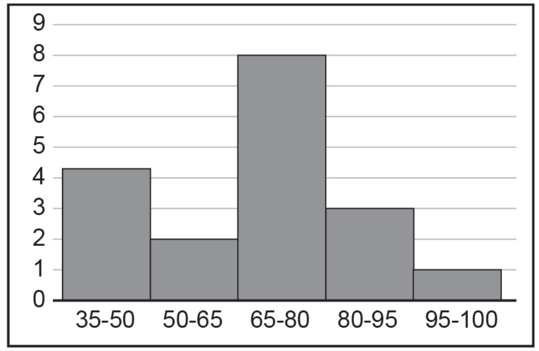


This type of graph is used to show the shape of the distribution, its central value, and its variability. In a box-and-whisker plot: the ends of the box are the upper and lower quartiles, so the box spans the interquartile range. the median is marked by a vertical line inside the box.

Grouped data

Representing grouped data graphically:

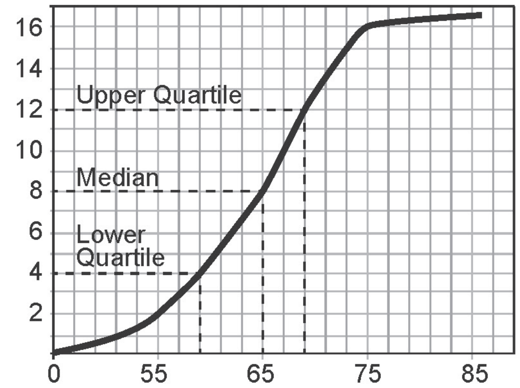
Histogram



Histograms display continuous data. Therefore the bars are touching.

Discrete data has clear separation between the different possible values, while continuous data doesn't. We use bar graphs for displaying discrete data, and histograms for displaying continuous data.

Ogive (Cumulative frequency curve)



- A way of representing grouped data
- Never goes down and should form an S-shape
- The horizontal axis will represent the data
- Only the upper boundary numbers will be represented. These are the x-co-ordinates of the points found
- The vertical axis will always represent the cumulative frequency – no matter what the situation is being represented
- Remember to ground the ogive using the lower boundary number of the first class interval with zero
- Can be used to estimate median, quartiles and percentiles

Measures of Central Tendency

Ungrouped data

Mean	Example: List of shoe sizes: 7,9,12,9,8,6,9,12,13,17
Most commonly used measure of central tendency Add all data and divide by number of items in data set. The mean is distorted by outliers	$\frac{7+9+12+9+8+6+9+12+13+17}{10}$ $= \frac{102}{10}$ $= 10,2$
Median	
Middlemost score (odd number) or average of the two middle scores (even number). Numbers need to be ordered	6 7 8 9 $\underline{9 \quad 9}$ 12 12 13 17 $\frac{9+9}{2} = \frac{18}{2} = 9$
Mode	
The most frequently occurring score Can have more than one mode	9

Grouped data

Estimate of the mean:

- Calculate the midpoint of each class
- Multiply each midpoint by the frequency for that interval
- Add up and divide by the total number of scores

The modal class:

- This is the interval in which the data occurs most frequently

The median:

- The best way to calculate the median is by drawing a cumulative frequency curve (Ogive
- A way of representing grouped data
- Never goes down and should form an S-shape
- Can also be used to estimate median, quartiles and percentiles

Measures of Dispersion (spread of data)

1. Range

The difference in the largest and the smallest value in the data set. The bigger the range the more spread out the data is.

2. Quartiles

Measures of dispersion around the median. The median divides the data into two halves. The lower and upper quartiles divide the data further into quarters.

To find: Lower quartile - Q_1: $\frac{1}{4}(n+1)$

Median - Q_2: $\frac{1}{2}(n+1)$

Upper quartile - Q_3: $\frac{3}{4}(n+1)$

Remember: This gives the *position*!

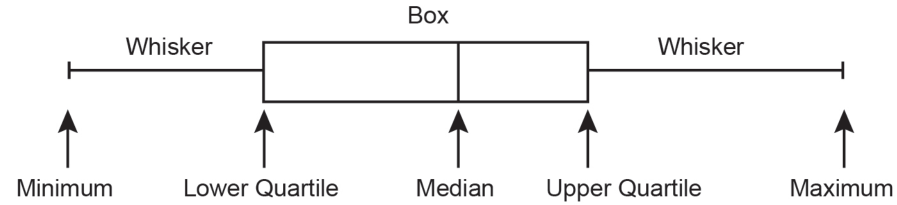
3. Inter-quartile range (IQR)

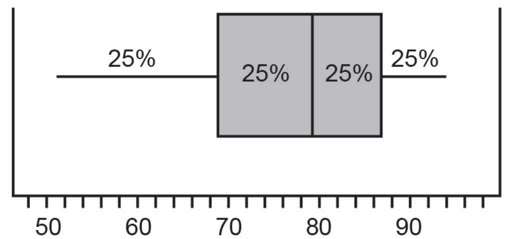
The difference between the upper quartile and lower quartile $(Q_3 - Q_1)$

4. Five number summary:

- Minimum: The smallest value in the set of data
- Lower quartile: The median of the lower half of the values
- Median: The value that divides the data into halves
- Upper quartile: The median of the upper half of the values
- Maximum: The largest value in the data.

The box and whisker plot is a graphical representation of the five number summary.





Variance and Standard deviation

- A way of measuring the spread of data
- Variance = average of squared differences of the mean
- Standard deviation = $\sqrt{variance}$

How standard deviation is found:

1. Work out the average (mean value) of your set of numbers
2. Work out the difference between each number and the mean
3. Square the differences
4. Add up the squares of all the differences

5. Divide this by the number of data in the set - this is called the **variance**.
6. Take the square root of the variance – this is the standard deviation.

If data is normally distributed

- Around 68% of data are within one standard deviation of the mean
- Around 95% of data are within two standard deviations of the mean
- Around 99% of data are within three standard deviations of the mean.

If the data is grouped the middle value of the interval must be used as well as the frequency for the calculation as above.

Skewed data and outliers

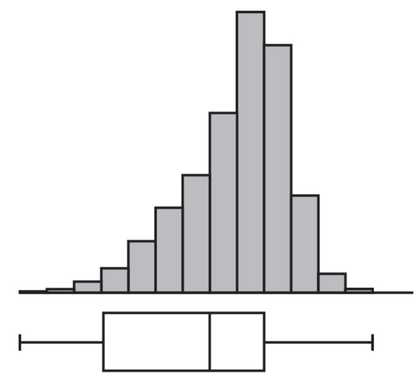
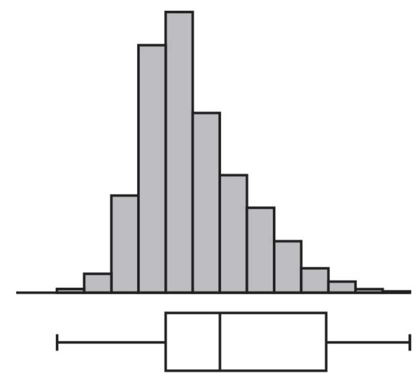
The mean is susceptible to the influence of outliers so if there are any outliers, the mean is not considered a good representation of the data.

If you have a normally distributed sample, the mean and median are both good measures of central tendency. (In perfectly symmetrical data the mean would equal the median)

If the data is skewed, the mean tends to be 'dragged' in the direction of the skewness. (In this case, the median is more likely to be a better representation of the data). Skewness exists if there are extreme scores or tail.

The more skewed the distribution, the greater the difference between the median and mean.

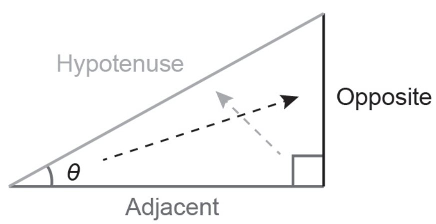
In most cases:

Negatively skewed (mean subtract median < 0)	Positively skewed (mean subtract median > 0)
Skewed to the left – the data is more spread out on the left (Longer tail on left = skewed to left)	Skewed to the right – the data is more spread out on the right. (Longer tail on right = skewed to the right)
mean < median < mode	mode < median < mean
	

TRIGONOMETRY

Right-angled triangles

SOHCAHTOA



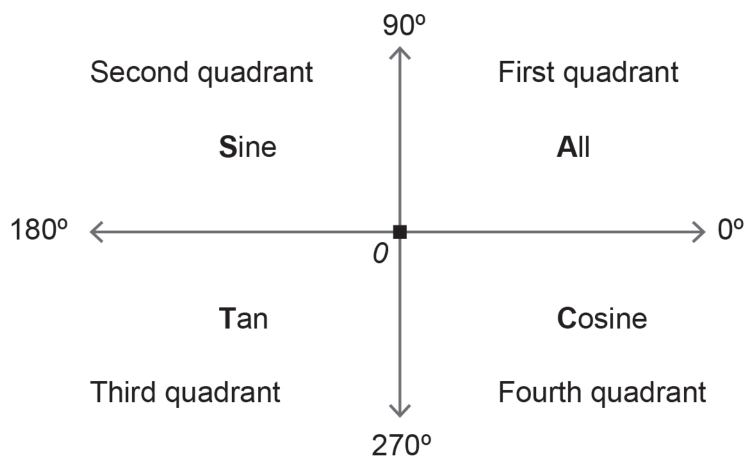
SOH Sin $\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$

CAH Cos $\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$

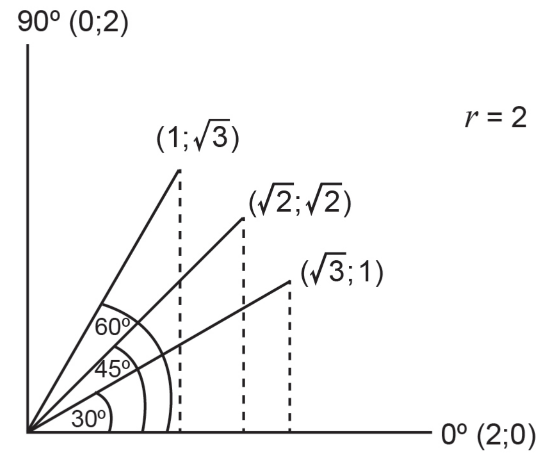
TOA Tan $\theta = \frac{\text{Opposite}}{\text{Adjacent}}$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

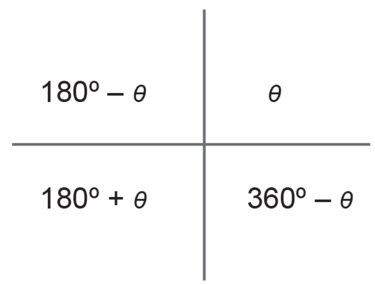
Quadrants



Special angles



Reductions



Step 1: re-name the angle

- The angle is in quadrant …
- Where we name angles..(according to above diagram)

Step 2: Reduce to the acute angle

- This angle is in quadrant..
- Where the trig ratio is…(positive or negative)

NOTE: Adding 360° or subtracting 360° from an angle does not change the ratio of the angle.

For example: tan 500º

tan 500º	(subtract 360°)
= tan 140º	(quadrant 2 ∴ 180º–..)
= tan(180º – 40º)	(quadrant 2 ∴ negative)
= –tan 40º	

Complementary angles

sin (90°– θ) = cos θ

sin (90°+ θ) = cos θ

cos (90°– θ) = sin θ

cos (90°+ θ) = -sin θ

Identities

$sin^2\theta + cos^2\theta = 1$ $\qquad$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$

Tips to prove identities:

- Change any tan function to $\frac{sin}{cos}$
- If there are any fractions to be added or subtracted, find LCD and simplify
- Consider the new numerator (after adding or subtracting) and check for factorising opportunities

Using diagrams to determine numerical values of ratios (Pythagoras questions)

Steps:

- Using BOTH pieces of information, decide which quadrant you need to work in
- Make a sketch, drawing the triangle in the correct quadrant.
- Fill in the two known sides from the given information
- Use Pythagoras to find the third side
- Summarise the information you now know regarding what x, y and r are all equal to
  (Be careful of signs here!)
- Use this information to complete the question using substitution.

NB: Need to know trig ratios in terms of x, y and r.

Don't even consider the 'question' (find…) until the groundwork is done.

General solutions

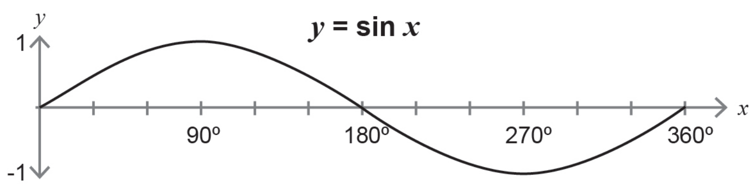
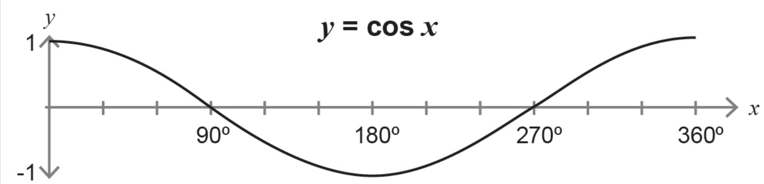
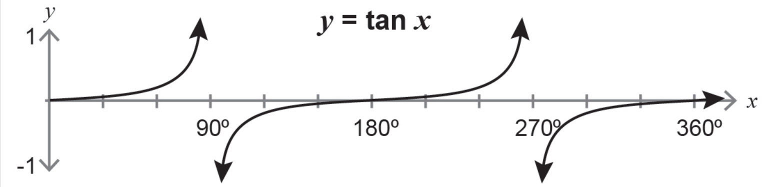
Steps:

- Make the trig function the subject of the formula
- Use the 2^{nd} function on the calculator: (shift ; trig function ; ratio) to find the reference angle (If the trig function is negative, do not use this when finding the reference angle).
- Note whether the function is positive or negative
- Choose the quadrants accordingly and find the general solutions according to the quadrants
- Use the appropriate reductions to represent angles in the chosen quadrants.
- Use k to show that it is a general solution and if required, substitute integers to find specific solutions.

Examples:

Find the general solution: $cos\ \theta = 0{,}85$ (shift ; cos ; 0,85) $RA = 31{,}79°$ (ratio is positive - ∴ quadrant 1 and 4) $\theta = 31{,}79° + k.360°$ OR $\theta = 360° - 31{,}79° + k.360°$ $\theta = 328{,}21° + k.360°$ $k \in Z$	Solve for $x \in (-180°;360°)$ $2\sin(x + 20°) = -1{,}53$ $\sin(x + 20°) = -\frac{1{,}53}{2}$ (shift ; sin ; $\frac{1{,}53}{2}$) [Do not use the negative sign] $\therefore RA = 49{,}91°$ (ratio is negative ∴ quadrant 3 and 4) $x + 20° = 180° + 49{,}91° + k.360°$ $x = 209{,}91° + k.360°$ OR $x + 20° = 360° - 49{,}91° + k.360°$ $x = 290{,}09° + k.360°$ $k \in Z$ $\therefore x \in \{209{,}91°; 290{,}09°\}$

Trig graphs

Sine graph	A function of the form $y = sin\ x$ with a period of 360º 
Cosine graph	A function of the form $x = cos\ x$ with a period of 360º 
Tangent graph	A function of the form $y = tan\ x$ with a period of 180º 

Period: The number of degrees it takes for the graph to complete a pattern before it gets repeated

Amplitude: The maximum deviation from the x-axis.
Can be found by using: ½ (distance between maximum and minimum values)

Vertical shifts of the sine, cosine and tangent graphs

$y = sin\ x + q$ $\quad y = cos\ x + q$ $\quad y = tan\ x + q$

'q' represents the units the basic graph shifts vertically (up or down)
It will change the maximum and minimum value and therefore the range.
It will NOT change the amplitude or period.
The vertical distance (size) remains the same.

Amplitude shifts of the sine and cosine graphs:

$y = a \sin x \qquad y = a \cos x$

The graph is stretched or squashed from its original position. The vertical distance (size) changes – it becomes longer or shorter.

The value of 'a':

- gives the new amplitude. If 'a' is negative, this affects the direction of the graph
- changes the maximum and minimum value and therefore the range.
- does NOT change the period. It remains 360°.

Horizontal shifts

$y = \sin (x + p) \qquad y = \cos (x + p) \qquad y = \tan (x + p)$

p represents the horizontal (left or right) shift of the basic graph

If $p < 0$, the graph shifts to the right

If $p > 0$, the graph shifts to the left

Period changes

$y = \sin bx \qquad y = \cos bx \qquad y = \tan bx$

b affects the period of the graph.

$$\frac{\text{original period}}{b} = \text{new period}$$

(b could also be seen as 'the number of full graphs that can be seen in the original period)

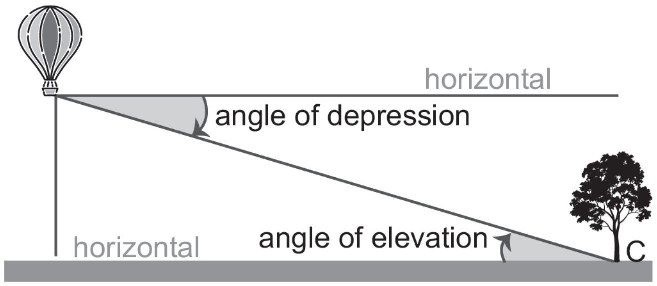
For example, for the function $y = \sin 3x$, the new period is $\frac{360°}{3} = 120°$ and if the function was drawn over 360°, there would be 3 sine curves visible.

Summary

$y = a \sin(b\theta + p) + q$
$y = a \cos(b\theta + p) + q$
$y = a \tan(b\theta + p) + q$

a	Amplitude	Stretches ($a > 1$) or squashes ($0 < a < 1$) or reflects in the x-axis (flips over) if $a < 0$
b	Period	Distance in degrees to complete a cycle. If $b = 1$, then period is 360° for sin & cos graph & 180° for tan graph To find 'new' period if b ≠ 1, divide regular period by b
p	Vertical shift	Number of units shifted up or down the y-axis
q	Horizontal shift	Number of degrees shifted left or right on the x-axis

2-dimensional problems



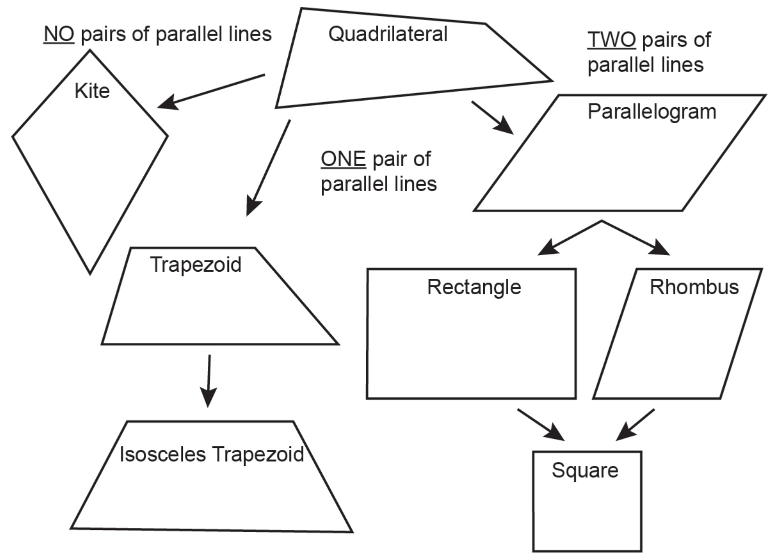
Example Find the height of the tree 	$tan\ 38^{\circ} = \frac{tree}{4,2}$ $4,2 \times tan\ 38^{\circ} = tree$ $3,28m = tree$

Sine, Cosine and Area rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	Look for pairs with opposite side and angle. If you're missing only one of the 4 values when looking at 2 pairs, use Sine rule
$a^2 = b^2 + c^2 - 2bc \cos A$	Use if given 2 sides and included angle or 3 sides
$\frac{1}{2}$ ab sin C	Use to find area Need 2 sides and included angle

EUCLIDEAN GEOMETRY AND MEASUREMENT

Family tree of quadrilaterals showing how they relate to each other



Definitions of the 6 quadrilaterals

Parallelogram	A quadrilateral with both pairs of opposite sides parallel
Rectangle	A parallelogram with 4 right angles
Rhombus	A parallelogram with 4 equal sides
Square	A parallelogram with 4 equal sides and 4 right angles
Kite	A quadrilateral with 2 pairs of adjacent sides equal and no opposite sides equal.
Trapezium	A quadrilateral with one pair of opposite sides parallel

Properties of quadrilaterals

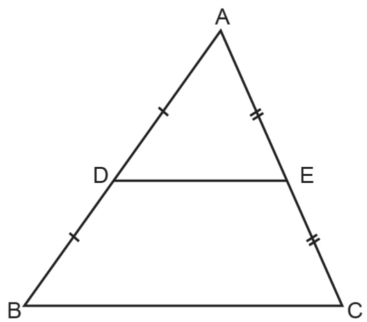
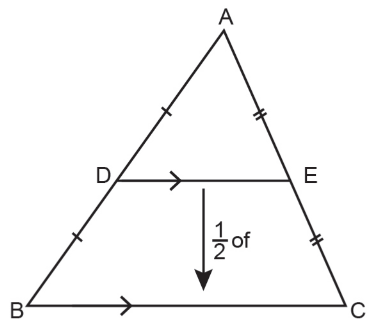
PROPERTY	PARALLELOGRAM	RECTANGLE	RHOMBUS	SQUARE
Opposite sides parallel	✓	✓	✓	✓
Opposite angles equal	✓	✓	✓	✓
Opposite sides equal	✓	✓	✓	✓
Diagonals bisect each other	✓	✓	✓	✓
Diagonals are equal		✓		✓
Diagonals are perpendicular			✓	✓
Diagonals bisect opposite angles			✓	✓
All sides equal			✓	✓
All angles right angles		✓		✓

How to prove a quadrilateral is a:

Parallelogram ● both pairs of opposite sides parallel or ● both pairs of opposite sides equal or ● one pair of opposite sides equal and parallel or ● diagonals bisect each other or ● opposite angles equal	**Rectangle** It must be a parallelogram with: ● equal diagonals or ● one right angle
Rhombus It must be a parallelogram with: ● 4 equal sides or ● diagonals bisect at right angles	**Square** It must be a ● rhombus with one right angle or ● rectangle with 2 adjacent sides equal

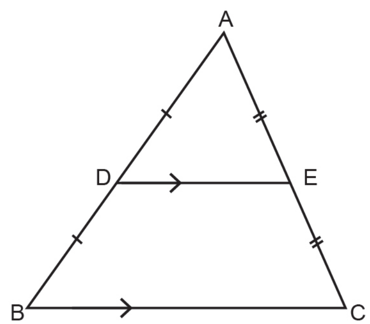
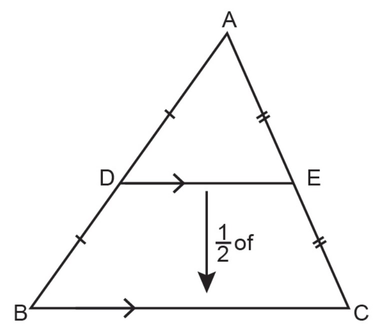
The midpoint theorem

The line joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side. (Abbreviated reason – midpt theorem)

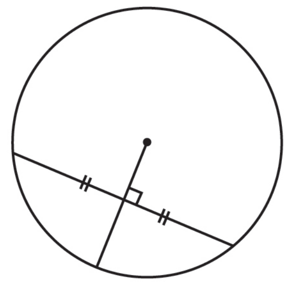
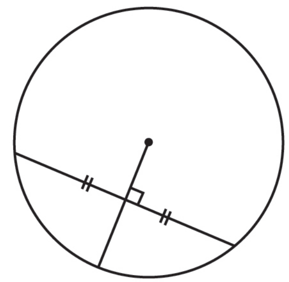
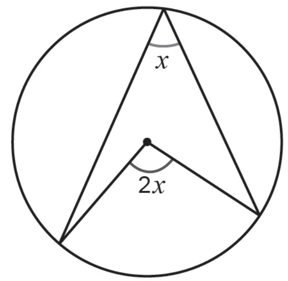
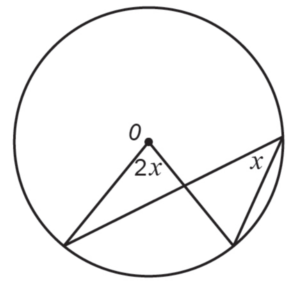
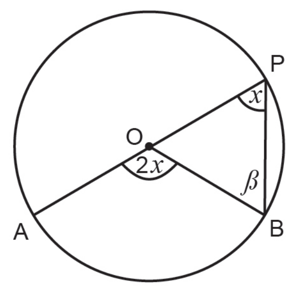
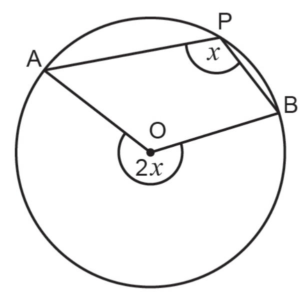
Given:	Then:
	

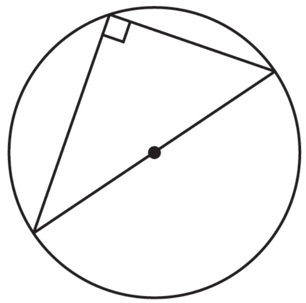
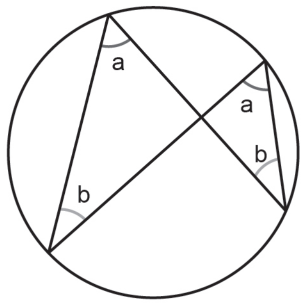
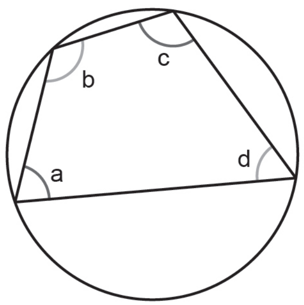
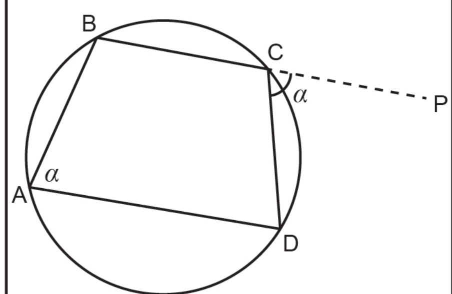
Converse of midpoint theorem

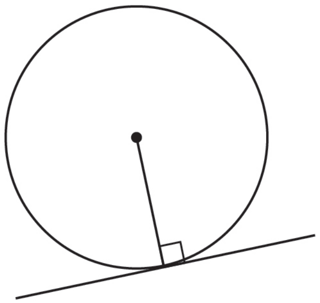
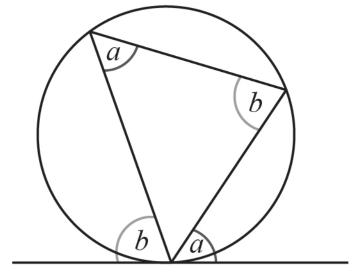
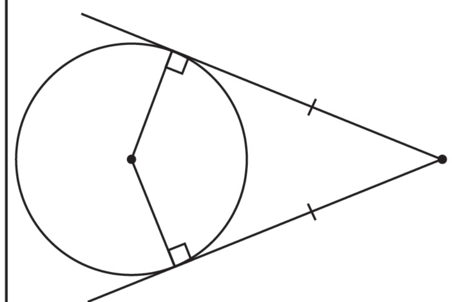
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side. (Abbreviated reason – line through midpt || to 2nd side.

Given:	Then:
	

Circle Geometry

The perpendicular line from the centre of a chord bisects the cord	The line from the centre to the midpoint of a chord is perpendicular to the chord
	
The angle at the centre of a circle is twice the angle at the circumference of a circle	
	
	

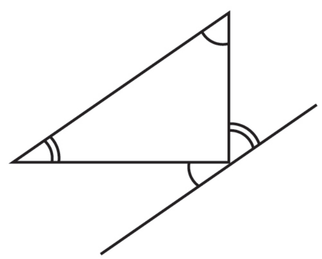
The angle in a semi-circle is always a right angle	Angles subtended by a chord (or arc) in the same segment are equal.
	
Opposite angles of a cyclic quadrilateral are supplementary	The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle
 $a + c = 180^\circ$ $b + d = 180^\circ$	

A tangent is perpendicular to the radius	The angle between a tangent and a chord is equal to the angle subtended by the chord in the opposite segment
	
Two tangents drawn from a common point to a circle are equal in length	
	

Tips to consider when you're stuck:

If you must prove:

- sides equal: look for the two angles that should be equal. If this doesn't seem possible, use congruency if the angles are in 2 different triangles.
- that a quad is a cyclic quad: look for
  - (i) ext < = to opp int <
  - (ii) opp <'s = 180
  - (iii) line subtends equal <'s
- 2 lines parallel: look for
  - (i) alt <'s equal
  - (ii) corres <'s equal
  - (iii) co-int <'s = 180º
- that a line is a tangent to an 'invisible' circle:
  - (i) look for the tan-chord theorem 'diagram' and prove the appropriate angles equal



  - (ii) 90º angle where radius meets the line

Be careful of a quad in a circle with the centre as one of the points. It is NOT a cyclic quad. You will probably use the angle at the centre is twice the angle at the circumference, but with the REFLEX angle.

If you are given:

• Parallel lines: you WILL use either (i) alt <'s (ii) corres <'s (iii)co-int <'s	• The centre of a circle: Look for: (i) < in semi-circle (ii) < at centre = 2x < at circ (iii)radius / chord (perp)
• A cyclic quad: Look for: (i) ext < = int opp < (ii) opp <'s = 180 (iii)<'s in same segment	• Tangent: Look for: (i) tan/chord (ii) tan $\perp$ rad (or diameter)
• 2 tangents from same point: Mark them equal and look for equal angles from isosceles triangle formed.	

MEASUREMENT

Volume

The space taken up by a 3D object. To find volume, the area of the base is multiplied by the perpendicular height. This only works for right prisms

VOLUME OF:	AREA OF BASE x HEIGHT
Cube	$(l \times l) \times ht = l \times l \times l = l^3$
Rectangular prism	$(l \times b) \times h = lbh$
Triangular prism	$(\frac{1}{2} b \times h) \times H$ Note: h = height of Δ H = height of prism
Cylinder	$\pi r^2 \times ht = \pi r^2 h$

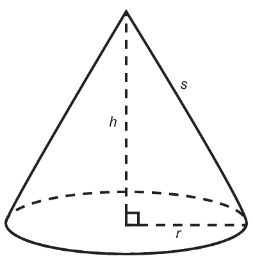
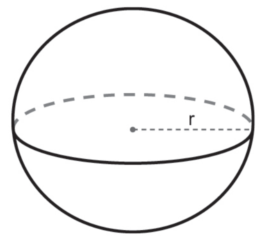
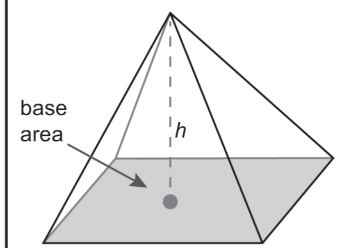
Surface Area

The area taken up by the net of a 3D solid. The sum of the area of all the faces. The following basic shape formulae are needed to find the area of the faces on any 3-dimensional object.

SHAPE	AREA FORMULA
Square	$l \times l = l^2$
Rectangle	$l \times b$
Triangle	$\frac{1}{2}b \times \perp$ height
Circle	πr^2

Cones, pyramids and spheres

(These formulae are given in an assessment)

3D object	Surface Area	Volume
Cone 	$\pi rs + \pi r^2$ (the slant height is sometimes named l)	$\frac{1}{3}\pi r^2 h$
Sphere 	$4\pi r^2$ (the slant height is sometimes named l)	$\frac{4}{3}\pi r^3$
Pyramid 	Sum of the areas of: ● the base and ● the triangles* * the number of triangles depends on the type of base	$\frac{1}{3}$(area of base) $\times h$ (remember that the base could be any polygon but generally the square, rectangle and triangle would be used)

The effect on volume when multiplying any dimension by a constant factor k:

- If only one dimension is changed by a value of k, the volume will be k times bigger
- If only two dimensions are changed by a value of k, the volume will be k^2 times bigger
- If all three dimensions are changed by a value of k, the volume will be k^3 times bigger

RESOURCE 10

REVISION: Paper 2 2017

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

NOVEMBER 2017

MARKS: 150

TIME: 3 hours

This question paper consists of 13 pages and an answer book of 24 pages.

IMAT2

EASTERN CAPE

Please turn over

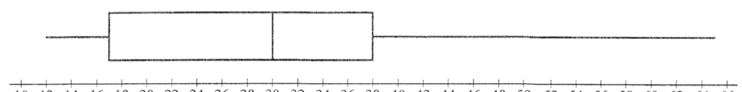
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. Round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. Write neatly and legibly.

QUESTION 1

1.1 Mr Brown conducted a survey on the amount of airtime (in rands) EACH student had on his or her cellphone. He summarised the data in the box and whisker diagram below.



1.1.1 Write down the five-number summary of the data. (2)

1.1.2 Determine the interquartile range. (1)

1.1.3 Comment on the skewness of the data. (1)

1.2 A group of 13 students indicated how long it took (in hours) before their cellphone batteries required recharging. The information is given in the table below.

5	8	10	17	20	29	32	48	50	50	63	y	107

1.2.1 Calculate the value of y if the mean for this data set is 41. (2)

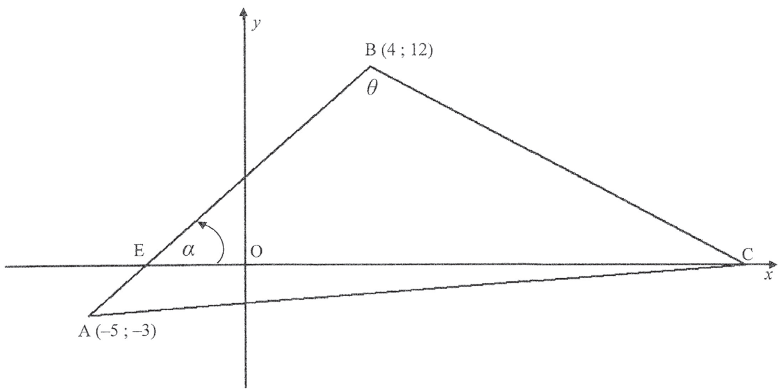
1.2.2 If $y = 94$, calculate the standard deviation of the data. (1)

1.2.3 The mean time before another group of 6 students needed to recharge the batteries of their cellphones was 18 hours. Combine these groups and calculate the overall mean time needed for these two groups to recharge the batteries of their cellphones. (3)

[10]

QUESTION 4

C, a point on the x-axis, $A(-5\,;-3)$ and $B(4\,;12)$ are the vertices of a triangle.
AB intersects the x-axis at E.
$A\hat{B}C=\theta$ and $B\hat{E}C=\alpha$.



4.1 Calculate the gradient of AB. (2)

4.2 Determine the coordinates of point E. (3)

4.3 Determine the size of α. Round off to the nearest whole number. (2)

4.4 If $\theta=76°$, determine the equation of the line through A parallel to BC. (5)
[12]

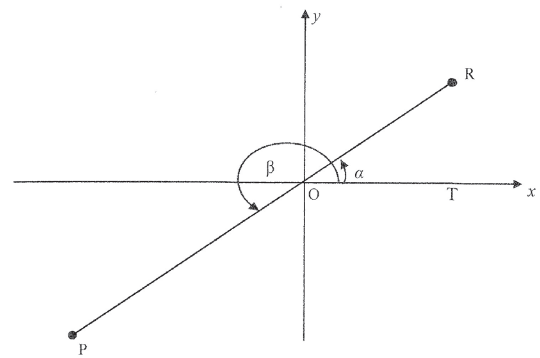
QUESTION 5

5.1 Simplify fully: $\sin(90°-x).\cos(180°+x)+\tan x.\cos x.\sin(x-180°)$ (6)

5.2 Prove, WITHOUT using a calculator, that

$$\frac{\sin 315°.\tan 210°.\sin 190°}{\cos 100°.\sin 120°}=\frac{-\sqrt{2}}{3}$$ (6)

5.3 In the diagram below, R is a point in the first quadrant such that $T\hat{O}R=\alpha$.
RO is extended to P such that $OP=2\,RO$ and $T\hat{O}P=\beta$.
It is given that $\sin\alpha=\frac{3}{5}$.



WITHOUT using a calculator, determine:

5.3.1 The value of $\tan\alpha$ (3)

5.3.2 The value of $\sin\beta$ (3)

5.3.3 The coordinates of P (4)

5.4 Prove the identity: $\frac{\sin\theta-\tan\theta.\cos^2\theta}{\cos\theta-1+\sin^2\theta}=\tan\theta$ (4)
[26]

QUESTION 6

6.1 Determine the general solution for $\sin(x-30°) = \cos 2x$ (5)

6.2 Consider the functions $f(x)=\sin(x-30°)$ and $g(x)=\cos 2x$

6.2.1 Write down the period of g. (1)

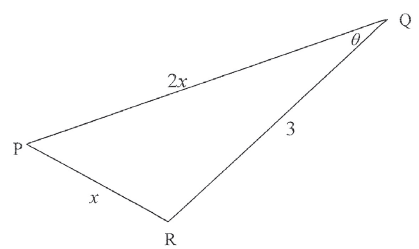
6.2.2 State the range of f. (2)

6.2.3 On the grid provided in the ANSWER BOOK, draw the graphs of f and g for $x\in[-90°;180°]$.
Clearly show ALL intercepts with the axes, turning points and end points. (5)

6.2.4 Write down the x-coordinates of the points of intersection of f and g in the interval $x\in[-90°;180°]$. (3)
[16]

QUESTION 7

In ΔPQR, QR = 3 units, PR = x units, PQ = $2x$ units and $P\hat{Q}R = \theta$.



7.1 Show that $\cos\theta = \dfrac{x^2+3}{4x}$ (3)

7.2 If $x = 2{,}4$ units:

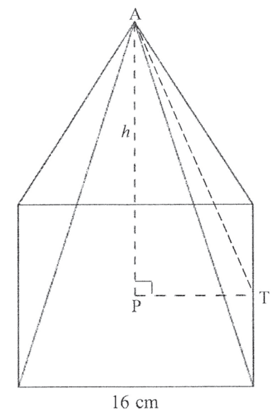
7.2.1 Calculate θ (3)

7.2.2 Calculate the area of ΔPQR (2)

7.3 Calculate the values of x for which the triangle exists. (4)
[12]

QUESTION 8

A pyramid with a square base with a side length of 16 cm is sketched below. P lies on the square base directly below A.
The volume of the pyramid is 640 cm^3.



Volume of pyramid $= \frac{1}{3}Ah$

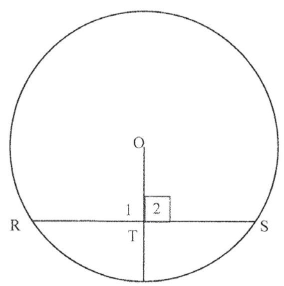
8.1 Show that the perpendicular height of the pyramid, AP, is 7,5 cm. (2)

8.2 Hence, determine the total surface area of the pyramid. (4)
[6]

Give reasons for your statements and calculations in QUESTIONS 9, 10, 11 and 12.

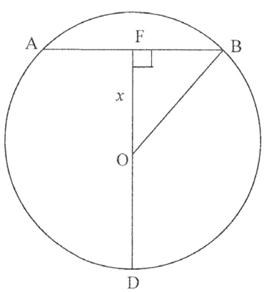
QUESTION 9

9.1 In the diagram below, O is the centre of the circle and point T lies on chord RS. Prove the theorem which states that if $OT \perp RTS$ then $RT = TS$.



(5)

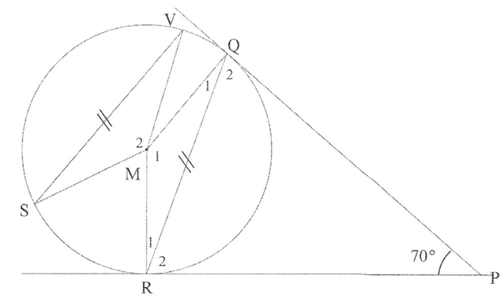
9.2 In the diagram, O is the centre of circle ABD. F is a point on chord AB such that $DOF \perp AB$. $AB = FD = 8$ cm and $OF = x$ cm.



Determine the length of the radius of the circle. (5)
[10]

QUESTION 10

M is the centre of the circle SVQR having equal chords SV and QR.
RP and QP are tangents to the circle at R and Q respectively such that $R\hat{P}Q = 70°$.



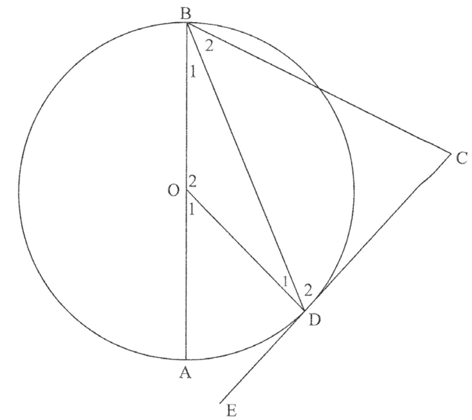
10.1 Calculate the size of $\hat{R}_2$. (4)

10.2 Calculate the size of $\hat{Q}_1$. (2)

10.3 Determine the size of $\hat{M}_2$. (3)
[9]

QUESTION 11

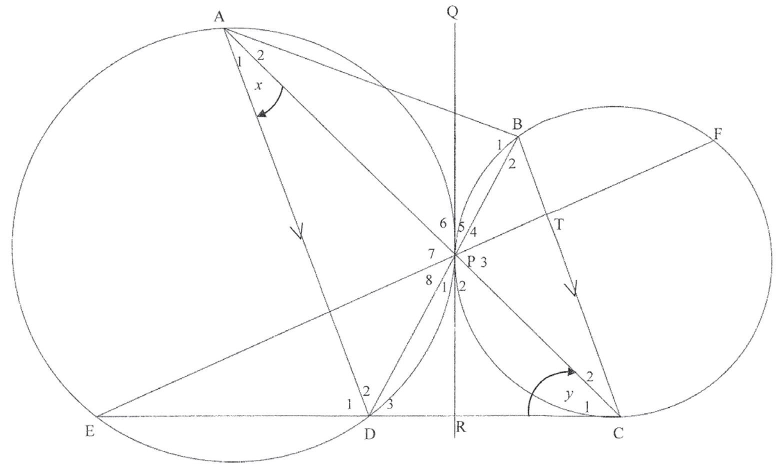
In the diagram below, O is the centre of the circle. CDE is a tangent to the circle at D.
DB bisects $A\hat{B}C$. Let $\hat{B}_1 = x$



11.1	Prove that BC || OD	(4)
11.2	Show that $\hat{C} = 90°$	(3)
		[7]

QUESTION 12

In the diagram below, two circles touch each other externally at point P.
QPR is a common tangent to both circles at P. EDRC is a tangent to circle PBFC at C.
$R\hat{C}A = y$ and $D\hat{A}C = x$. AD || BC.



12.1	Name, with reasons, FOUR other angles equal to x.	(7)
12.2	Show that $E\hat{P}A = x + y$	(4)
12.3	Determine the numerical value of $x + y$, if it is given that DCTP is a cyclic quadrilateral.	(4)
		[15]
	TOTAL:	**150**

RESOURCE 11

REVISION: Paper 1 Exemplar

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 11

MATHEMATICS P1

EXEMPLAR 2013

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages.

Please turn over

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $(2x-1)(x+5)=0$ (2)

1.1.2 $2x^2-4x+1=0$ (Leave your answer in simplest surd form.) (3)

1.2 Simplify, without the use of a calculator, the following expressions fully:

1.2.1 $125^{\frac{2}{3}}$ (2)

1.2.2 $(3\sqrt{2}-12)(2\sqrt{2}+1)$ (3)

1.3 Given: $\dfrac{x^2-x-6}{3x-9}$

1.3.1 For which value(s) of x will the expression be undefined? (2)

1.3.2 Simplify the expression fully. (3)
[15]

QUESTION 2

2.1 Given: $(x+2)(x-3)<-3x+2$

2.1.1 Solve for x if: $(x+2)(x-3)<-3x+2$ (4)

2.1.2 Hence or otherwise, determine the sum of all the integers satisfying the expression $x^2+2x-8<0$. (3)

2.2 Given: $\dfrac{4^{x-1}+4^{x+1}}{17\,.\,12^x}$

2.2.1 Simplify the expression fully. (4)

2.2.2 If $3^{-x}=4t$, express $\dfrac{4^{x-1}+4^{x+1}}{17\,.\,12^x}$ in terms of t. (1)

2.3 Solve for x and y from the given equations:

$3^y=81^x$ and $y=x^2-6x+9$ (7)
[19]

QUESTION 3

3.1 The solution to a quadratic equation is $x = \dfrac{3 \pm \sqrt{4-8p}}{4}$ where $p \in \mathbf{Q}$.

Determine the value(s) of p such that:

3.1.1 The roots of the equation are equal (2)

3.1.2 The roots of the equation are non-real (2)

3.2 Given: $\sqrt{5-x} = x+1$

3.2.1 Without solving the equation, show that the solution to the above equation lies in the interval $-1 \leq x \leq 5$. (3)

3.2.2 Solve the equation. (5)

3.2.3 Without any further calculations, solve the equation $-\sqrt{5-x} = x+1$. (1)
[13]

QUESTION 4

4.1 Melissa has just bought her first car. She paid R145 000 for it. The car's value depreciates on the straight-line method at a rate of 17% per annum. Calculate the value of Melissa's car 5 years after she bought it. (2)

4.2 An investment earns interest at a rate of 8% per annum compounded quarterly.

4.2.1 At what rate is interest earned each quarter of the year? (1)

4.2.2 Calculate the effective annual interest rate on this investment. (2)

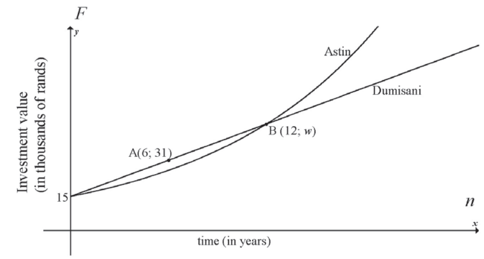
4.3 R14 000 is invested in an account.

The account earns interest at a rate of 9% per annum compounded semi-annually for the first 18 months and thereafter 7,5% per annum compounded monthly.

How much money will be in the account exactly 5 years after the initial deposit? (5)
[10]

QUESTION 5

The graphs below represent the growth of two investments, one belonging to Dumisani and one belonging to Astin. Both investments earn interest annually (only).



5.1 What is the value of both initial investments? (1)

5.2 Does Dumisani's investment earn simple or compound interest? (1)

5.3 Determine Dumisani's interest rate. (2)

5.4 Hence or otherwise, calculate the interest rate on Astin's investment. Give your answer correct to ONE decimal place. (4)
[8]

QUESTION 6

6.1 Given: $\frac{1}{2}; \frac{1}{4}; \frac{1}{8}; \dots ; \frac{1}{1\,024}$

6.1.1 Explain how you will determine the 4^{th} term of the sequence. (2)

6.1.2 Write a formula for the n^{th} term of the sequence. (2)

6.1.3 Determine the number of terms in the sequence. (2)

6.2 Given the linear pattern: 156 ; 148 ; 140 ; 132 ; …

6.2.1 Write down the 5^{th} term of this number pattern. (1)

6.2.2 Determine a general formula for the n^{th} term of this pattern. (2)

6.2.3 Which term of this linear number pattern is the first term to be negative? (3)

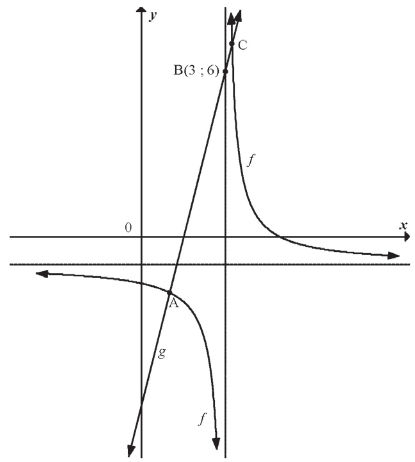
6.2.4 The given linear number pattern forms the sequence of first differences of a quadratic number pattern $T_n = an^2 + bn + c$ with $T_5 = -24$. Determine a general formula for T_n. (5)
[17]

QUESTION 7

A given quadratic pattern $T_n = an^2 + bn + c$ has $T_2 = T_4 = 0$ and a second difference of 12. Determine the value of the 3rd term of the pattern. **[6]**

QUESTION 8

The sketch below represents the graphs of $f(x) = \frac{2}{x-3} - 1$ and $g(x) = dx + e$.
Point B (3 ; 6) lies on the graph of g and the two graphs intersect at points A and C.



8.1 Write down the equations of the asymptotes of f. (2)

8.2 Write down the domain of f. (2)

8.3 Determine the values of d and e, correct to the nearest integer, if the graph of g makes an angle of 76° with the x-axis. (3)

8.4 Determine the coordinates of A and C. (6)

8.5 For what values of x is $g(x) \geq f(x)$? (3)

8.6 Determine an equation for the axis of symmetry of f which has a positive slope. (3)
[19]

QUESTION 9

Given: $f(x) = -x^2 + 2x + 3$ and $g(x) = 1 - 2^x$

9.1 Sketch the graphs of f and g on the same set of axes. (9)

9.2 Determine the average gradient of f between $x = -3$ and $x = 0$. (3)

9.3 For which value(s) of x is $f(x).g(x) \geq 0$? (3)

9.4 Determine the value of c such that the x-axis will be a tangent to the graph of h, where $h(x) = f(x) + c$. (2)

9.5 Determine the y-intercept of t if $t(x) = -g(x) + 1$ (2)

9.6 The graph of k is a reflection of g about the y-axis. Write down the equation of k. (1)
[20]

QUESTION 10

Sketch the graph of $f(x) = ax^2 + bx + c$ if it is also given that:

- The range of f is $(-\infty;7]$
- $a \neq 0$
- $b < 0$
- One root of f is positive and the other root of f is negative. **[4]**

QUESTION 11

Given: $P(W) = 0,4$
$P(T) = 0,35$
$P(T \text{ and } W) = 0,14$

11.1 Are the events W and T mutually exclusive? Give reasons for your answer. (2)

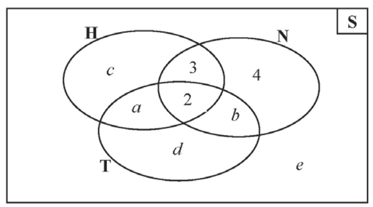
11.2 Are the events W and T independent? Give reasons for your answer. (3)
[5]

QUESTION 12

12.1 A group of 33 learners was surveyed at a school. The following information from the survey is given:

- 2 learners play tennis, hockey and netball
- 5 learners play hockey and netball
- 7 learners play hockey and tennis
- 6 learners play tennis and netball
- A total of 18 learners play hockey
- A total of 12 learners play tennis
- 4 learners play netball ONLY

12.1.1 A Venn diagram representing the survey results is given below. Use the information provided to determine the values of a, b, c, d and e.



(5)

12.1.2 How many of these learners do not play any of the sports on the survey (that is netball, tennis or hockey)? (1)

12.1.3 Write down the probability that a learner selected at random from this sample plays netball ONLY. (1)

12.1.4 Determine the probability that a learner selected at random from this sample plays hockey or netball. (1)

12.2 In all South African schools, EVERY learner must choose to do either Mathematics or Mathematical Literacy.

At a certain South African school, it is known that 60% of the learners are girls. The probability that a randomly chosen girl at the school does Mathematical Literacy is 55%. The probability that a randomly chosen boy at the school does Mathematical Literacy is 65%.

Determine the probability that a learner selected at random from this school does Mathematics. (6)

[14]

TOTAL: 150

RESOURCE 12

REVISION: Memorandum Paper 1 Exemplar

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P1

EXEMPLAR 2013

MEMORANDUM

MARKS: 150

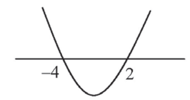
This memorandum consists of 13 pages.

Please turn over

QUESTION 1

1.1.1	$(2x-1)(x+5)=0$ $x=\frac{1}{2}$ **OR** $x=-5$	✓✓ answers (2)
1.1.2	$2x^2-4x+1=0$ $x=\frac{-(-4)\pm\sqrt{(-4)^2-4(2)(1)}}{2(2)}$ $x=\frac{4\pm\sqrt{8}}{4}$ $x=\frac{4\pm\sqrt{4\,.\,2}}{4}$ $x=\frac{4\pm2\sqrt{2}}{4}$ $x=\frac{2(2\pm\sqrt{2})}{4}$ $x=\frac{2\pm\sqrt{2}}{2}$	✓ substitution into correct formula ✓✓ answers (3)
1.2.1	$125^{\frac{2}{3}}$ $=(5^3)^{\frac{2}{3}}$ $=5^2$ $=25$	✓ 5^3 ✓answer (accept 25 or 5^2) (2)
1.2.2	$(3\sqrt{2}-12)(2\sqrt{2}+1)$ $=6.\,2+3\sqrt{2}-24\sqrt{2}-12$ $=-21\sqrt{2}$	✓ $6.2+3\sqrt{2}$ ✓ $-24\sqrt{2}-12$ ✓answer (3)
1.3.1	$3x-9=0$ $3x=9$ $x=3$	✓ $3x-9=0$ ✓ answer (2)
1.3.2	$\frac{x^2-x-6}{3x-9}=\frac{(x-3)(x-2)}{3(x-3)}$ $=\frac{x-2}{3}$	✓ $(x-3)(x-2)$ ✓ $3(x-3)$ ✓answer (3) **[15]**

QUESTION 2

2.1.1	$(x+2)(x-3)<-3x+2$ $x^2-x-6+3x-2<0$ $x^2+2x-8<0$ $(x+4)(x-2)<0$ + 0 − 0 + (−4, 2) or  $-4<x<2$	✓ standard form ✓ factors ✓ $-4<x$ ✓ $x<2$ (4)
2.1.2	$x^2+2x-8<0$ $-4<x<2$ Sum of integers $=(-3)+(-2)+(-1)+(0)+(1)$ $=-5$	✓ $-4<x<2$ ✓ $-3,-2,-1,0,1$ ✓answer (3)
2.2.1	$\frac{4^{x-1}+4^{x+1}}{17\,.\,12^x}=\frac{4^x\,.\,4^{-1}+4^x.4^1}{17\,.\,3^x\,.\,4^x}$ $=\frac{4^x(4^{-1}+4)}{17\,.\,3^x.4^x}$ $=\frac{4^x\left(\frac{1}{4}+4\right)}{17\,.\,3^x.4^x}$ $=\frac{\left(\frac{17}{4}\right)}{17.3^x}$ $=\frac{1}{4}.3^{-x}$ or $\frac{1}{4.3^x}$	✓ factorise numerator ✓ $3^x\,.\,4^x$ ✓ simplification of numerator to $\frac{17}{4}$ ✓ answer (4)
2.2.2	$\frac{4^{x-1}+4^{x+1}}{17\,.\,12^x}=\frac{1}{4}.3^{-x}$ $=\frac{1}{4}.4t$ $=t$	✓answer (1)

2.3	$3^y = 81^x$ and $y = x^2 - 6x + 9$ $3^y = 3^{4x}$ $y = 4x$ $4x = x^2 - 6x + 9$ $0 = x^2 - 10x + 9$ $0 = (x-9)(x-1)$ $x = 9$ or 1 $y = 4(9)$ or $4(1)$ $= 36$ or 4 $(x; y) = (9\ ;\ 36)$ or $(1\ ;\ 4)$	✓ $3^y = 3^{4x}$ ✓ $y = 4x$ ✓ $4x = x^2 - 6x + 9$ ✓standard form ✓factors ✓x-values ✓y-values (7) **[19]**

QUESTION 3

3.1.1	$4 - 8p = 0$ $-8p = -4$ $p = \frac{1}{2}$	✓ $4 - 8p = 0$ ✓ answer (2)
3.1.2	$4 - 8p < 0$ $p > \frac{1}{2}$	✓ $4 - 8p < 0$ ✓ answer (2)
3.2.1	$\sqrt{5-x} = x + 1$ $5 - x \geq 0$ and $x + 1 \geq 0$ $x \leq 5$ and $x \geq -1$ Hence $-1 \leq x \leq 5$	✓ $5 - x \geq 0$ ✓ $x + 1 \geq 0$ ✓and (3)
3.2.2	$5 - x = x^2 + 2x + 1$ $x^2 + 3x - 4 = 0$ $(x+4)(x-1) = 0$ $x = -4$ or $x = 1$ Since $-1 \leq x \leq 5$, $x = 1$ only	✓ square both sides ✓ standard form ✓ factors ✓ answers ✓ selection of 1 (5)
3.2.3	$x = -4$	✓ answer (1) **[13]**

QUESTION 4

4.1	$A = P(1 - in)$ $= 145000[1 - (0{,}17)(5)]$ $= \text{R}\,21\ 750$	✓ substitution ✓ answer (2)
4.2.1	$\frac{8\%}{4} = 2\%$ per quarter	✓ answer (1)
4.2.2	$A = P(1+i)^n$ After 1 year, $A = P(1 + i_{eff})^1$ and $A = P(1 + 0.02)^4$ Hence $1 + i_{eff} = (1 + 0.02)^4$ $i_{eff} = (1 + 0.02)^4 - 1$ $= 0{,}0824$ The effective interest rate is 8,24% p.a.	✓ $1 + i_{eff} = (1 + 0{,}02)^4$ ✓ answer (2)
4.3	$A = 14\,000\left(1 + \frac{0{,}09}{2}\right)^3\left(1 + \frac{0{,}075}{12}\right)^{42}$ $= \text{R}\,20\ 755{,}08$	✓ $\frac{0{,}09}{2}$ ✓ $14\,000\left(1 + \frac{0{,}09}{2}\right)^3$ ✓ $\frac{0{,}07}{12}$ ✓42 ✓answer (5) **[10]**

QUESTION 5

5.1	R 15 000	✓ answer (1)
5.2	Simple interest	✓ answer (1)
5.3	$A = P(1+in)$ $31 = 15(1+6i)$ $\frac{31}{15} = 1+6i$ $i = \left(\frac{31}{15}-1\right) \div 6$ $= \frac{8}{45}$ $= 0{,}1778$ $= 17{,}78\%$	✓ substitution of (6 ; 31) into correct formula ✓answer (2)
5.4	$A = P(1+in)$ $w = 15(1+0{,}1778 \times 12)$ $= 47$ $A = P(1+i)^n$ $47 = 15(1+i)^{12}$ $\sqrt[12]{\frac{47}{15}} = 1+i$ $i = \sqrt[12]{\frac{47}{15}} - 1 = 0{,}09985 = 9{,}99\%$	✓$w = 47$ ✓ substitutes (12 ; w) ✓ $\sqrt[12]{\frac{47}{15}}$ ✓answer (4) **[8]**

QUESTION 6

6.1.1	Multiply $\frac{1}{8}$ by $\frac{1}{2}$	✓ multiply $\frac{1}{8}$ ✓ $\frac{1}{2}$ (2)
6.1.2	$T_n = \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$ **OR** $T_n = \left(\frac{1}{2}\right)^n$ **OR** $T_n = 2^{-n}$	✓ $a = \frac{1}{2}$ ✓ $\left(\frac{1}{2}\right)^{n-1}$ (2) ✓✓ answer (2) ✓✓ answer (2)
6.1.3	Continuing the pattern: $\frac{1}{2};\frac{1}{4};\frac{1}{8};\frac{1}{16};\frac{1}{32};\frac{1}{64};\frac{1}{128};\frac{1}{256};\frac{1}{512};\frac{1}{1024}$ Hence $n = 10$ **OR** $\frac{1}{2^n} = \frac{1}{1024}$ $2^{-n} = 2^{-10}$ $n = 10$	✓ expand sequence ✓$n = 10$ (2) ✓ $\frac{1}{2^n} = \frac{1}{1024}$ ✓$n = 10$ (2)
6.2.1	124	✓answer (1)
6.2.2	$T_n = -8n + 164$	✓ $-8n$ ✓$+164$ (2)
6.2.3	$-8n + 164 < 0$ $164 < 8n$ $20{,}5 < n$ Hence T_{21} is the first term to be negative.	✓ $-8n+164<0$ ✓ $20{,}5 < n$ ✓answer (3)

6.2.4	$2a = -8$ $a = -4$ $3a + b = 156$ $3(-4) + b = 156$ $b = 168$ $T_5 = -24$ $-4(5)^2 + 168(5) + c = -24$ $c = -764$ $T_n = -4n^2 + 168n - 764$	✓ $a = -4$ ✓ substitutions ✓ $b = 168$ ✓ substitution ✓ $c = -764$ (5)
	OR $T_5 = -24$ (given) $T_6 = -24 + 124$ $= 100$ $T_n = -4n^2 + bn + c$ $-24 = -4(5)^2 + b(5) + c$ $76 = 5b + c$ ... (1) $100 = -4(6)^2 + b(6) + c$ $244 = 6b + c$ ... (2) $168 = b$ ... (2) – (1) $c = -764$	✓ $T_6 = 100$ ✓ $a = -4$ ✓ substitutions ✓ $b = 168$ ✓ $c = -764$ (5)
	OR $T_5 = -24$ (given) $T_4 = -24 - 132$ $= -156$ $T_n = -4n^2 + bn + c$ $-24 = -4(5)^2 + b(5) + c$ $76 = 5b + c$ ... (1) $-156 = -4(4)^2 + b(4) + c$ $-92 = 4b + c$ ... (2) $168 = b$ ... (1) – (2) $c = -764$	✓ $T_4 = -156$ ✓ $a = -4$ ✓ substitutions ✓ $b = 168$ ✓ $c = -764$ (5) **[17]**

QUESTION 7

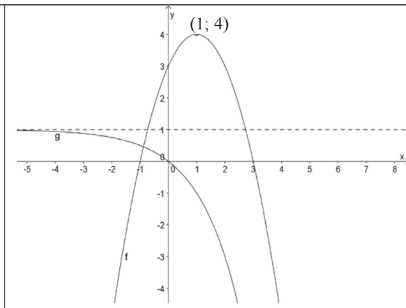
T_1 0 x T_3 y 0 12 12	✓introduce variables
$y = x + 12$ $T_3 = 0 + x = x$ **AND** $T_3 + y = 0$ $y = -x$	✓ $T_3 = x$ ✓ $y = -x$
AND $y = x + 12$	✓ $y = x + 12$
Hence $-x = x + 12$ $-2x = 12$ $x = -6$	✓ $-x = x + 12$ ✓answer **[6]**
OR **2a = 12** **a = 6** $T_n = 6n^2 + bn + c$	✓ $a = 6$
$n = 2$ and $n = 4$: $24 + 2b + c = 0$ $96 + 4b + c = 0$ $72 + 2b = 0$ $2b = -72$ $b = -36$	✓ $24 + 2b + c = 0$ ✓ $96 + 4b + c = 0$ ✓ $b = -36$
$24 - 72 + c = 0$ $c = 48$	✓ $c = 48$
$T_n = 6n^2 - 36n + 48$ $T_3 = 6(3)^2 - 36(3) + 48$ $= 102 - 108$ $= -6$	✓answer **[6]**

QUESTION 8

8.1	$x = 3$ $y = -1$	✓ answer ✓ answer (2)
8.2	$R\,;\, x \neq 3$ **OR** $(-\infty\,;3)\cup(3\,;\infty)$	✓ R ✓ $x \neq 3$ (2) ✓ $(-\infty\,;3)$ ✓ $(3\,;\infty)$ (2)
8.3	$d = \tan 76°$ $d = 4$ $6 = 4(3)+e$ $e = -6$ $g(x) = 4x-6$	✓ $d = \tan 76°$ ✓ $d = 4$ ✓ $e = -6$ (3)
8.4	$\frac{2}{x-3} - 1 = 4x-6$ $\frac{2}{x-3} = 4x-5$ $2 = 4x(x-3)-5(x-3)$ $2 = 4x^2-12x-5x+15$ $0 = 4x^2-17x+13$ $0 = (4x-13)(x-1)$ $x = \frac{13}{4}$ or $x = 1$ $y = 4\left(\frac{13}{4}\right)-6$ or $y = 4(1)-6$ $y = 7$ or $y = -2$ Points of intersection are A $(1\,;-2)$ and C $\left(\frac{13}{4}\,;7\right)$	✓ equating ✓ simplification ✓ standard form ✓ factors ✓ x-values ✓ y-values (6)
8.5	$1 \le x < 3$ or $x \ge \frac{13}{4}$ **OR** $x \in [1\,;3)\cup\left[\frac{13}{4}\,;\infty\right)$	✓ $1 \le x$ ✓ $x < 3$ ✓ $x \ge \frac{13}{4}$ (3)
8.6	$y = (x-3)-1$ $y = x-4$ **OR** $y = x+c$ Substitute $(3;\,-1)$ $-1 = 3+c$ $c = -4$ $y = x-4$	✓ $x-3$ ✓ -1 ✓ answer (3) ✓✓ substitute $(3\,;-1)$ ✓ answer (3) **[19]**

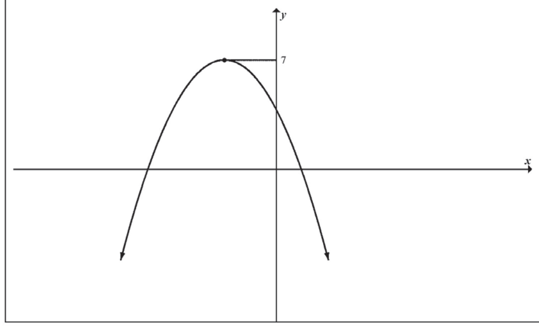
 Please turn over

QUESTION 9

9.1	 $x^2-2x-3=0$ $(x-3)(x+1)=0$ $x=3$ *or* $x=-1$ $x = \frac{-2}{2(-1)} = 1$ $y = -(1)^2+2(1)+3 = 4$ $0 = 1-2^x$ $2^x = 2^0$ $x = 0$ $y = 1-2^0$ $y = 0$	*f*: ✓ shape ✓ x-int ✓ y-int ✓✓ turning point *g*: ✓ shape ✓ x-int ✓ y-int ✓ asymptote (9)
9.2	Average gradient $= \frac{f(0)-f(-3)}{0-(-3)}$ $= \frac{3-(-12)}{3}$ $= 5$	✓ correct formula ✓ $f(-3) = -12$ ✓ answer (3)
9.3	$-1 \le x \le 0$ or $x \ge 3$	✓ $-1 < x$ ✓ $x < 0$ ✓ $x > 3$ (3)
9.4	Given: $f(x)+c=0$ has one solution/equal roots i.e. $f(x) = -c$ has one solution $\Rightarrow -c = f(1) = 4$ $\Rightarrow c = -4$ **OR** h is f translated 4 units down y-intercept of h will then be at -1 $\therefore 3+c=-1$ $c=-4$	✓ $-c = f(1)$ ✓ answer (2) ✓ $3+c=-1$ ✓ answer (2)
9.5	$(0\,;1)$	✓✓ $(0\,;1)$ (2)
9.6	$k(x) = 1-2^{-x}$	✓ answer (1) **[20]**

 Please turn over

QUESTION 10

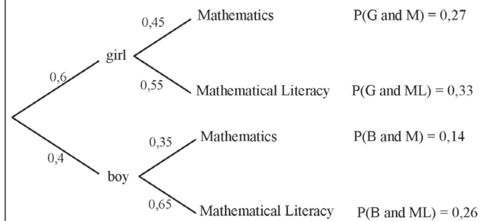
Range of $f\ (-\infty;7]$ $\Rightarrow y$-part of turning point [Max value of $f(x)$] is 7 $a<0$ and shape ∩ $b<0$ $\Rightarrow$ b negative $\Rightarrow$ axis of symmetry on left of y-axis roots real, unequal & opposite signs $\Rightarrow$ x-ints on opposite sides of y-axis 	✓ shape ✓turning point at $y = 7$ ✓ axis of symmetry on left of y-axis ✓ roots are on opposite sides **[4]**

QUESTION 11

11.1	No, W and T are **not** mutually exclusive Because P(W and T) $\neq 0$ **OR** No, W and T are **not** mutually exclusive Because P(W or T) = $0{,}61 \neq 0{,}75$ = P(W) + P(T)	✓not mutually exclusive ✓ P(W and T) $\neq 0$ (2) ✓not mutually exclusive ✓ P(W or T) $\neq$ P(W) + P(T) (2)
11.2	P(W and T)= 0,14 (given) and P(W)×P(T)= 0,4 × 0,35 =0,14 $\Rightarrow$P(W and T) = P(W)×P(T) Therefore yes, W and T are independent events	✓ P(W)×P(T) = 0,14 ✓ P(W and T) = P(W)×P(T) ✓ conclusion (yes) (3) **[5]**

 Please turn over

QUESTION 12

12.1.1	$a = 5$ $b = 4$ $c = 8$ $d = 1$ $e = 6$	✓ $a = 5$ ✓ $b = 4$ ✓ $c = 8$ ✓ $d = 1$ ✓ $e = 6$ (5)
12.1.2	6	✓answer (1)
12.1.3	$\frac{4}{33}$	✓ answer (1)
12.1.4	$\frac{4+3+2+a+b+c}{33} = \frac{26}{33}$	✓ answer (1)
12.2	 P(Mathematics) = P(G and M) + P(B and M) $= (0{,}6)(0{,}45) + (0{,}4)(0{,}35)$ $= 0{,}27 + 0{,}14$ $= 0{,}41$	✓0,4 ✓0,45 ✓0,35 ✓P(G and M) = 0,27 ✓P(B and M) = 0,14 ✓answer (6) **[14]**

TOTAL: 150

RESOURCE 13

REVISION: Paper 2 Exemplar 2013

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 11

MATHEMATICS P2

EXEMPLAR 2013

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages and 3 diagram sheets.

Please turn over

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. THREE diagram sheets for QUESTION 2.1, QUESTION 2.2, QUESTION 9.2, QUESTION 10.1, QUESTION 10.2 and QUESTION 11.2 are attached at the end of this question paper. Write your name on these diagram sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

The data below shows the number of people visiting a local clinic per day to be vaccinated against measles.

5	12	19	29
35	23	15	33
37	21	26	18
23	18	13	21
18	22	20	

1.1 Determine the mean of the given data. (2)

1.2 Calculate the standard deviation of the data. (2)

1.3 Determine the number of people vaccinated against measles that lies within ONE standard deviation of the mean. (2)

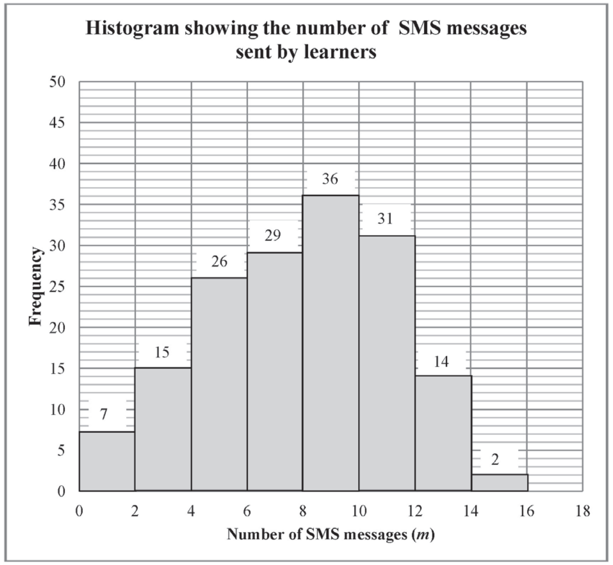
1.4 Determine the interquartile range for the data. (3)

1.5 Draw a box and whisker diagram to represent the data. (3)

1.6 Identify any outliers in the data set. Substantiate your answer. (2)
[14]

QUESTION 2

A group of Grade 11 learners were interviewed about using a certain application to send SMS messages. The number of SMS messages, m, sent by each learner was summarised in the histogram below.



2.1 Complete the cumulative frequency table provided in DIAGRAM SHEET 1. (2)

2.2 Use the grid provided in DIAGRAM SHEET 2 to draw an ogive (cumulative frequency curve) to represent the data. (3)

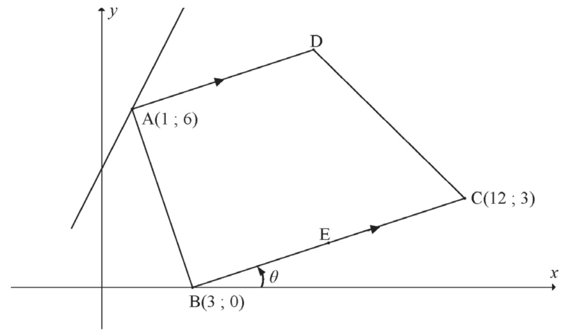
2.3 Use the ogive to identify the median for the data. (1)

2.4 Estimate the percentage of the learners who sent more than 11 messages using this application. (2)

2.5 In which direction is the data skewed? (1)
[9]

QUESTION 3

A(1 ; 6), B(3 ; 0), C(12 ; 3) and D are the vertices of a trapezium with AD || BC. E is the midpoint of BC. The angle of inclination of the straight line BC is θ, as shown in the diagram.



3.1 Calculate the coordinates of E. (2)

3.2 Determine the gradient of the line BC. (2)

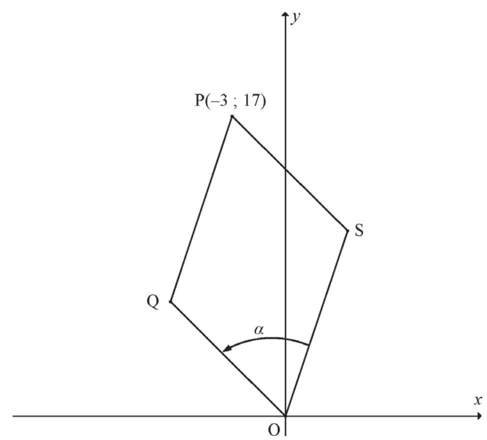
3.3 Calculate the magnitude of θ. (2)

3.4 Prove that AD is perpendicular to AB. (3)

3.5 A straight line passing through vertex A does not pass through any of the sides of the trapezium. This line makes an angle of 45° with side AD of the trapezium. Determine the equation of this straight line. (5)
[14]

QUESTION 4

In the diagram below, P(–3 ; 17), Q, O and S are the vertices of a parallelogram. The sides OS and OQ are defined by the equations $y = 6x$ and $y = -x$ respectively. $Q\hat{O}S = \alpha$.



4.1 Determine the equation of QP in the form $y = mx + c$. (3)

4.2 Hence, determine the coordinates of Q. (4)

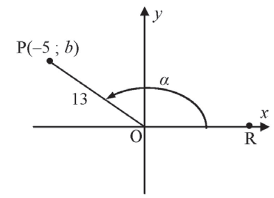
4.3 Calculate the length of OQ. Leave your answer in simplified surd form. (2)

4.4 Calculate the size of α. (3)

4.5 If $OS = \sqrt{148}$ units, calculate the length of QS. (3)
[15]

QUESTION 5

5.1 In the figure below, the point P(–5 ; b) is plotted on the Cartesian plane. OP = 13 units and $R\hat{O}P = \alpha$.



Without using a calculator, determine the value of the following:

5.1.1 $\cos\alpha$ (1)

5.1.2 $\tan(180° - \alpha)$ (3)

5.2 Consider: $\dfrac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)}$

5.2.1 Simplify $\dfrac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)}$ to a single trigonometric ratio. (5)

5.2.2 Hence, or otherwise, **without using a calculator,** solve for θ if $0° \le \theta \le 360°$:

$\dfrac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)} = 0{,}5$ (3)

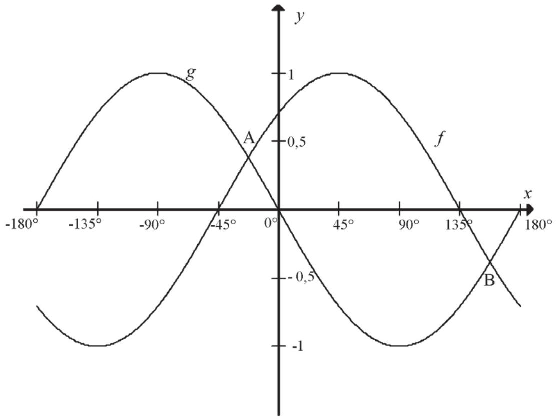
5.3 5.3.1 Prove that $\dfrac{8}{\sin^2 A} - \dfrac{4}{1 + \cos A} = \dfrac{4}{1 - \cos A}$. (5)

5.3.2 For which value(s) of A in the interval $0° \le A \le 360°$ is the identity in QUESTION 5.3.1 undefined? (3)

5.4 Determine the general solution of $8\cos^2 x - 2\cos x - 1 = 0$. (6)
[26]

QUESTION 6

In the diagram below, the graphs of $f(x) = \cos(x + p)$ and $g(x) = q \sin x$ are shown for the interval $-180° \leq x \leq 180°$.



6.1 Determine the values of p and q. (2)

6.2 The graphs intersect at A(–22,5° ; 0,38) and B. Determine the coordinates of B. (2)

6.3 Determine the value(s) of x in the interval $-180° \leq x \leq 180°$ for which $f(x) - g(x) < 0$. (2)

6.4 The graph f is shifted 30° to the left to obtain a new graph h.

6.4.1 Write down the equation of h in its simplest form. (2)

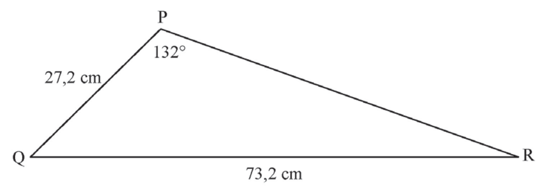
6.4.2 Write down the value of x for which h has a minimum in the interval $-180° \leq x \leq 180°$. (1)

[9]

QUESTION 7

7.1 Prove that in any acute-angled ΔABC, $\frac{\sin A}{a} = \frac{\sin C}{c}$. (5)

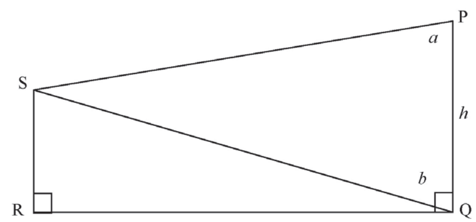
7.2 In ΔPQR, $\hat{P} = 132°$, PQ = 27,2 cm and QR = 73,2 cm.



7.2.1 Calculate the size of $\hat{R}$. (3)

7.2.2 Calculate the area of ΔPQR. (3)

7.3 In the figure below, $S\hat{P}Q = a$, $P\hat{Q}S = b$ and PQ = h. PQ and SR are perpendicular to RQ.



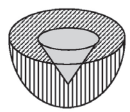
7.3.1 Determine the distance SQ in terms of a, b and h. (3)

7.3.2 Hence show that $RS = \frac{h \sin a \cos b}{\sin(a+b)}$. (3)

[17]

QUESTION 8

A solid metallic hemisphere has a radius of 3 cm. It is made of metal A. To reduce its weight a conical hole is drilled into the hemisphere (as shown in the diagram) and it is completely filled with a lighter metal B. The conical hole has a radius of 1,5 cm and a depth of $\frac{8}{9}$ cm.



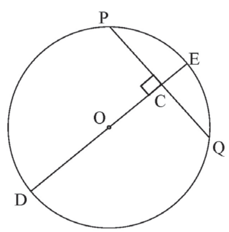
Calculate the ratio of the volume of metal A to the volume of metal B. **[6]**

QUESTION 9

9.1 Complete the statement so that it is valid:

The line drawn from the centre of the circle perpendicular to the chord … (1)

9.2 In the diagram, O is the centre of the circle. The diameter DE is perpendicular to the chord PQ at C. DE = 20 cm and CE = 2 cm.



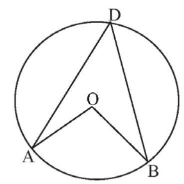
Calculate the length of the following with reasons:

9.2.1 OC (2)

9.2.2 PQ (4)
[7]

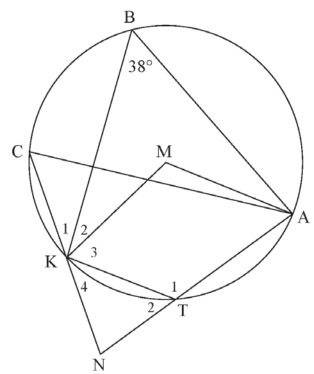
QUESTION 10

10.1 In the diagram, O is the centre of the circle and A, B and D are points on the circle. Use Euclidean geometry methods to prove the theorem which states that $A\hat{O}B = 2A\hat{D}B$.



(5)

10.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle. AT produced and CK produced meet in N. Also NA = NC and $\hat{B} = 38°$.



10.2.1 Calculate, with reasons, the size of the following angles:

(a) $K\hat{M}A$ (2)

(b) $\hat{T}_2$ (2)

(c) $\hat{C}$ (2)

(d) $\hat{K}_4$ (2)

10.2.2 Show that NK = NT. (2)

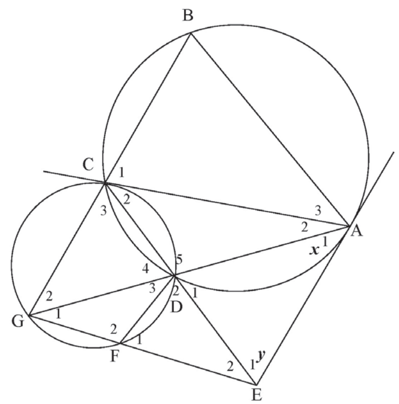
10.2.3 Prove that AMKN is a cyclic quadrilateral. (3)
[18]

QUESTION 11

11.1 Complete the following statement so that it is valid:

The angle between a chord and a tangent at the point of contact is … (1)

11.2 In the diagram, EA is a tangent to circle ABCD at A.
AC is a tangent to circle CDFG at C.
CE and AG intersect in D.



If $\hat{A}_1 = x$ and $\hat{E}_1 = y$, prove the following with reasons:

11.2.1 BCG || AE (5)

11.2.2 AE is a tangent to circle FED (5)

11.2.3 AB = AC (4)
[15]

TOTAL: 150

NAME OF LEARNER: ____________________

DIAGRAM SHEET 1

QUESTION 2.1

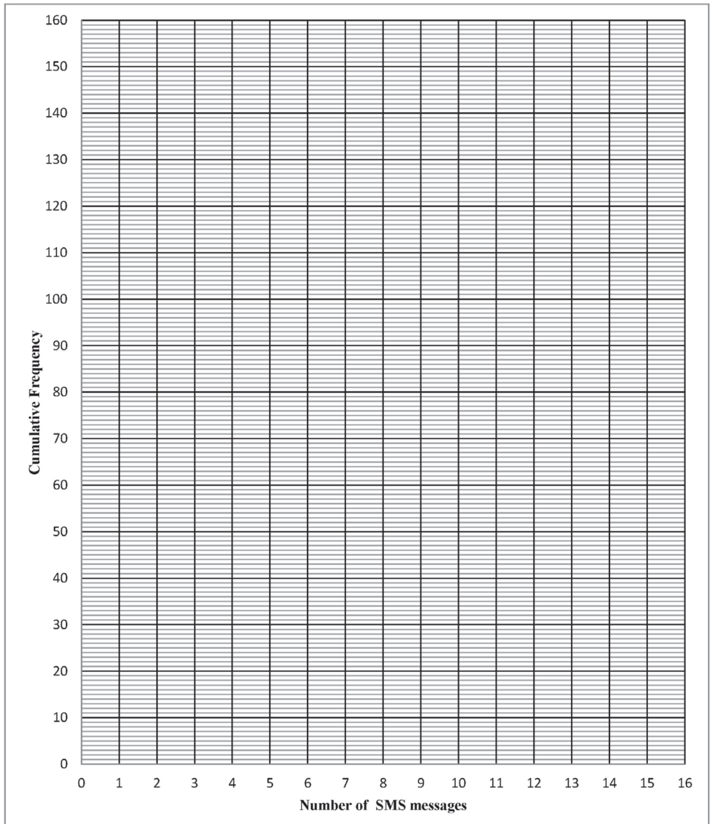
CLASS	FREQUENCY	CUMULATIVE FREQUENCY
$0 \le m < 2$		
$2 \le m < 4$		
$4 \le m < 6$		
$6 \le m < 8$		
$8 \le m < 10$		
$10 \le m < 12$		
$12 \le m < 14$		
$14 \le m < 16$		

Mathematics/P2 NSC – Grade 11 Exemplar DBE/2013

NAME OF LEARNER:

DIAGRAM SHEET 2

QUESTION 2.2

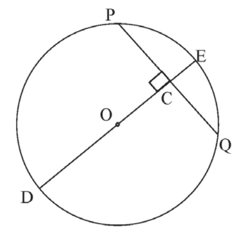


Mathematics/P2 NSC – Grade 11 Exemplar DBE/2013

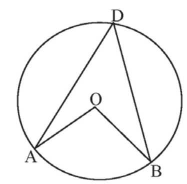
NAME OF LEARNER:

DIAGRAM SHEET 3

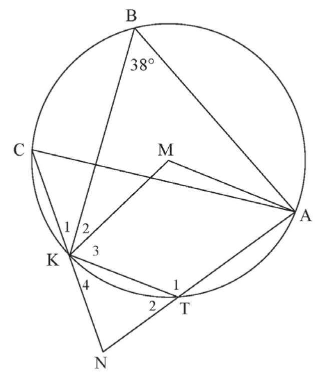
QUESTION 9.2



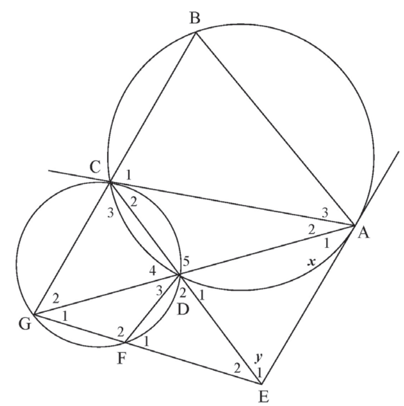
QUESTION 10.1



QUESTION 10.2



QUESTION 11.2



RESOURCE 14

REVISION: Memorandum Paper 2 Exemplar 2013

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 11

MATHEMATICS P2

EXEMPLAR 2013

MEMORANDUM

MARKS: 150

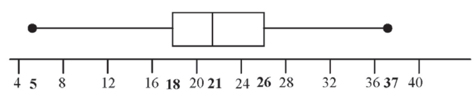
This memorandum consists of 13 pages.

Please turn over

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

QUESTION 1

1.1	Mean = $\frac{\sum_{i=1}^{n} x_i}{n} = \frac{408}{19} = 21{,}47$	✓ $\frac{408}{19}$ ✓answer (2)
1.2	Standard deviation = 7,81	✓✓answer (2)
1.3	The one standard deviation limits are $(\bar{x} - 1\sigma ; \bar{x} + 1\sigma)$ = (21,47 – 7,81; 21,47 + 7,81) = (13,66 ; 29,28) ∴ 13 people lie within 1 standard deviation of the mean.	✓ interval ✓13 people (2)
1.4	5 12 13 15 18 18 18 19 20 21 21 22 23 23 26 29 33 35 37 IQR = 26 – 18 = 8	✓ $Q_1 = 18$ ✓ $Q_3 = 26$ ✓IQR = 8 (3)
1.5		✓✓box ✓whiskers (3)
1.6	There is a marked difference between the lowest value (5) and the next lowest value (12) whilst the differences between all other data points are within at most 3 values. ∴ 5 is an outlier	✓reason ✓5 is an outlier (2) **[14]**

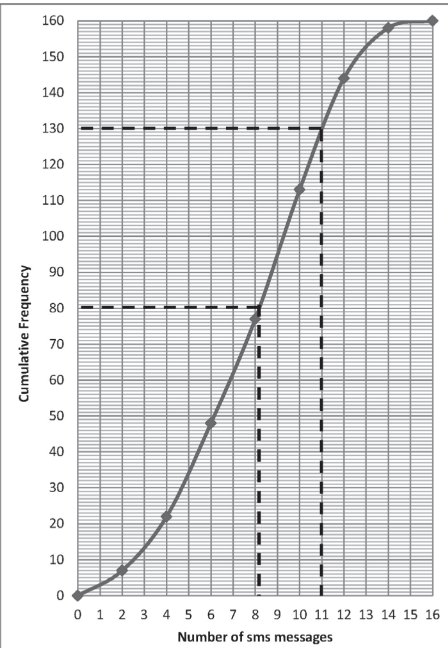
QUESTION 2

2.1

Class	Frequency	Cumulative frequency
$0 \le m < 2$	7	7
$2 \le m < 4$	15	22
$4 \le m < 6$	26	48
$6 \le m < 8$	29	77
$8 \le m < 10$	36	113
$10 \le m < 12$	31	144
$12 \le m < 14$	14	158
$14 \le m < 16$	2	160

✓ first three cumulative frequencies correct
✓remainder correct (total = 160) (2)

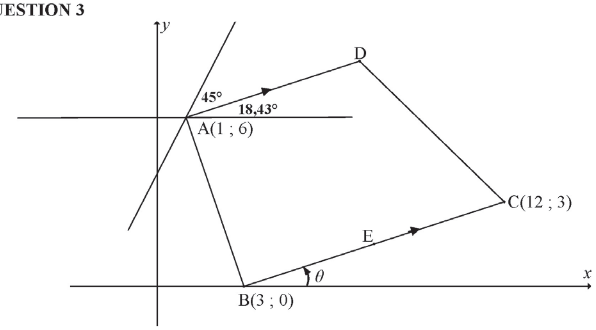
2.2



✓grounding at 0
✓plotting cumulative frequencies at upper limits
✓ smooth shape of curve (3)

2.3	The median for the data is approximately 8 messages.	✓Median (1)
2.4	Approximately 130 learners sent 11 or fewer messages. Therefore 30 learners sent more than 11 messages. $\frac{30}{160}\times 100\% = 18,75\%$	✓30 learners ✓answer (2)
2.5	Skewed to the left or negatively skewed	✓answer (1) **[9]**

QUESTION 3



3.1	$E\left(\frac{3+12}{2};\frac{0+3}{2}\right)$ $=\left(7\frac{1}{2};1\frac{1}{2}\right)$	✓ substitution into midpoint formula ✓answer (2)
3.2	$m_{BC}=\frac{3-0}{12-3}$ $=\frac{1}{3}$	✓ substitution into gradient formula ✓answer (2)
3.3	$\tan\theta = m_{BC}=\frac{1}{3}$ $\theta=\tan^{-1}\left(\frac{1}{3}\right)=18,43°$	✓ $\tan\theta = m_{BC}$ ✓answer (2)
3.4	$m_{AD}=m_{BC}=\frac{1}{3}$ AD‖BC, equal gradients $m_{AB}=\frac{6-0}{1-3}=-3$ $\therefore m_{AD}\times m_{AB}=\frac{1}{3}\times -3=-1$ $\therefore$ AD ⊥ AB	✓ $m_{AD}=\frac{1}{3}$ ✓ $m_{AB}=-3$ ✓ $m_{AD}\times m_{AB}=-1$ (3)
3.5	inclination of new line = 45° + 18,43° = 63,43° $\therefore \tan 63,43° = 2 = m_{line}$ $\therefore y-6=2(x-1)$ $y=2x+4$	✓18,43° ✓63,43° ✓$m=2$ ✓subst of (1 ; 6) ✓ equation (5) **[14]**

QUESTION 4

4.1	$m_{QP} = m_{OS} = 6$ QP‖OS, equal gradients $y - 17 = 6(x+3)$ $y = 6x + 35$	✓ $m_{QP} = 6$ ✓subst (–3 ; 17) into formula ✓equation (3)
4.2	$6x + 35 = -x$ $7x = -35$ $x = -5$ $y = -(-5) = 5$ **OR** $y = 6(-5) + 35 = 5$ $\therefore$ Q(–5 ; 5)	✓ setting up equation ✓$x = -5$ ✓ $y = 5$ ✓coordinates of Q (4)
4.3	$OQ^2 = (-5-0)^2 + (5-0)^2$ $= 50$ $OQ = \sqrt{50} = 5\sqrt{2}$ *units*	✓ substitution into distance formula ✓ $5\sqrt{2}$ (2)
4.4	$m_{OS} = 6$ $\therefore$ inclination of OS is $\tan^{-1}(4) = 80{,}54°$ $m_{OQ} = -1$ $\therefore$ inclination of QO is $180° - \tan^{-1}(1) = 135°$ $\alpha = 135° - 80{,}54...°$ $= 54{,}46°$	✓80,54° ✓135° ✓54,46° (3)
4.5	$QS^2 = OS^2 + OQ^2 - 2OS.OQ.\cos\alpha$ $= 148 + 50 - 2(\sqrt{148})(\sqrt{50}.\cos 54{,}46°$ $QS = 9{,}90$ units	✓correct use of cosine rule ✓substitution into formula ✓9,90 (3) **[15]**

QUESTION 5

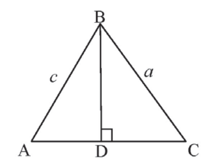
5.1.1	$\cos\alpha = -\frac{5}{13}$	✓ $-\frac{5}{13}$ (1)
5.1.2	$(-5)^2 + b^2 = 13^2$ $b^2 = 169 - 25 = 144$ $b = 12$ $\tan(180° - \alpha)$ $= -\tan\alpha$ $= -(-\frac{12}{5})$ $= \frac{12}{5}$	✓$b = 12$ ✓$-\tan\alpha$ ✓$\frac{12}{5}$ (3)
5.2.1	$\frac{\sin(\theta - 360°)\sin(90° - \theta)\tan(-\theta)}{\cos(90° + \theta)}$ $= \frac{\sin\theta\cos\theta(-\tan\theta)}{-\sin\theta}$ $= -\cos\theta\left(-\frac{\sin\theta}{\cos\theta}\right)$ $= \sin\theta$	✓✓✓ reductions ✓$\tan\theta = \frac{\sin\theta}{\cos\theta}$ ✓ $\sin\theta$ (5)
5.2.2	From 5.2.1: $\sin\theta = 0{,}5$ Ref $\angle = 30°$ $\therefore \theta = 30°$ or $\theta = 150°$	✓$\sin\theta = 0{,}5$ ✓30° ✓150° (3)

5.3.1	$LHS = \frac{8}{\sin^2 A} - \frac{4}{1+\cos A}$ $= \frac{8}{1-\cos^2 A} - \frac{4}{1+\cos A}$ $= \frac{8}{(1-\cos A)(1+\cos A)} - \frac{4}{1+\cos A}$ $= \frac{8-4(1-\cos A)}{(1-\cos A)(1+\cos A)}$ $= \frac{8-4+4\cos A}{(1-\cos A)(1+\cos A)}$ $= \frac{4(1+\cos A)}{(1-\cos A)(1+\cos A)}$ $= \frac{4}{1-\cos A} = RHS$	✓ $\sin^2 A = 1 - \cos^2 A$ ✓factorising ✓ addition ✓simplification ✓ factorising (5)
5.3.2	Identity is undefined when $\sin^2 A = 0$. That is when $\sin A = 0$ or $\cos A = \pm 1$ $\therefore$ A = 0° or A = 180° or A = 360°.	✓✓✓ each value (3)
5.4	$8\cos^2 x - 2\cos x - 1 = 0$ $(4\cos + 1)(2\cos x - 1) = 0$ $\cos x = -\frac{1}{4}$ or $\cos x = \frac{1}{2}$ $\therefore x = 104{,}48° + k.360°; k \in Z$ or $x = 60° + k.360°; k \in Z$ $x = 255{,}52° + k.360°; k \in Z$ $x = 300° + k.360°; k \in Z$	✓ factorising ✓values of cos$x$ ✓ 104,48° or 255,52° ✓ 60° or 300° ✓ + 360°.k ✓$k \varepsilon$ Z (6) **[26]**

QUESTION 6

6.1	$p = -45°$ $q = -1$	✓ value of p ✓ value of q (2)
6.2	B(157,5° ; – 0,38)	✓ value of x ✓ value of y (2)
6.3	$f(x) < g(x)$ when $-180° \le x < -22{,}5°$ or $157{,}5° < x \le 180°$	✓ $-180° \le x < -22{,}5°$ ✓ $157{,}5° < x \le 180°$ (2)
6.4.1	$h(x) = \cos(x - 45° + 30°)$ $= \cos(x - 15°)$	✓ + 30° ✓ simplest form (2)
6.4.2	$x = -135° - 30° = -165°$	✓ –165° (1) **[9]**

QUESTION 7

7.1	Draw BD ⊥ AC In ΔABD: sin A = $\frac{BD}{c}$ ∴ BD = c. sin A In ΔCBD: sin C = $\frac{BD}{a}$ ∴ BD = a. sin C ∴ c . sin A = a. sin C ∴ $\frac{\sin A}{a} = \frac{\sin C}{c}$		✓ construction ✓ sin A ✓ making BD the subject ✓ sin C ✓c . sin A = a. sin C (5)
7.2.1	$\frac{\sin R}{r} = \frac{\sin P}{p}$ $\frac{\sin R}{27{,}2} = \frac{\sin 132°}{73{,}2}$ $\sin R = \frac{27{,}2 \times \sin 132°}{73{,}2}$ $= 0{,}276...$ $\hat{R} = 16{,}03°$		✓ substitution into correct formula ✓making sin R the subject ✓16,03° (3)

7.2.2	$\hat{Q} = 180° - 132° - 16{,}03° = 31{,}97°$ area of PQR $= \frac{1}{2} pr.\sin Q$ $= \frac{1}{2}(73{,}2)(27{,}2).\sin 31{,}97°$ $= 527{,}10\ \text{cm}^2$	✓ $\hat{Q} = 31{,}97°$ ✓ substitution into correct formula ✓ 527,1 (3)
7.3.1	$P\hat{S}Q = 180° - (a+b)$ In ΔPSQ: $\frac{SQ}{\sin P} = \frac{PQ}{\sin P\hat{S}Q}$ $\frac{SQ}{\sin a} = \frac{h}{\sin[180° - (a+b)]}$ $\frac{SQ}{\sin a} = \frac{h}{\sin(a+b)}$ $SQ = \frac{h \sin a}{\sin(a+b)}$	✓ $P\hat{S}Q = 180° - (a+b)$ ✓ $\sin[180° - (a+b)] = \sin(a+b)$ ✓making SQ the subject (3)
7.3.2	$S\hat{Q}R = 90° - b$ In ΔRSQ: $\frac{RS}{SQ} = \sin S\hat{Q}R$ $RS = SQ.\sin(90° - b)$ $= \frac{h \sin a}{\sin(a+b)}.\cos b$ $= \frac{h \sin a.\cos b}{\sin(a+b)}$	✓ $S\hat{Q}R = 90° - b$ ✓ use sine ratio correctly ✓ $\sin(90° - b) = \cos b$ (3) **[17]**

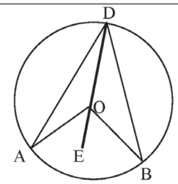
QUESTION 8

	Volume of hemisphere $= \frac{1}{2}\left[\frac{4}{3}\pi r^3\right]$ $= \frac{2}{3}\pi(3)^3$ $= 18\pi\ \text{cm}^3$ Volume of conical hole $= \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi(1{,}5)^2(\frac{8}{9})$ $= \frac{2}{3}\pi\ \ cm^3$ $\therefore \frac{\text{volume of metal A}}{\text{volume of metal B}} = \frac{17\frac{1}{3}\pi}{\frac{2}{3}\pi} = \frac{26}{1}$ Ratio of volume metal A : Volume metal B = 26 : 1	✓ substitution into correct formula ✓18 π ✓ substitution into correct formula ✓ $\frac{2}{3}\pi$ ✓ $17\frac{1}{3}\pi$ ✓ratio 26 : 1 (6) **[6]**

QUESTION 9

9.1	…bisects the chord.		✓ answer (1)
9.2.1	OE = 10 cm OC = OE – CE = 10 – 2 = 8 cm	… O midpoint of DE	✓ OE = 10 ✓ OC = 8 (2)
9.2.2	In ΔCOQ: $QC^2 = OQ^2 - OC^2$ $= (10)^2 - (8)^2$ = 36 QC = 6 cm ∴ PQ = 2QC PQ = 12 cm	… Theorem of Pythagoras … line drawn from centre ⊥ to chord bisects chord	✓Using Theorem of Pythagoras ✓ QC = 6 ✓ PQ = 12 (S) ✓ reason (4) **[7]**

QUESTION 10

10.1	 Construction: Produce DO to E Proof: In ΔOBD: $O\hat{B}D = O\hat{D}B$ …OD = OB = r $E\hat{O}B = 2 \times O\hat{D}B$ …exterior angle of triangle In ΔAOD: $O\hat{A}D = O\hat{D}A$ …OA = OD = r $E\hat{O}A = 2 \times O\hat{D}A$ …exterior angle of triangle $A\hat{O}B = E\hat{O}B + E\hat{O}A$ $= 2 \times O\hat{D}B + 2 \times O\hat{D}A$ $= 2(O\hat{D}B + O\hat{D}A)$ $= 2A\hat{D}B$	✓ construction ✓ $O\hat{B}D = O\hat{D}B$ ✓ $E\hat{O}B = 2 \times O\hat{D}B$ (S/R) ✓ $E\hat{O}A = 2 \times O\hat{D}A$ (S/R) ✓ $A\hat{O}B = E\hat{O}B + E\hat{O}A$ (5)
10.2.1(a)	$\hat{M} = 76°$ … ∠ at centre = 2(∠ at circumference)	✓ 76° ✓ reason (2)
10.2.1(b)	$\hat{T}_2 = 38°$ …ext∠ of cyc quad KTAB	✓ 38° ✓ reason (2)
10.2.1(c)	$\hat{C} = 38°$ … ext∠ of cyclic quad or ∠s in same segment	✓ 38° ✓ reason (2)
10.2.1(d)	$C\hat{A}N = \hat{C} = 38°$ …NA = NC $\hat{K}_4 = 38°$ …ext ∠ of cyclic quad CATK	✓ $C\hat{A}N = 38°$ (S/R) ✓ $\hat{K}_4 = 38°$ (2)
10.2.2	$\therefore \hat{K}_4 = \hat{T}_2$ ∴NK = NT …base ∠s equal	✓ statement ✓ reason (2)
10.2.3	$\hat{N} = 180° - (38° + 38°)$ …∠s of ΔKNT $= 104°$ $\hat{N} + K\hat{M}A = 104° + 76° = 180°$ ∴ AMKN is cyclic quad …opposite ∠s = 180°	✓ $\hat{N} = 104°$ (S/R) ✓ $\hat{N} + K\hat{M}A = 180°$ ✓reason (3) **[18]**

QUESTION 11

11.1	..... equal to the angle subtended by the same chord in the alternate segment.	✓ alternate segment (1)
11.2.1	$\hat{A}_1 = \hat{C}_2 = x$ …tangent chord theorem $\hat{C}_2 = \hat{G}_2 = x$ …tangent chord theorem $\therefore \hat{A}_1 = \hat{G}_2 = x$ ∴ BCG ∣∣ EA …alternate ∠s =	✓ $\hat{A}_1 = \hat{C}_2 = x$ ✓ reason ✓ $\hat{C}_2 = \hat{G}_2 = x$ ✓ reason ✓ conclusion with reason (5)
11.2.2	$\hat{E}_1 = \hat{C}_3 = y$ …alternate ∠s; BG∣∣EA $\hat{F}_1 = \hat{C}_3 = y$ …ext∠ of cyclic quad CDFG $\therefore \hat{E}_1 = \hat{F}_1 = y$ ∴ EA is a tangent …converse tangent-chord theorem	✓ $\hat{E}_1 = \hat{C}_3 = y$ (S/R) ✓ $\hat{F}_1 = \hat{C}_3 = y$ (S) ✓ reason ✓ $\hat{E}_1 = \hat{F}_1 = y$ ✓reason (5)
11.2.3	$\hat{B} = C\hat{A}E$ …tangent-chord theorem $\hat{C}_1 = C\hat{A}E$ … alternate ∠s; BG∣∣EA $\hat{C}_1 = \hat{B}$ ∴ AB = AC …base ∠s =	✓ $C\hat{A}E = \hat{B}$ ✓reason ✓ $\hat{C}_1 = C\hat{A}E$ (S/R) ✓ reason (4) **[15]**

TOTAL: 150

RESOURCE 15

REVISION: Test Term 4

QUESTION	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1 – 3	Statistics	20	
4–5	Equations and Exponents	30	
	TOTAL	50	

ANSWER QUESTIONS 1 – 3 ON THE QUESTION PAPER

INSTRUCTIONS:

1. If necessary, round answers off to two decimal places.
2. Calculators may be used.
3. Formulae:

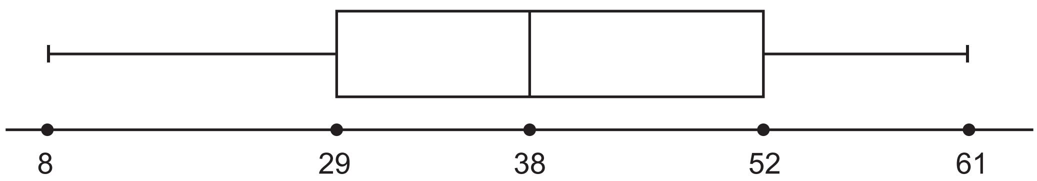
$$\frac{\sum_{i=1}^{n} x_i}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} \left(x_i - \overline{x}\right)^2}{n}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

QUESTION 1 **5 MARKS**

The following box and whisker diagram was obtained from the marks achieved by 100 learners in a Mathematics test. The test was out of 65.



1.1 Write down the median mark achieved. (1)

1.2 What is the range of marks? (1)

1.3 What is the inter quartile range? (2)

1.4 Assuming that only two learners achieved 52 out of 65, how many learners achieved a mark higher than 52? (1)

1.1		1.2	
1.3		1.4	

QUESTION 2 **6 MARKS**

A group of Grade 12 learners were asked how many hours studying they did prior to their final mathematics examination.

The results were as follows:

14 7 9 12 19 10 16 15 3 21

2.1 Calculate the mean and standard deviation of these times. (4)

2.2 How many learners' studying times lie outside one standard deviation from the mean? (2)

2.1	
2.2	

QUESTION 3 **9 MARKS**

The table below shows the results from a survey of the amount of money spent buying data in one month for a group of 100 Grade 11 learners.

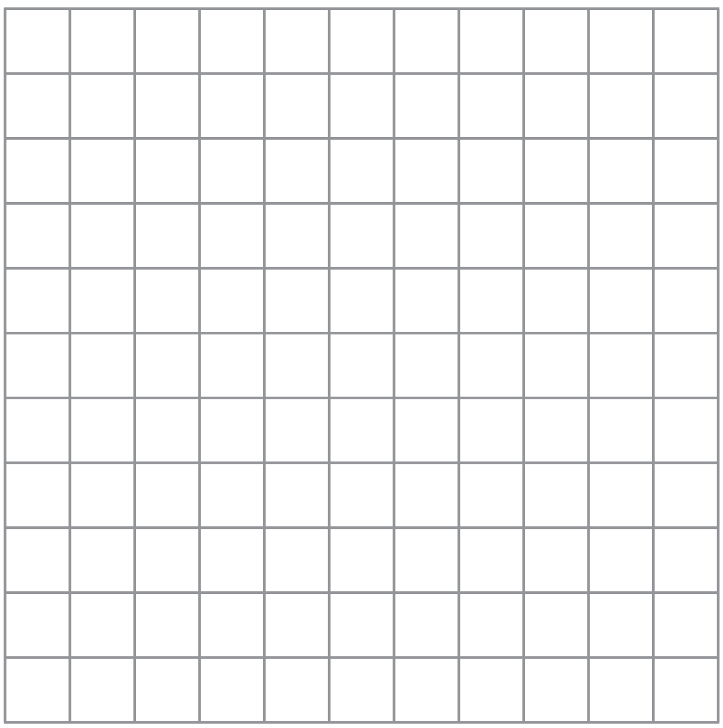
Expenditure in Rand	**Frequency**	**Cumulative Frequency**
$0 \leq x < 50$	2	
$50 \leq x < 100$	22	
$100 \leq x < 150$	58	
$150 \leq x < 200$	14	
$200 \leq x < 250$	4	

3.1 Complete the table. (1)

3.2 Determine the estimated mean for the set of data.

(2)

3.3 Draw an ogive curve (cumulative frequency graph) for the data.



3.4 Use the graph to estimate the median expenditure on data for this group of learners indicating where you read off the answer (mark it with an A).
Estimated median: _______________ (2)

QUESTION 4 **24 MARKS**

Solve for x:

4.1 $(3x + 1)(x - 4) = 0$ (2)

4.2 $\frac{5x^2 - 47}{x^2 + x - 6} = \frac{4}{x + 3}$

(write your answer correct to TWO decimal places) (5)

4.3 $\sqrt{3x^2 - 11x} = x - 2$ (5)

4.4 $(x - 1)(x - 4) \leq 18$ (4)

Solve for x and y:

4.5 $xy - 2y = x + 1$ and $5 = 3x - y$ (8)

QUESTION 5 **6 MARKS**

Simplify, without the use of a calculator:

5.1 $\frac{12^{3x}.4^{-x}}{27^{x+1}.16^{x}}$ (4)

5.2 $\frac{7^{270} + 7^{272}}{7^{270}}$ (2)

RESOURCE 16

REVISION: Memorandum Test Term 4

QUESTION	DESCRIPTION	MAXIMUM MARK	ACTUAL MARK
1 – 3	Statistics	20	
4–5	Equations and Exponents	30	
	TOTAL	50	

ANSWER QUESTIONS 1 – 3 ON THE QUESTION PAPER

INSTRUCTIONS:

1. If necessary, round answers off to two decimal places.
2. Calculators may be used.
3. Formulae:

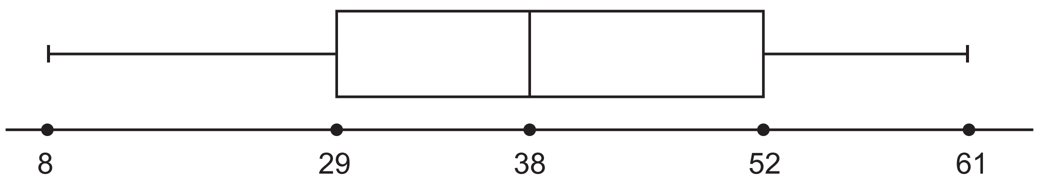
$$\frac{\sum_{i=1}^{n} x_i}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2}{n}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

QUESTION 1 **5 MARKS**

The following box and whisker diagram was obtained from the marks achieved by 100 learners in a Mathematics test. The test was out of 65.



1.1 Write down the median mark achieved. (1)

1.2 What is the range of marks? (1)

1.3 What is the inter quartile range? (2)

1.4 Assuming that only two learners achieved 52 out of 65, how many learners achieved a mark higher than 52? (1)

1.1	38 ✓	1.2	61 – 8 = 53 ✓
1.3	52 – 29 ✓ = 23 ✓	1.4	23 ✓

QUESTION 2 **6 MARKS**

A group of Grade 12 learners were asked how many hours studying they did prior to their final mathematics examination.

The results were as follows:

14 7 9 12 19 10 16 15 3 21

2.1 Calculate the mean and standard deviation of these times. (4)

2.2 How many learners' studying times lie outside one standard deviation from the mean? (Show working) (2)

2.1	Mean: 12,6 ✓ ✓ Std deviation: 5,24 ✓ ✓
2.2	12,6 – 5,24 = 7,36 12,6 + 5,24 = 17,84 ✓ There are 4 learners who lie outside 1 std dev from the mean ✓

QUESTION 3 **9 MARKS**

The table below shows the results from a survey of the amount of money spent buying data in one month for a group of 100 Grade 11 learners.

Expenditure in Rand	Frequency	Cumulative Frequency	
$0 \leq x < 50$	2	2	
$50 \leq x < 100$	22	24	
$100 \leq x < 150$	58	82	✓
$150 \leq x < 200$	14	96	
$200 \leq x < 250$	4	100	

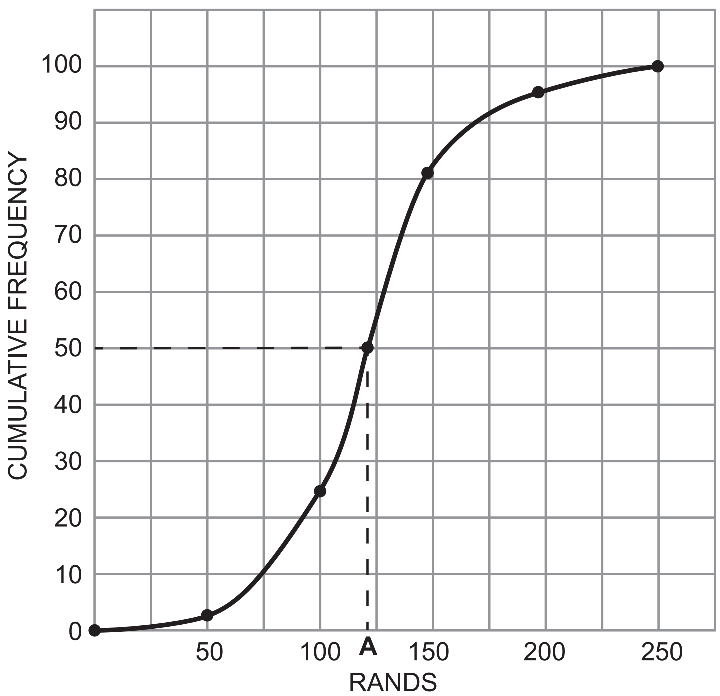
3.1 Complete the table. (1)

3.2 Determine the estimated mean for the set of data.

$\bar{x}$ ✓ = 123 ✓ ✓

(2)

3.3 Draw an ogive curve (cumulative frequency graph) for the data.



✓ headings

✓ ✓ correct points

✓ median - A

3.4 Use the graph to estimate the median expenditure on data for this group of learners indicating where you read off the answer (mark it with an A).
Estimated median: ≈120 ✓ (2)

QUESTION 4 **24 MARKS**

Solve for x:

4.1 $(3x + 1)(x - 4) = 0$ (2)

$x = \frac{1}{3}$ ✓ or $x = 4$ ✓

4.2 $\frac{5x^2 - 47}{x^2 + x - 6} = \frac{4}{x + 3}$

(write your answer correct to TWO decimal places) (5)

$\frac{5x^2 - 47}{(x^2 + 3)(x - 2)✓} = \frac{4}{x + 3}$

LCD = $(x + 3)(x - 2)$; $x \neq -3;2$

$5x^2 - 47 = 4(x - 2)$

$5x^2 - 47 - 4x + 8 = 0$

$5x^2 - 4x - 39 = 0$ ✓

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = -(4) \pm \frac{\sqrt{(-4)^2 - 4(5)(-39)}}{2(5)}$ ✓

$x = 3{,}22$ ✓ or $x = -2{,}42$ ✓

4.3 $\sqrt{3x^2 - 11x} = x - 2$ (5)

$3x^2 - 11x = x^2 - 4x + 4$ ✓

$2x^2 - 7x - 4 = 0$ ✓

$(2x + 1)(x - 4)$✓ $= 0$

$x = \frac{1}{2}$ or $x = 4$ ✓ (both solutions)

N|S $\therefore x = 4$

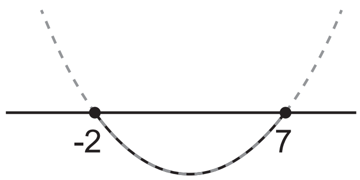
4.4 $(x-1)(x-4) \leq 18$ (4)

$x^2 - 5x + 4 - 18 \leq 0$

$x^2 - 5x - 14 \leq 0$ ✓

$(x-7)(x+2)$ ✓ ≤ 0

CV's: 7 ;-2 ✓



$-2 \leq x \leq 7$ ✓ ✓

Solve for x and y:

4.5 $xy - 2y = x + 1$ and $5 = 3x - y$ (8)

$y = 3x - 5$ ✓

$x(3x-5) - 2(3x-5) = x + 1$ ✓

$3x^2 - 5x - 6x + 10 = x + 1$

$3x^2 - 12x + 9 = 0$ ✓

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$ ✓

$x = 3$ ✓ or $x = 1$ ✓

$\therefore y = 4$ ✓ or $y = -2$ ✓

QUESTION 5 **6 MARKS**

Simplify, without the use of a calculator:

5.1 $\frac{12^{3x}.4^{-x}}{27^{x+1}.16^{x}}$ (4)

$=\frac{(2^2.3)^{3x}.(2^2)^{-x}}{(3^3)^{x+1}.2^{4x}}$ ✓

$=\frac{2^{6x}.3^{3x}.2^{-2x}}{3^{3x+3}.2^{4x}}$ ✓

$=\frac{2^{4x}.3^{3x}}{3^{3x+3}.2^{4x}}$

$=2^{4x-4x}.3^{3x-3x-3}$ ✓

$=2^{0}.3^{-3}$

$=\frac{1}{3^3}=\frac{1}{27}$ ✓

5.2 $\frac{7^{270}+7^{272}}{7^{270}}$ (2)

$=\frac{7^{270}(1+7^2)}{7^{270}}$ ✓

$= 1 + 7^2 = 50$ ✓