MATHEMATICS

RESOURCE PACK

GRADE 12 TERM 4

Summary notes – Paper 1

ALGEBRAIC EXPRESSIONS AND EXPONENTS

Products

General Rule with brackets: Use the distributive law.

Factors

To factorise an expression is the opposite operation to finding the product.

1. Common factor (including grouping and sign changing)

   Example: $3x^2 - 9x^3 = 3x^2(1 - 3x)$

2. Grouping
   - Four or more terms usually requires grouping

   Example, $6p^3 - 4q^3 + 3p^2q - 8pq^2$ (6 & 3 gives the same ratio as 8 & 4)

   $= 3p^2(2p + q) - 4q^2(q + 2p)$ $\quad [2p + q = q + 2p]$

   $= (2p + q)(3p^2 - 4q^2)$

3. Difference of two squares
   - Always two terms separated by a minus sign

     Example, $(3a+b)^2 - 16$

     $= [(3a + b) + 4][(3a + b) - 4]$

4. Sum and Difference of two cubes
   - Always two terms separated by a plus or minus sign
   - Both terms must be perfect cubes
   - Factors will always be a binomial and a trinomial
   - Binomial bracket: cube root each term and keep same sign
   - Trinomial bracket:

     1st term: Square 1st term from binomial bracket

     2nd term: Find product of 2 terms from binomial bracket and change the sign

     3rd term: Square the 2nd term from the binomial bracket

   Example, $27x^3 + 64y^3$

   $= (3x + 4y)(9x^2 + - 12xy + 16y^2)$

5. Trinomials
   - Always three terms and factorises into two factors (hence the two brackets)

     If coefficient of x^2 is not 1:
   - choose the appropriate signs to match the product of the last term
   - find factors of the first term and last term
   - use cross multiplication to find the factors that work

REMEMBER: ALWAYS look for a highest common factor first.

Algebraic fractions

Multiplication and Division

- For division, change to multiplication and reciprocate
- Factorise all numerators and denominators fully
- Simplify by looking for common factors in any numerator and denominator (remember: you cannot simplify 'next to' an addition or subtraction).

Addition and Subtraction

- Ensure all denominators are fully factorised
- Find LCD (lowest common denominator)
- Change numerators accordingly to ensure equivalent fractions
- Collect like terms

Completing the square

Completing the square is a technique used to express quadratic expressions in the form of:

$$a(x \pm p)^2 + q$$

Steps to completing the square:

1. Take out the coefficient of x^2 if it is not 1
2. Add and immediately subtract (half the coefficient of x)²
3. Factorise (the newly formed perfect square trinomial) and distribute the coefficient.

Example:

Complete the square on the expression: $2x^2 - 10x + 4$

1.		$2(x^2 - 5x + 2)$
	Find ½ the coefficient of x	$\frac{1}{2}(-5) = \frac{-5}{2}$
	and square it	$\frac{25}{4}$
2.	Add and subtract (to keep the expression the same)	$2\left(x^2 - 5x + \mathbf{\frac{25}{4}} - \mathbf{\frac{25}{4}} + 2\right)$
	Note the *perfect square trinomial* you have created	$2\left(\mathbf{x^2 - 5x + \frac{25}{4}} - \frac{25}{4} + 2\right)$
3.	Factorise the perfect square trinomial and collect other 2 like terms	$2\left[\left(x - \mathbf{\frac{5}{2}}\right)^2 - \frac{17}{4}\right]$ ↙ **This will always be half the coefficient of x**
	Remove the outer brackets by distributing the coefficient of x^2	$2\left(x - \frac{5}{2}\right)^2 - \frac{17}{2}$

EXPONENTS AND SURDS

Definition/law	Example	Explanation
$x^a \times x^b = x^{a+b}$	$2^3 \times 2^2 \times 2$ $= 2^{3+2+1}$ $= 2^6$	When multiplying like bases keep the bases the same and add the exponents.
$\frac{x^a}{x^b}$ $= x^{a-b}$	$\frac{6x^6}{2x^2} = 3x^4$	When dividing like bases keep the base and subtract the exponent. Divide numbers as per normal.
$(x^a)^b = x^{ab}$	$(-2a^2b^3)^4$ $= (-2)^2 \times a^{2\times2} \times b^{3\times2}$ $= 4a^4b^6$	When raising exponents to a power, keep the base and multiply the exponents.

$(xy)^a = x^a y^a$ or $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$	$(a^4 b)^3$ $= a^{12} b^3$ $\left(\frac{a^3}{b}\right)^3 = \frac{a^9}{b^3}$	When more than one base is raised to an exponent, each base is raised to the exponent. When a fraction is raised to an exponent, the numerator and denominator must be raised to that exponent.
$x^0 = 1$	$(x^4 + 4)^0 \times 3^0$ $= 1 \times 1 = 1$	Any base raised to the power of zero is equal to 1. ($x \neq 0$ as 0^0 is undefined)
$x^{-a} = \frac{1}{x^a}$	$3x^{-2} = \frac{3}{x^2}$ and $\frac{3}{x^{-2}} = 3x^2$	A base raised to a negative exponent is equal to its reciprocal raised to the same positive exponent.
$\sqrt[n]{xy} = \sqrt[n]{x} \times \sqrt[n]{y}$	$\sqrt{18} = \sqrt{9} \times \sqrt{2}$ $= 3\sqrt{2}$	When surds are multiplied they can be split apart and rooted individually.
$\sqrt[mn]{x}$ $= \sqrt[m]{\sqrt[n]{x}}$	$\sqrt[6]{64}$ $= \sqrt[3\times2]{64}$ $= \sqrt[3]{\sqrt{62}}$ $= \sqrt[3]{8} = 2$	When taking the root of a root, it is the same as taking the single root to the product of both roots.

Fractions with exponents

Type 1: When the numerator and/or denominator has more than one term, factorising is required. To find a common factor, Law 1 needs to be used in reverse ($2^{x+1} = 2^x . 2^1$).

This makes finding the HCF and knowing what remains when it has been taken out much easier.

For example:

$\frac{3^{1+x} - 5.3^x}{(3^x.6)}$

$= 31.3x - 5.3x3x.6$ Use inverse of Law 1

$= 3x3 - 53x.6$ Find HCF and factorise

$= -\frac{2}{6}$ Simplify

$= -\frac{1}{3}$

Type 2: When there is only one term in the numerator and denominator, each base must be written as a product of prime factors then the laws and definitions of exponents are used to simplify.

For example:

$\frac{3^{n+1}.4^{n-1}}{2^n.6^{n-1}}$

$= \frac{3^{n+1}.(2^2)^{n-1}}{2^n.(2.3)^{n-1}}$ (prime factors)

$= \frac{3^{n+1}.2^{2n-2}}{2^n.2^{n-1}.3^{n-1}}$

$= \frac{3^{n+1}.2^{2n-2}}{2^{2n-1}.3^{n-1}}$

$= 3^{n+1-(n-1)}.2^{2n-2(2n-1)}$

$= 3^{n+1-n+1}.2^{2n-2-2n+1}$

$= 3^2.2^{-1}$

$= \frac{9}{2}$

Simplification of surds

A surd is the root of a number that would result in an irrational number.

For example: $\sqrt{3}$ is a surd as the answer is irrational.

$\sqrt{9}$ has a rational answer (3).

Further examples:

a) $\sqrt{12}-\sqrt{48}+\sqrt{75}$

$=\sqrt{4\times 3}-\sqrt{16\times 3}+\sqrt{25\times 3}$ (break down into the product of a perfect square and another factor

$=2\sqrt{3}-4\sqrt{3}+5\sqrt{3}$ (square root)

$=3\sqrt{3}$ (simplify)

b) $(\sqrt{5}-2)(\sqrt{5}+2)$ (Difference of 2 squares)

$= 5 - 4$

$= 1$

Nature of roots

In the quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, the discriminant (it discriminates between different types of solutions), $b^2 - 4ac$ is used to find the nature of the roots.

$b^2 - 4ac > 0 \rightarrow$ two real roots

$b^2 - 4ac = 0 \rightarrow$ one real root (the roots are equal)

$b^2 - 4ac < 0 \rightarrow$ non-real roots

EQUATIONS AND INEQUALITIES

1. **Linear equations**
   - Remove brackets (using distributive law) and collect like terms on each side.
   - Get all the terms with the variable in them on LHS and all constants on RHS (but remember, whatever is done to one side of the equation must be done to the other side to keep the equation balanced).
   - Collect like terms on each side again and get the variable on its own using division.
2. **Equations with fractions**
   - Find LCD. Multiply ALL terms throughout equation by LCD to remove all fractions (no more denominators).
   - There should be NO fractions AT ALL in the next step.
   - Continue the same as for linear equations.
3. **Quadratic equations**
   - Recognisable by the "square". You should be expecting two answers.
   - Get ALL terms on LHS so that RHS = 0.
   - Factorise the LHS fully.
   - Find the two possible solutions using the concept that two factors multiplied to equal zero will mean that each one of the factors could possible equal zero.
4. **Simultaneous equations** (Given two equations with two variables to solve for – usually a quadratic at Grade 11 level)
   - Get ONE of the variables by itself in ONE of the equations.

- Use this information to substitute back into the second equation. You should now have an equation with only one unknown variable.
- Solve for this variable.
- Substitute the variable found back into the first equation and solve for the second variable.

5. Exponential equations

- Bases must be the same to solve exponential equations – if the bases are the same, the exponents will be the same.
- If bases are not the same, use prime factors to make them the same.

6. Literal equations

- Treat as if it is an ordinary linear equation first (try and ignore the fact that there are many variables and few or no numbers).
- Focus on the variable you have been asked to solve for.
- Get all terms with this variable in on one side and all terms without this variable in on the other side.
- If the variable you are solving for is in more than one term (and they're all on one side now), factorise by taking this variable out as a common factor.
- Divide both sides by any other variables 'in the way' and get the variable you're solving for on its own.

7. Equations involving surds

- Isolate the surd.
- Square both sides.
- Solve for x.
- Check your answer.

8. Equations with rational exponents

These are exponents with fractions. If an equation is in the form: $x^{\frac{a}{b}} = y$:

- there will be a positive and negative solution if a is even and b odd.
- there will be one solution if a is odd.

Examples:

$x^{\frac{2}{3}} - 16 = 0$ $x^{\frac{2}{3}} = 2^4$ (a is even b is odd $\therefore \pm$ solutions) $(x^{\frac{2}{3}})^{\frac{3}{2}} = \pm(2^4)^{\frac{3}{2}}$ (both sides raised to $\frac{3}{2}$) $x = \pm 2^6$ $x = \pm 64$	$3^{\frac{x}{4}} = 27$ $3^{\frac{x}{4}} = 3^3$ (convert 27 to a prime base) $\frac{x}{4} = 3$ (if the bases are the same the exponents must be equal) $x = 12$

9. Equations involving factorising

$3^{x+2} + 3^{x+3} - 3^x = 105$ $3^x.3^2 + 3^x.3^3 - 3^x = 105$ (inverse of Law 1) $3^x(3^2 + 3^3 - 1) = 105$ (factorise) $3^x(35) = 105$ (divide both sides by 35) $3^x = 3$ $x = 1$	$3^{2x} - 10.3^x + 9 = 0$ $((3^x)^2)$- (use k method) Let $k = 3^x$ $k^2 - 10k + 9 = 0$ $(k - 9)(k - 1) = 0$ (factorise) $k = 9$ or $k = 1$ $\therefore 3^x = 9$ or $3^x = 1$ $3^x = 3^2$ $3^x = 3^0$ (any number raised to the power of zero =1) $\therefore$ $x = 2$ or $x = 0$

10. Linear inequalities

- Treat the same as a linear equation
- IF it is required to divide by a negative integer to get the variable alone, the sign (< or >) needs to be changed.
- These solutions may need to be represented on a number line.

11. Quadratic inequalities

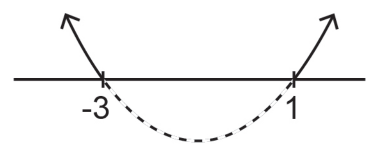
Points to remember when solving inequalities:

- If you multiply or divide by a negative number, the sign changes (< becomes > etc)
- Because you are solving for a variable, you can NEVER multiply or divide by a variable in an inequality as you don't know whether it is positive, negative or zero.

<u>Example</u>

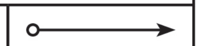
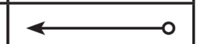
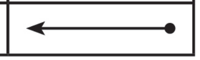
$$x^2 + 2x - 3 \geq 0$$
$$(x + 3)(x - 1) \geq 0$$

- Find the critical values (these are NOT the solutions – merely the values that will assist as they are in a quadratic equation)
- Mark the critical values on a number line (remember that these values represent the x-intercepts of the quadratic function)
- Sketch the function
- Find the part of the function that matches the inequality in the question (in this case greater than or equal to zero. This is the positive part of the function above the x-axis)



Solution: $x \leq -3$ or $x \geq 1$

Inequalities, Interval Notation and Representation on a number line

Inequality sign	words	Open/closed dot	
>	Greater than	Open	
≥	Greater than or equal to	Closed	
<	Less than	Open	
≤	Less than or equal to	Closed	

Interval Notation is used to represent a set of Real Numbers as it is impossible to list them.

PATTERNS, SEQUENCES AND SERIES

Sequence: A set of numbers written in order according to a mathematical rule.

The terms of a sequence are indicated by the symbol T_n

Example, T_2 is the second term of the sequence.

T_n, the n^th^ term gives the rule for the sequence.

A sequence that goes up or down in equal steps is called an arithmetic sequence.

In an arithmetic sequence, a constant value is either added or subtracted to generate the next term in the sequence.

The difference between any 2 terms in an arithmetic sequence is known as the common difference.

Linear patterns

All these patterns have a common difference between each term. In other words,

$$T_2 - T_1 = T_3 - T_2$$

The general term for a linear pattern can be written as

$$T_n = an + q \quad \text{or} \quad T_n = an + b$$

This form is like the standard form of the straight-line graph which shows it is a linear pattern.

To find the general pattern (also known as the n^{th} term) use: $T_n = a + (n - 1)d$ $(a = T_1 \quad d$ = common difference) Example: Find the general term for the pattern 4 7 10 13… Common difference: 3 (7 – 4 = 3 and 10 – 7 = 3)	$T_n = a + (n - 1)d$ $T_n = 4 + (n - 1)(3)$ $T_n = 4 + 3n - 3$ $\therefore T_n = 3n + 1$
Given the position, looking for the term: substitute n with position given and find T_n Example. Find the 20^{th} term of the above pattern	$T_n = 3n + 1$ $T_{20} = 3(20) + 1$ $= 61$
Given the term, looking for the position: make an equation and solve for n (substitute in T_n) Example: In which position will the term 151 be in the above pattern?	$T_n = 3n + 1$ $151 = 3n + 1$ $50 = n$ 151 is the 50^{th} term

Quadratic sequences

In a quadratic sequence the *second difference* is constant.

The first differences of a quadratic sequence form a linear sequence.

8   20   38   62

12   18   24   (first line of difference)

6   6 ⟶ second difference

$T_n = an^2 + bn + c$

$a \rightarrow$ half the second difference (2a = 2nd difference)

$b \rightarrow$ a constant ($3a + b$ = 1st term of 1st line of difference)

$c \rightarrow$ a constant ($a + b + c$ = 1st term of original sequence)

$n \rightarrow$ position of the n^{th} term in the sequence

For the above example:

$2a = 6$ $a = 3$	$3a + b = 12$ $3(3) + b = 12$ $9 + b = 12$ $b = 3$	$a + b + c = 8$ $3 + 3 + c = 8$ $6 + c = 8$ $c = 2$
$T_n = 3n^2 + 3n + 2$		

NB: You need at least 4 terms before you can assume that you have a quadratic sequence. If you are given 3 terms, a geometric sequence should be considered.

Geometric patterns (exponential)

In a geometric sequence the ratio is constant (each term is multiplied by the ratio to find the following term)

General term: $T_n = a.r^{n-1}$ and $r = \frac{T_2}{T_1} = \frac{T_3}{T_2}$

$a \rightarrow$ is the first term $r \rightarrow$ ratio

$n \rightarrow$ position of the n^{th} term in the sequence

To find the general term of these patterns, use: $T_n = a.r^{n-1}$

(where: a = the first term and r = common ratio)

∴ for the pattern: 4 8 16 32

$T_n = 4.2^{n-1}$

$= 2^2.2^{n-1}$

$= 2^{2+n-1} = 2^{n+1}$

If it seems that you are dividing each time to get the next term, it means you are multiplying by a fraction (inverse function of division is multiplication)

Series and Sequences

A sequence is an ordered list of numbers.

A series is a sequence in which all the terms have been added together.

$a \rightarrow$ first term $d \rightarrow$ common difference $r \rightarrow$ constant ratio	$T_1 \rightarrow$ Term 1 $S_1 \rightarrow$ Sum of first term (so $T_1 = S_1$) $S_n \rightarrow$ sum to n terms ($S_3 = T_1 + T_2 + T_3$)

Arithmetic Sequences and Series

- Has a constant difference ($T_2 - T_1 = T_3 - T_2$)
- Can be written as: $T_n = a + (n - 1)d$
- The sum of the first n terms of a sequence is given by:
  $S_n = \frac{n}{2}[2a + d(n - 1)]$ OR $S_n = \frac{n}{2}(a + l)$

Geometric Sequences and Series

- Has a constant ratio
- Can be written as $T_n = a.r^{n-1}$
- The sum of the first n terms of a sequence is given by:
  $S_n = \frac{a(r^n - 1)}{r - 1}$ OR $S_n = \frac{a(1 - r^n)}{1 - r}$
- If $-1 < r < 1$ then it makes sense to find the sum to infinity of the geometric series. The formula for this infinite sum is: $S_\infty = \frac{a}{r - 1}$

Infinite Series

This is a series that goes on without ending. There is no last term.
An infinite series cannot be evaluated by adding terms, since it is not possible to add infinitely many non-zero numbers.

Infinite geometric series

An infinite arithmetic series will always *diverge*, whereas an infinite geometric series can (under certain circumstances) *converge*.

Sum to infinity of a geometric series

For a geometric series to converge, the constant ratio must lie between –1 and 1 ($-1 < r < 1$). This ensures that r^n gets closer and closer to zero as n gets bigger.
Although an infinite geometric series never ends, it is possible to calculate a value when the series converges (it approaches a specific value).
If a series keeps growing infinitely bigger or infinitely smaller, it is NOT possible to calculate a value for them and we say that this type of series diverges.
Example of diverging infinite geometric series:

$2 + 4 + 8 + 16$……. (infinitely bigger)

OR

$-2 - 4 - 8 - 16$……. (infinitely smaller)

Example of a converging infinite geometric series:

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} \ldots\ldots\ldots$$

(the more terms that are added, the closer the last term gets to zero; it therefore converges and the final answer to this series CAN be calculated)

Sigma notation

$\sum$ - signifies that we are dealing with the <u>sum</u> of the terms of a sequence (a different way of writing a series)

$$\sum_{k=1}^{n} T_k$$

This is a short way of writing: $T_1 + T_2 + T_3 \ldots T_n$

Example:

$$\sum_{n-1}^{5}(4n-1)$$

This means:

- You will build the terms of the sequence using the general term $(4n-1)$
- You will start building the sequence by using the number below the sigma sign (in this case $n = 1$)
- Continue to build the sequence using all the natural numbers until you reach the number shown above the sigma sign (in this case $n = 5$)

The series is 3 + 7 + 11 + 15 + 19 = 55

Therefore,

$$\sum_{n=1}^{5}(4n-1) = 55$$

Sigma notation helps in writing down the sum of:

- An arithmetic series

$$\sum_{k=0}^{n-1}(a+kd) = \frac{n}{2}(2a+d(n-1))$$

- A Geometric series

$$\sum_{k=0}^{n-1}a.r^k = \frac{a(1-r^n)}{1-r}$$

If the last term of a sequence is known, you can use

$$S = \sum_{i=l}^{n}a_i = \frac{n}{2}(a+l)$$

If the last term is unknown you use

$$S = \sum_{t=l}^{n}a + d(i-l) = \frac{n}{2}[2a+(n-1)d]$$

Tips to answering questions in this section:

- Read the question carefully to ensure you know whether you are dealing with an arithmetic or a geometric sequence.
- When given sigma notation, write down the first 3 terms of the sequence. This will help you find a and d (arithmetic) or a and r (geometric).
- Always write down the list of all possible variables and fill in any values given.
- Words such as sum/total/evaluate all indicate that the sum formulae will be used.
- When working with sum to infinity, the condition $-1 < r < 1$ is often used in the question.

POLYNOMIAL FUNCTIONS

Long division

Example: $(2x^3 - 3x^2 + x + 15) \div (2x+3)$

$$\begin{array}{r} x^2 - 3x + 5 \\ 2x+3 \overline{)\, 2x^3 - 3x^2 + x + 15} \\ \underline{-2x^3 + 3x^2} \qquad\qquad \\ -6x^2 + x \qquad \\ \underline{-(-6x^2 - 9x)} \qquad \\ 10x + 15 \\ \underline{-(10x + 15)} \\ 0 \end{array}$$

$\therefore 2x^3 - 3x^2 + x + 15 = (2x+3)(x^2 - 3x + 5)$

Remember that variables must be in descending powers and if one or more terms are missing, a place holder is required.

Remainder Theorem

If $f(x)$ (a polynomial) is divided by $(x - a)$ (a linear polynomial) then the remainder is $f(a)$.

Explained in 'numbers' :

If $\frac{x^3 - 16x + 4}{x - 2}$

$f(x) = x^3 - 16x + 4$ $\therefore$ remainder is $f(2)$ (make $x - 2 = 0$ and solve)

$f(2) = 2^3 - 16(2) + 4 = -20$

<u>Example 1</u>

If $f(x) = -2x^3 + ax^2 - 4x + 3$ is divided by $(x + 3)$, the remainder is 15.

Find 'a'

Step 1: Let the divisor equal zero and solve for x.

$x + 3 = 0$

$x = -3$

Step 2:

Find $f(-3)$ and make this equal to the remainder. Use this to find 'a'.

$f(-3) = -2(-3)^3 + a(-3)^2 - 4(-3) + 3$

$15 = -2(-3)^3 + a(-3)^2 - 4(-3) + 3$

$15 = -54 + 9a + 12 + 3$

$54 = 9a$

$6 = a$

<u>Example 2</u>

The remainder when $g(x) = 16ax^3 - 2bx + 5$ is divided by $(x + 1)$ is -9.

If it is divided by $(2x - 1)$, the remainder is 6. Find 'a' and 'b'.

Step 1: Make the divisors equal to zero and solve for x

$x + 1 = 0$

$x = -1$

$2x - 1 = 0$

$x = \frac{1}{2}$

Step 2: Follow step 2 from previous example, but with BOTH pieces of information.

$g(x) = 16ax^3 - 2bx + 5$

$g(-1) = 16a(-1)^3 - 2b(-1) + 5$

$-9 = 16a(-1)^3 - 2b(-1) + 5$

$-9 = -16a + 2b + 5$

$-14 = -16a + 2b$

$-7 = -8a + b$

$g(x) = 16ax^3 - 2bx + 5$

$g\left(\frac{1}{2}\right) = 16a\left(\frac{1}{2}\right)^3 - 2b\left(\frac{1}{2}\right) + 5$

$6 = 16a\left(\frac{1}{2}\right)^3 - 2b\left(\frac{1}{2}\right) + 5$

$6 = 2a - b + 5$

$1 = 2a - b$

Solve simultaneously.

$-7 = -8a + b$

$8a - 7 = b$

$1 = 2a - b$

$1 = 2a - (8a - 7)$

$1 = -6a + 7$

$-6a = -6a$

$a = 1$

$\therefore 8(1) - 7 = b$

$1 = b$

Factor Theorem

When a polynomial has been divided by another polynomial and the remainder is zero, the divisor must be a factor of the dividend.

<u>Example 1</u>

Show that $(x + 1)$ is a factor of $f(x) = 2x^3 - 2x^2 - 10x - 6$

(Use the remainder theorem and show that the remainder is zero)

$f(-1) = 2(-1)^3 - 2(-1)^2 - 10(-1) - 6$

$f(-1) = 0$ $\quad\quad \therefore x + 1$ is a factor of $f(x)$.

<u>Example 2</u>

Determine the value of 'p' if $(x + 2)$ is a factor of $x^3 + px + (3 - p)$

[Remember: 'is a factor' means after division the remainder is zero]

Step 1: Make divisor equal to zero and solve

$x = -2$

Step 2: Find $f(-2)$, make it equal to zero and solve for missing variable

$$f(-2) = (-2)^3 + p(-2) + (3 - p)$$
$$0 = -8 - 2p + 3 - p$$
$$p = -\frac{5}{3}$$

Solving cubic equations

Steps to follow:

- Write in standard form
- Factorise (using division and theorems learnt)
- Make factors equal to zero and solve for each factor (cubic equation – 3 solutions)
- You may need to use the quadratic formula

If there are only two terms – one a cube and one a constant – you do not need to factorise.

Example $\quad x^3 + 10 = 0$

$$x^3 = -10$$
$$x = \sqrt[3]{-10} = -2{,}154\ \ldots$$

LOGS

Logs are directly related to exponents.

If $m^n = a$ then $log_m a = n$

Examples (exponential form ↔ log form)

$100 = 10^2$ $\therefore log_{10} 100 = 2$	$25 = 5^2$ $\therefore log_5 25 = 2$
$log_2 32 = 5$ $\therefore 2^5 = 32$	$log_7 49 = 2$ $\therefore 7^2 = 49$

Laws, Definitions and deductions

1. $log_a mn = log_a m + log_a n$
2. $log_a \frac{m}{n} = log_a m - log_a n$
3. $log_a x^m = m\ log_a x$

(These 3 laws only work if the bases are the same)

4. $log\ 1 = 0$

(Any base, except zero, to the power zero = 1)

5. $log_b a = \frac{log\ a}{log\ b}$
6. $log_a a = 1$
7. $log_{\frac{1}{x}} a = -log_x a$

NOTES:

- A log is an exponent $\therefore$ all logs must have a base
- You cannot take the log of zero or a negative number
- Whatever is inside a log must always be greater than zero
  ($\log_a m \rightarrow m > 0$)
  (This is especially important when solving equations)
- $\log a > 0$ for $a > 1$ and $\log a < 0$ for $0 < a < 1$
- $\log\frac{27}{3} \neq \frac{\log 27}{\log 3}$

Equations

Exponential equations/calculator work

$4^x = 5$	Can't get the same base so use logs
$\log 4^x = \log 5$	Use log laws
$x \log 4 = \log 5$	Divide on both sides to solve
$x = \frac{\log 5}{\log 4}$ $= 1{,}16$	Calculator work

When solving log equations, write down any restrictions first.
(Remember: whatever is inside a log must always be greater than zero)

Exponential equations with equal bases

$\log_3 x + \log_3(x + 6) - 3 = 0$	State the restrictions
$x > 0$ and $x + 6 > 0$ $x > -6$ $\therefore x > 0$	Get all log terms on LHS
$\log_3 x + \log_3(x + 6) = 3$	Use log laws to simplify
$\log_3 x(x + 6) = 3$	Use rules to remove logs
$3^3 = x(x + 6)$	Solve resulting equation
$x^2 + 6x - 27 = 0$ $(x + 9)(x - 3) = 0$ $x = -9$ or $x = 3$	Check restrictions and choose final answer
$\therefore x = 3$	

FINANCE AND GROWTH

Simple Interest $A = P(1 + in)$ **Compound Interest** $A = P(1 + i)^n$	A = Final amount P = Principal amount i = interest rate n = number of times interest is calculated* * In simple interest it is always annually.
Hire Purchase (buying an item from a shop on credit – you are officially hiring the item until the final payment when you have finally purchased it)	● Always use simple interest formula $A = P(1 + i.n)$ ● If insurance is required, it is always on the total purchase price regardless of deposits paid ● Deposits are subtracted from purchase price to find amount needed to be 'borrowed'.
Inflation (The increase in an item over the course of time)	● Always use compound interest formula ● When working towards a previous time period then you usually have 'A' and are looking for 'P'.
Exchange rates	The rate of one country's money against another country's money. Use ratios to convert between one currency and another.

Effective and nominal Interest (Nominal Interest is what you are quoted from the bank/institution. Effective interest is what you *actually* gain (if it's a savings situation) or *actually* pay (if it's a loan situation) due to the interest being compounded.	To convert between nominal and effective: $i_{eff} + 1 = \left(1 + \frac{i}{m}\right)^m$ i_{eff} = effective rate i_{nom} = nominal rate m = number of compounding periods/year
Depreciation Assets (cars, machinery etc) reduce in value over time.	Depreciation on a straight-line balance (it will eventually be worth nothing): $A = P(1 - i.n)$ Depreciation on a reducing balance: $A = P(1 - i)^n$

Reducing balance loans

When taking a loan from a bank, interest is paid on the reducing balance. In other words, the lower the balance, the less interest you pay. If you pay extra money into these loans, you can save a lot of money on interest as well as pay the loan off sooner.

This type of loan is covered in Annuities.

Annuities

An annuity is an investment. It is made up of a series of equal payments (or repayments) at regular intervals and subject to a rate of interest.

Annuities

Regular payments are made by the investor into the annuity in order to accumulate money.	Regular payments are made from the annuity to the investor until there is no money left in the account.
$F = \frac{x[(1+i)^n - 1]}{i}$	The amount of money needed in the account to cover all the payments is called the present value of the annuity. $P = \frac{x[1-(1+i)^{-n}]}{i}$
i = *interest rate per payment interval*	i = *interest rate per payment interval*
n = *number of regular payments*	n = *number of regular payments*
x = *amount of regular payment*	x = *amount of regular payment*
(These payments are made by the investor)	(These payments are made to the investor)
This is an INCREASING annuity	

Bond Repayments

This is when you borrow money to buy a property, at a given interest rate and then make regular payments to pay off the original amount of money borrowed as well as any interest that has arisen.

The amount to be paid each month is calculated using:

$$P = \frac{x[1-(1+i)^{-n}]}{i}$$

It is the same formula as the present value annuity because you can think of a bond as an annuity with a known present value.

Summary of the three situations that affect the value of *n*

1	Pay at the end of each month (one month after the annuity begins) until the final month. On a timeline: T_1 No complications – use the formula as it is.
2	Pay now (immediately) and then at the end of every month. On a timeline: T_0 There will be one extra payment (n + 1)
3	Pay now and continue paying a month in advance. On a timeline: T_0 The final payment will 'sit' on the bank for another month and earn some interest. Treat it like the first situation BUT 'grow' the answer using compound interest for another month.

Sinking Funds

This is a fund that a business may set up to pay for future expenses. A fixed amount is paid in on a regular basis and interest is earned. A sinking fund is an increasing annuity.

FUNCTIONS

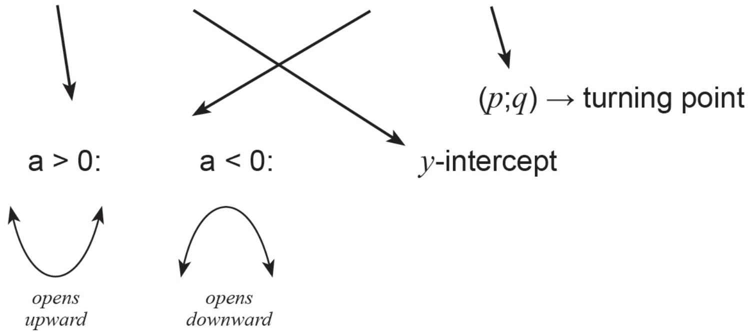
There are two types of functions:

ONE-TO-ONE	MANY-TO-ONE
A single x-value for a particular y-value	More than one x-value for a particular y-value
THERE CAN ONLY BE ONE y-VALUE	

The straight-line graph (Linear function)

Standard form:	$y = ax + q$ Gradient: $a > 0$: / $a < 0$: \\ Vertical shift (up/down): y-intercept
To draw	Find the x-intercept (make $y = 0$) Find the y-intercept (make $x = 0$)
To find the equation	● Given the y-intercept and another point: Substitute y-intercept for 'q' Substitute other point $(x;y)$ to find 'a' ● Given two points Use two points to find gradient ('a') Use any point to substitute and find 'q' Note: check the values of 'a' & 'q' according to what they represent (for example, if you have found that $a < 0$, check that the line has a negative slope)
Domain and Range	Domain (all possible x-values on the function): $x \in R$ Range (all possible y-values on the function): $y \in R$
Other	● If 2 lines are parallel, then $m_1 = m_2$ ● If 2 lines are perpendicular, then $m_1 \times m_2 = -1$ ● A line perpendicular to the x-axis and parallel to the y-axis (a vertical line): the equation will be in the form $y = c$ ● A line perpendicular to the y-axis and parallel to the x-axis (a horizontal line): the equation will be in the form $x = c$

The parabola (Quadratic function)

Standard form:	$y = ax^2 + bx + c$ or $y = a(x - p)^2 + q$ 
To draw	Find the x-intercept (make $y = 0$) Find the y-intercept (make $x = 0$) Find the axis of symmetry: $x = \frac{-b}{2a}$ Find the turning point: substitute the value of x from the axis of symmetry into the equation to find the corresponding y-value.

To find the equation	• Given the x-intercepts and another point: $y = a(x - x_1)(x - x_2)$ Substitute x-intercepts for x_1 and x_2 Substitute other point $(x ; y)$ to find 'a' • Given turning point and another point: $y = a(x - p)^2 + q$ Substitute the turning point for p and q Substitute other point $(x ; y)$ to find 'a' Note: check the values of 'a' according to what it represents (for example, if you have found that $a < 0$, check that the parabola opens downwards/is upside down)
Domain and Range	Domain (all possible x-values on the function): $x \in R$ Range (all possible y-values on the function): If $a > 0$: $y \in [q ; \infty)$ If $a < 0$: $y \in (-\infty ; q]$
Other	Parabolas can have a minimum or a maximum value. • If $a > 0$, there is a minimum value The minimum value is $y = q$ • If $a < 0$, there is a maximum value The maximum value is $y = q$

Examples: Finding equations of parabolas

Finding the equation of a parabola given the TURNING POINT and another point.

Use	Example:
$y = a(x-p)^2 + q$	The turning point of a parabola is (2 ; 6) and it also passes through the point (5 ; -30). Find the equation of the parabola.
Substitute turning point into $(p ; q)$	$y = a(x-2)^2 + 6$
Substitute other point $(x ; y)$	$-30 = a(5-2)^2 + 6$ (1*)
Solve for a	$-30 = a(9) + 6$ $-4 = a$
Substitute 'a' back into (1*)	$y = -4(x-2)^2 + 6$
Multiply out and collect like terms ($y = ax^2 + bx + c$)	$y = -4(x^2 - 4x + 4) + 6$ $y = -4x^2 + 16x - 16 + 6$ $y = -4x^2 + 16x - 10$

Finding the equation of a parabola given the x-INTERCEPTS and another point.

Use	Example:
$y = a(x-x_1)(x-x_2)$	A parabola passes through the point (-2 ; 0), (5 ; 0) and (0 ; 5). Find the equation of the graph.
Substitute 2 values of x-intercepts into x_1 and x_2	$y = a(x+2)(x-5)$ (1)*
Multiply out	$y = a(x^2 - 3x - 10)$ (1)*
Substitute the other coordinate $(x ; y)$ and solve for a.	$5 = a(-10)$ $\frac{-1}{2} = a$
Substitute 'a' back into (1)*	$y = \frac{-1}{2}(x^2 - 3x - 10)$
Multiply out and collect like terms ($y = ax^2 + bx + c$)	$y = \frac{-1}{2}x^2 + \frac{3}{2}x + 5$

Minimum and maximum values of quadratic expressions

Once you have completed the square of an expression and have it in the form:

$$a(x-p)^2 + q$$

If $a > 0$, there will be a MINIMUM value
If $a < 0$, there will be a MAXIMUM value

This will be the MAXIMUM or MINIMUM value

Example 1: $-2(x-1)^2 + 4$

Has a maximum value

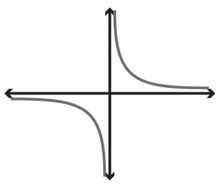
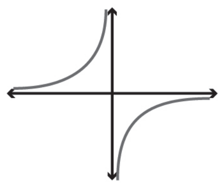
which is

Example 2: $\frac{1}{2}(x-5)^2 - 5$

Has a minimum value

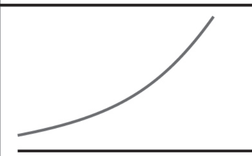
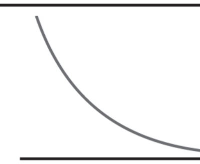
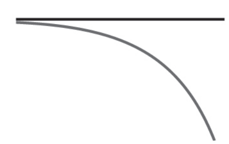
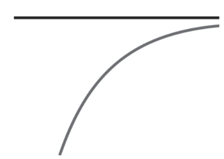
which is

The hyperbola (Hyperbolic function)

Standard form:	$y = \frac{a}{x-p} + q$ q: Vertical shift (up/down) Horizontal asymptote ($y = q$) p: Horizontal shift (left/right) Vertical asymptote ($x = p$) $a > 0$:  $a < 0$: 
To draw	Draw in the horizontal asymptote: $y = q$ Draw in the vertical asymptote: $x = p$ Find the x-intercept (make $y = 0$) Find the y-intercept (make $x = 0$)
To find the equation	Given the asymptotes and another point: Substitute the value of the asymptotes for 'p' and 'q' Substitute other point (x ; y) to find 'a' Note: check the values of 'a' according to what it represents (for example, if you have found that $a < 0$, check that the hyperbola is in the correct quadrants)
Domain and Range	Domain (all possible x-values on the function): $x \in R; x \neq p$ Range (all possible y-values on the function): $y \in R; y \neq q$
Other	A hyperbola has two axes of symmetry. ● One has a gradient of '1' and the other has a gradient of '-1' ● They both pass through the point where the asymptotes meet. (p ; q)

The exponential graph (Exponential function)

Standard form:	$y = a.b^{x-p} + q$ $(a \neq 0 ; b \neq 0)$ p: Horizontal shift (left/ right) q: Vertical shift (up/down) Horizontal asymptote ($y = q$)

	b > 1	0 < b < 1
a > 0		
a < 0		

To draw	Decide whether it lies above or below the asymptote. (If $a > 0$, it lies above the asymptote and if $a < 0$ it lies below the asymptote) Decide whether it is increasing or decreasing by considering 'a' and 'b'. Draw in the horizontal asymptote: $y = q$ Find the y-intercept (make $x = 0$) Find the x-intercept (make $y = 0$) Find a few more points if necessary with possible x-values.
To find the equation	The amount of information required is always directly related to the number of variables missing. You are likely to be given the 'format' of the graph. Given the asymptote and another point: Substitute the value of the asymptote for 'q' Substitute other point $(x ; y)$ to find 'a' or 'b' Simultaneous equations may be necessary.
Domain and Range	Domain (all possible x-values on the function): $x \in R$ Range (all possible y-values on the function): If $a > 0$: $y \in [q ; \infty)$ If $a < 0$: $y \in (-\infty ; q]$

General information regarding functions

1. Find the values of x for which:

$f(x) = g(x)$	Make the equations equal and solve for x. If the coordinates are asked for, substitute the x-value(s) into any function and solve for y.
For each of the questions below: ● First find the part of the graph that answers the question (highlight it if possible) ● Find the x-values that correspond to the part of the graph that satisfies the statement.	
$f(x) > g(x)$	Where is the function $f(x)$ greater than (in other words above) the function $g(x)$.
$f(x) < g(x)$	Where is the function $f(x)$ less than (in other words below) the function $g(x)$.
$f(x) \geq g(x)$	Where is the function $f(x)$ greater than (in other words above) or equal to the function $g(x)$.
$f(x) \leq g(x)$	Where is the function $f(x)$ less than (in other words below) or equal to the function $g(x)$.

2. Average Gradient

This is the average gradient between two points on a curve

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

3. Transformations of functions
   - Reflections

Reflection in the x-axis ($y = 0$)	Rule: $(x\,;y) \rightarrow (x\,;-y)$ In other words – leave the x-value the same and change the y-value to negative
Reflection in the y-axis ($x = 0$)	Rule: $(x\,;y) \rightarrow (-x\,;y)$ In other words – leave the y-value the same and change the x-value to negative

INVERSE FUNCTIONS

An inverse function is a reflection of the function in the line $y = x$

Rule: $(x\,;y) \rightarrow (y\,;x)$

Example: $f(x) = -3x + 4$

Use the rule for reflecting in the line $y = x$

$$x = -3y + 4$$

Make y the subject (std form)

$$3y = -x + 4$$

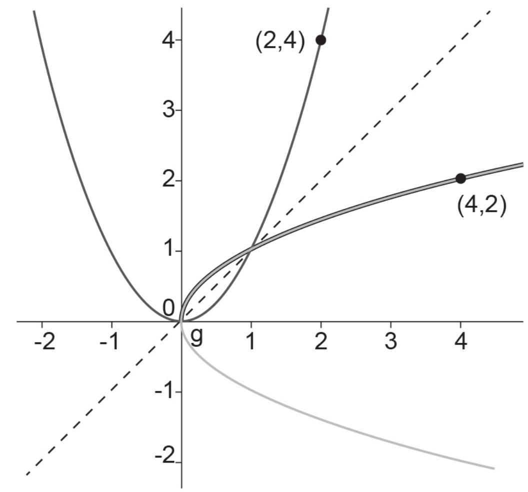
$$y = -\frac{1}{3}x + 4$$

$f(x) \rightarrow$ denotes a function $\qquad f^{-1}(x) \rightarrow$ denotes the inverse of a function

So: If $f(x) = -3x + 4$ then $f^{-1}(x) = -\frac{1}{3}x + 4$

Quadratic functions and their inverses

$f(x) = \frac{1}{2}x^2$

$\therefore\ x = \frac{1}{2}y^2$ $2x = y^2$ $\therefore\ y = \pm\sqrt{2x}$	Sketch:  The inverse of $f(x)$ is NOT a function. There would have to be restrictions on the domain of the original function for the inverse to be a function.

Exponential functions and their inverses (logarithmic functions)

$y = a^x$ (exponential)

Its inverse:

$x = a^y$

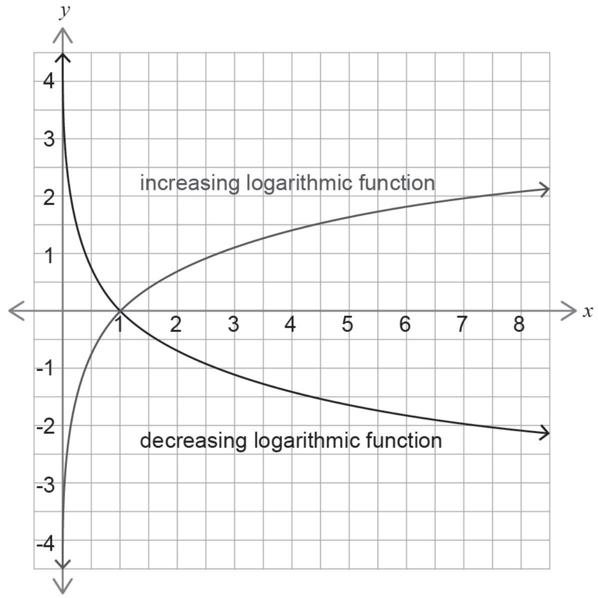
$\therefore$ y = $\log_a x$ $\quad$ ($x > 0$; remember that any positive number raised to an exponent cannot be negative AND $a \neq 0$; $a \neq 1$; $a > 0$)

Steps to sketching log functions.

- Test for shape

  $0 < a < 1$ (decreasing)

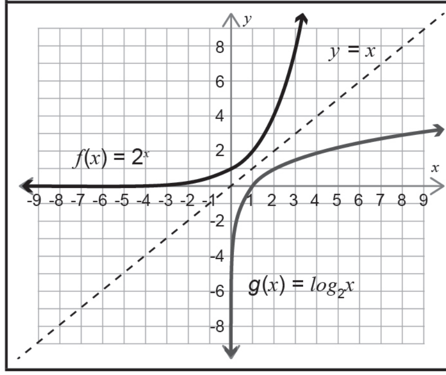
  $a > 1$ (increasing)



- Find x-intercept ($y = 0$)
- Substitute in one other value for another point.

Example:

Sketch the graphs of: (i) $f(x) = 2^x$ (ii) $g(x) = f^{-1}(x)$ (iii) $y = x$

$f(x) = 2^x$ $a > 1$ $\therefore$ increasing y-intercept ($x = 0$): $(2)^0 = 1$ Another point: If $x = -2$; $y = \frac{1}{4}$ $(-2 ; \frac{1}{4})$	$f^{-1}(x)$: $x = 2^y$ $y = log_2 x$ $y = log_2 x$ $0 < a < 1$ $\therefore$ decreasing x-intercept ($y = 0$): $0 = log_2 x$ $(2)^0 = x$ $x = 1$ Another point: If $x = 2$; $y = 1$ $(2 ; 1)$
	The line $y = x$ is the axis of symmetry for functions and their inverses.

Summary of functions and inverses (f^{-1})

	Function (f)	Inverse (f^{-1})	Standard Form
Linear	$y = mx + q$	$x = my + q$	$y = \frac{x-q}{m}$
Example	$y = 3x + 2$	$x = 3y + 2$	$y = \frac{1}{3}x - \frac{2}{3}$
Quadratic	$y = ax^2$	$x = ay^2$	$y = \pm\sqrt{\frac{x}{a}}$
Example	$y = 3x^2$	$x = 3y^2$	$y = \pm\sqrt{\frac{x}{3}}$
Exponential	$y = a^x$	$x = a^y$	$y = log_a x$
Example	$y = 3^x$	$x = 3^y$	$y = log_3 x$ (x > 0)

Remember: The inverse of a function is sometimes not a function

DIFFERENTIAL CALCULUS

Limits

The limit of a function, $f(x)$, is the value of k that the function approaches from both LEFT and RIGHT as x tends to a given value n.

$$\lim_{x \to n} f(x) = k$$

[The limit of the function as x tends to n is equal to k]

Example, $\lim_{x \to 3}(x - 4) = -1$

The derivative of a function ***represents the gradient of the tangent at any given point*** on the function.

Derivatives

The derivative is the rate of change at a point. It is therefore the gradient at a point (or the instantaneous speed when discussing distance and time)

The derivative of $f(x)$ is: $f'(x) = \lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$

[The gradient at any point on $f(x)$]

- The derivative of any constant is zero
- $f'(x)$ gives the value of the slope (gradient) of the tangent line to the graph $y = f(x)$ at x.

REMEMBER:

$\frac{\Delta y}{\Delta x}$ is the average gradient (approximate slope) of $f(x)$ between two points

$\frac{dy}{dx}$ is the actual slope of $f(x)$ at a point

If $y = 2x^2$ then $\frac{dy}{dx} = 2x$

This is the 'formula' for the gradient of the tangent at any point on the curve $y = 2x^2$.

The gradient can be found for any value of x.

If $f'(x) = 0$ then the tangent line is horizontal (it has a gradient of zero) at the point where $x = a$. The tangent line will be at a turning point in this case.

Notation for the derivative

$f'(x)$ (f prime x or f dash x – the derivative of the function x)

$\frac{dy}{dx}$ (derivative of the function y in terms of x)

$\frac{d[f(x)]}{dx}$ and $D_x[f(x)]$ can also be used.

Finding the derivative from first principles

- DO NOT use the rules (shortcut) unless it is at the end to check your answer.
- Always write down the function ($f(x)$)
- Find $f(x + h)$. To do this replace any x in the function with $(x + h)$ and multiply out and simplify.
- Write down the formula
- Replace $f(x)$ and $f(x + h)$ with what you found in the previous step
- Simplify
- Watch your setting out!!
- Keep $\lim_{h \to 0}$ until the VERY last step when you finally replace the h with zero.
- Keep equal signs down the left and underneath each other from the beginning until the final step.

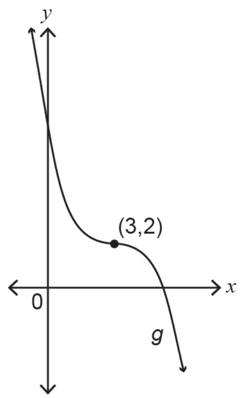
Finding the derivative of more complex functions using the rules of differentiation

- Split expression up into multiple terms
- Remove brackets
- There must be no variables in the denominator – use the rules of exponents to 'move' the variables to the numerator position
- There must be no surds. Change into exponential form
- Differentiate each term
  Rule: Multiply exponent with coefficient and subtract one from exponent.
  $$f(x) = x^a \quad \therefore \quad f'(x) = ax^{a-1}$$

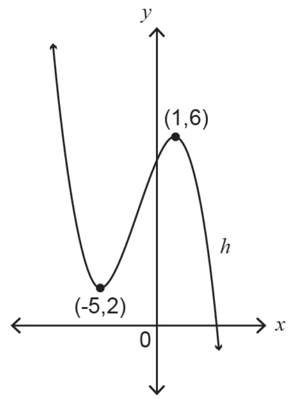
Sketching a cubic graph

- Make a rough sketch. If $a > 0$, the function will start by increasing. If $a < 0$, the function will start by decreasing.
- Find y-intercept (make $x = 0$)
- Find x-intercepts (make $y = 0$ and solve for x). If there is a repeated x-intercept, it will also be a turning point.
- Find the stationary points.
  Find the derivative of the function. Make it equal to zero and solve for x. Substitute these values back into ORIGINAL function to find corresponding y-values.
- Find the point of inflection.
  This is where the graph changes from concave up to concave down.
  Find the second derivative (derivative of the derivative). Make it equal to zero and solve for x. Sub this value back into ORIGINAL function to find corresponding y-value.

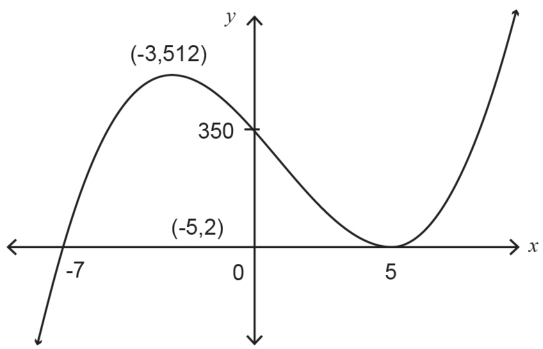
- Consider the x-intercepts. If there is only one the graph may look like this:



or



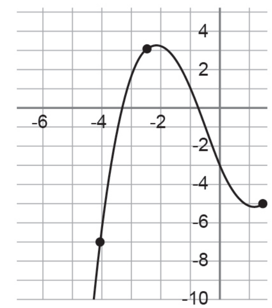
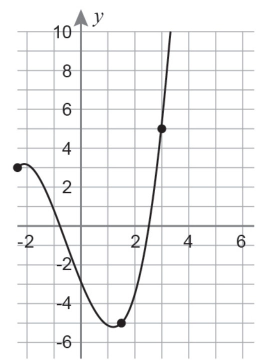
If two of the x-intercepts are the same, one of them (the repeated factor) will be the turning point and may look like this:

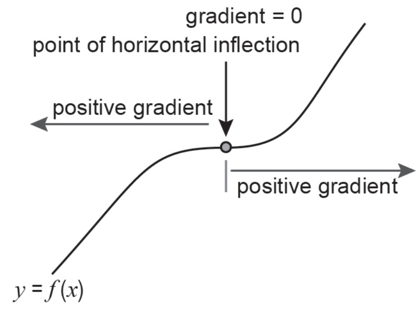


What the derivative tells us about the function

- It tells you if the function is increasing ($f'(x) > 0$) or decreasing ($f'(x) < 0$).
- If $f'(x) = 0$ there is a stationary point at x (and the tangent is horizontal) → [Not increasing or decreasing]

There are 3 main possibilities in this case:

The sign of $f'(x)$ changes from positive to negative	Local maximum at x Was increasing now decreasing Turning Point	
OR The sign of $f'(x)$ changes from negative to positive	Local minimum at x Was decreasing, now increasing Turning Point	

OR The sign of $f'(x)$ does not change	Was increasing, still increasing (as in example alongside) Was decreasing, still decreasing Point of inflection	

Remember: The derivative represents the gradient of the tangent to the function at ANY POINT.

If it is zero, it means the tangent is a horizontal line (flat) which in turn means it is at a stationary point.

Tangents to curves

To find the gradient of a tangent at a point:

- Differentiate the function
- Substitute the x-coordinate from the given point.

To find the equation of a tangent at a point:

- Find the gradient at the point
- Use $y - y^1 = m(x - x^1)$ with the gradient and the point to find the equation.

Optimisation

If you have a formula for what you are trying to maximise or minimise you can use calculus to help you find where your formula has a maximum or minimum.

Steps to follow:

- Decide what you are trying to optimise and find a formula for it.
- Isolate one of the variables, using the information given (same constraints/equations linking the variables). You can't differentiate a formula with more than one variable.
- Differentiate your formula with respect to the one variable and find the stationary points (make the derivative equal to zero and solve)
- Check which one gives you the required optimisation

Example: Find the maximum area of a rectangle whose perimeter is 100 metres

Determine the function that you need to optimize.

In the example, we need to optimize the area (A) of a rectangle, which is the product of its length l and breadth b. The function in this example is $A = lb$.

Identify the constraints to the optimization problem.

In our sample problem, the perimeter of the rectangle must be *100 metres*.

This will be useful in the next step.

Express that function in terms of a single variable upon which it depends, using algebra.

For this example, we're going to express the function in a single variable. "l"

1. A rectangle's perimeter is the sum of its sides, that is,
   $100m = 2l + 2b$
2. Subtract $2l$ from both sides of this equation, $100m - 2l = 2b$
3. Divide each side by 2: $b = 50m - l$
4. Substitute $50m - l$ for "b" in $A = lb$:

$$A = l\,(50m - l) = 50m\,l - l^2$$

Now the function that needs to be optimised is in terms of ONE variable. This is ESSENTIAL.

Calculate the derivative of the function with respect to a variable to find the critical points.

The derivative $\frac{dA}{dl} = 50m - 2l$

Set the function to zero and solve for the variable.

$$\frac{dA}{dl} = 0$$

$$0 = 50m - 2l$$

$$\therefore l = 25m.$$

Use the value found to calculate the corresponding optimal value of the function.

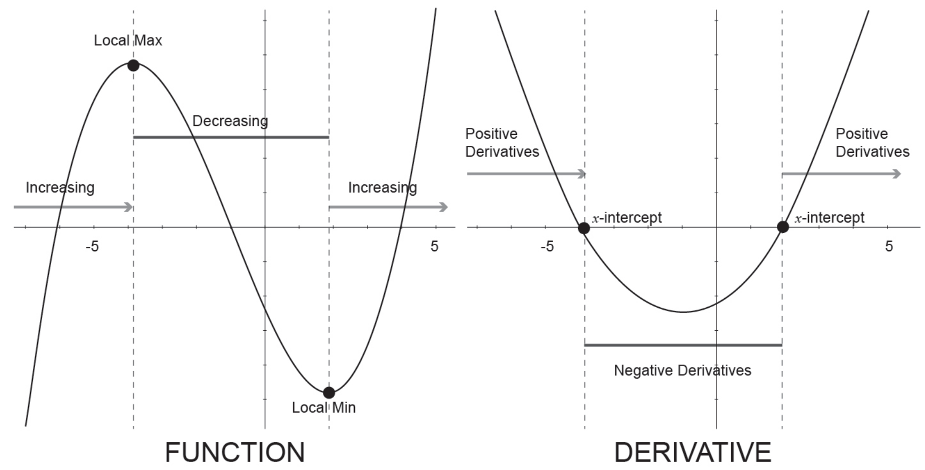
$$A = 50m\,l - l^2$$

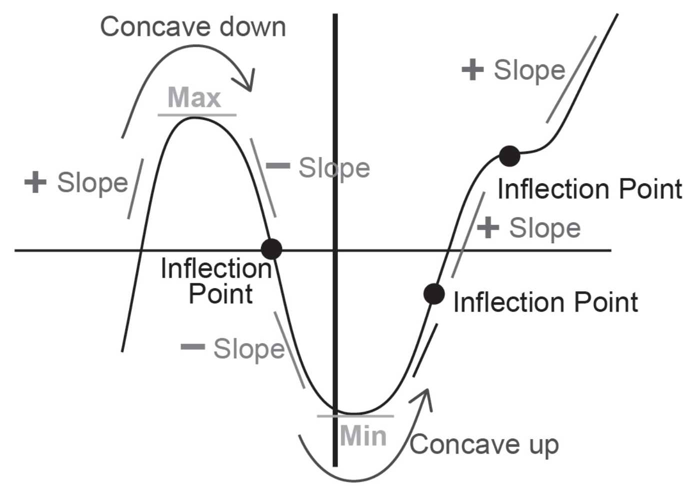
$$= 50m\,(25m) - (25m)^2$$

$$= 625m^2.$$

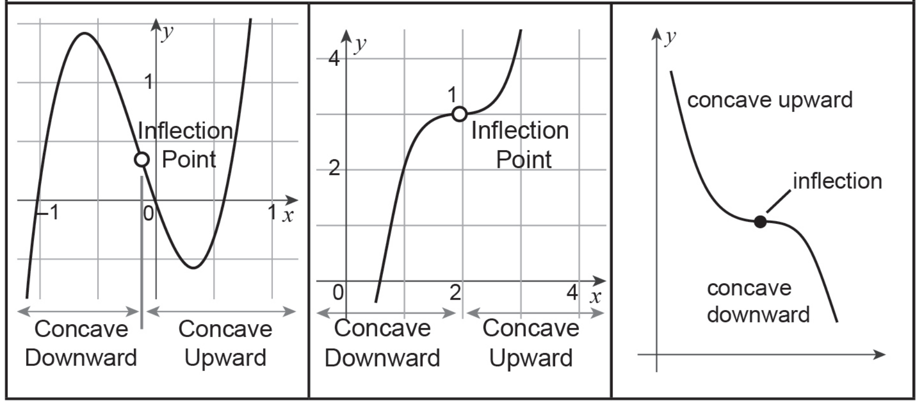
Function v Derivative

Compare and contrast the function and its derivative.





Points of Inflection



PROBABILITY

Probability is the likelihood of something happening or being true. Probability is assigned a value from 0 (impossible) to 1 (certain). The probabilities of the possible outcomes in a sample space must sum up to 1.

Probability of an event occurring and sample space

The probability of Event A occurring is: P(A) = $\frac{n(A)}{n(S)}$

In general, A is the total number of ways a specific event can occur. S is the total number of possible outcomes for the entire event.

Notation

A = {1;2;3;4;5} represents Event (Set) A.

- $n(A) \rightarrow$ the number of items in set A.
- $P(A) \rightarrow$ The probability of Event A occurring
- $P(A') \rightarrow$ The probability of Event A **NOT** occurring. It is also known as the complement of A
- $P(A \text{ or } B)$ = P(A ∪ B)→ The probability of A or B occurring. ∪ is the symbol for **'or'** and is also known as **union**.
- $P(A \text{ and } B)$ = P(A ∩ B)→ The probability of A and B occurring. ∩ is the symbol for **'and'** and is also known as **intersection**.

Inclusive events

Two events that can occur at the same time are inclusive.

$P(A \cap B) \neq 0$ $\qquad$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Mutually Exclusive events

Two events that are mutually exclusive cannot occur at the same time. There is no intersection.

$P(A \cap B) = 0$ $\qquad$ $P(A \text{ or } B) = P(A) + P(B)$

Exhaustive events

Two events A and B are exhaustive if together they cover all the elements of the sample space.

$P(A \text{ or } B) = 1$

Complementary events

Mutually exclusive, exhaustive events are **complementary** events. They are the only two possible outcomes. If one event does not occur, the other event must occur

$P(\text{not } A) = P(A') = 1 - P(A)$

The addition rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If the events are mutually exclusive then: $P(A \cup B) = P(A) + P(B)$, because $P(A \text{ and } B) = 0$.

Venn diagrams

Venn diagrams are a graphical way of representing a sample space and its events. If two events can *both* happen at the *same time*, then a Venn diagram is a good way to represent the situation.

Tree diagrams

When there are 2 or more consecutive events taking place, it is often useful to represent the possible solutions on a tree diagram. Tree diagrams are constructed by showing all possible events. They can be used for dependent or independent events. When dealing with tree diagrams always multiply along the branches (horizontal) and add probabilities moving down branches (vertical) at the end. Write the probability of an event occurring at the top of the branches and the actual event at the end of the branch.

Contingency tables

Contingency tables are statistical tables that show the relationship between 2 or more variables. They are often used to determine whether events are independent or not.

Dependent and independent events

Two events are independent if the outcome of one event does not affect the probability of another event occurring. If the outcome of one event changes the probability that another event occurs, the events are said to be dependent.

The product rule:

Test for independent events: $P(A \text{ and } B) = P(A) \times P(B)$

Remember: $P(A \text{ and } B)$ can be written as $P(A \cap B)$

The fundamental counting principle

The fundamental counting principle is a principle used to determine the number of different ways to accomplish different tasks. It states that:

If there are m different ways to perform a task and n different ways to perform a different task, the total number of different ways in which both tasks can be performed is $m \times n$.

Example

A local ice-cream parlour has 4 different flavours of ice-cream with a choice of 8 different toppings. If a customer chooses 1 topping and one flavour, how many different choices of dessert are there?

flavour   topping

_4_ × _8_ There are 32 choices.

Factorial notation

The factorial (!) of a natural number n is the product of the positive integers less than or equal to n.

$4! = 4 \times 3 \times 2 \times 1$

$n! = n \times (n-1) \times (n-2) \times (n-3) \times (n-4) \times \ldots \times 1$

$0! = 1$

Example: In how many different ways can the letters of the word EIGHT be arranged (repetition of letters are not permitted)?

_5_ × _4_ × _3_ × _2_ × _1_ There are 5! Ways (120)

Example:

Two different Science books, four different Geography books and two different Maths books are placed on a shelf.

How many ways can they be arranged? (8!)

How many ways can they be arranged if the books of the same subject must be placed together?

3! (for the three groups of books)

2! (the ways the 2 Science books can be arranged)

2! (the ways the 2 Maths books can be arranged)

4! (the ways the 4 Geography books can be arranged)

$\therefore 3! \times 2! \times 2! \times 4! = 576$

Repetition

When letters are repeated in a word, they could make the same word twice.

To alleviate this problem, divide by the factorial of the repeats.

Example: In how many different ways can the letters of the word SUCCESSFUL be arranged (repetition of letters are not permitted)?

$\underline{10} \times \underline{9} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

There are 10! ways IF there were no repeated letters.

Due to the repeated letters, there are $\frac{10!}{2!3!2!} = 151\ 200$

(2! For the 2 C's; 3! For the 3 S's; 2! For the 2 U's)

RESOURCE 2

PAST PAPER 1 2017: Week 1

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P1

NOVEMBER 2017

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages and 1 information sheet.

Please turn over

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 11 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs et cetera that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x^2 + 9x + 14 = 0$ (3)

1.1.2 $4x^2 + 9x - 3 = 0$ (correct to TWO decimal places) (4)

1.1.3 $\sqrt{x^2 - 5} = 2\sqrt{x}$ (4)

1.2 Solve for x and y if:
$3x - y = 4$ and $x^2 + 2xy - y^2 = -2$ (6)

1.3 Given: $f(x) = x^2 + 8x + 16$

1.3.1 Solve for x if $f(x) > 0$. (3)

1.3.2 For which values of p will $f(x) = p$ have TWO unequal negative roots? (4)
[24]

QUESTION 2

2.1 Given the following quadratic number pattern: 5 ; –4 ; –19 ; –40 ; …

2.1.1 Determine the constant second difference of the sequence. (2)

2.1.2 Determine the n^{th} term (T_n) of the pattern. (4)

2.1.3 Which term of the pattern will be equal to $-25\,939$? (3)

2.2 The first three terms of an arithmetic sequence are $2k - 7$; $k + 8$ and $2k - 1$.

2.2.1 Calculate the value of the 15^{th} term of the sequence. (5)

2.2.2 Calculate the sum of the first 30 even terms of the sequence. (4)
[18]

QUESTION 3

A convergent geometric series consisting of only positive terms has first term a, constant ratio r and n^{th} term, T_n, such that $\sum_{n=3}^{\infty} T_n = \frac{1}{4}$.

3.1 If $T_1 + T_2 = 2$, write down an expression for a in terms of r. (2)

3.2 Calculate the values of a and r. (6)
[8]

QUESTION 4

Given: $f(x) = -ax^2 + bx + 6$

4.1 The gradient of the tangent to the graph of f at the point $\left(-1 ; \frac{7}{2}\right)$ is 3.

Show that $a = \frac{1}{2}$ and $b = 2$. (5)

4.2 Calculate the x-intercepts of f. (3)

4.3 Calculate the coordinates of the turning point of f. (3)

4.4 Sketch the graph of f. Clearly indicate ALL intercepts with the axes and the turning point. (4)

4.5 Use the graph to determine the values of x for which $f(x) > 6$. (3)

4.6 Sketch the graph of $g(x) = -x - 1$ on the same set of axes as f. Clearly indicate ALL intercepts with the axes. (2)

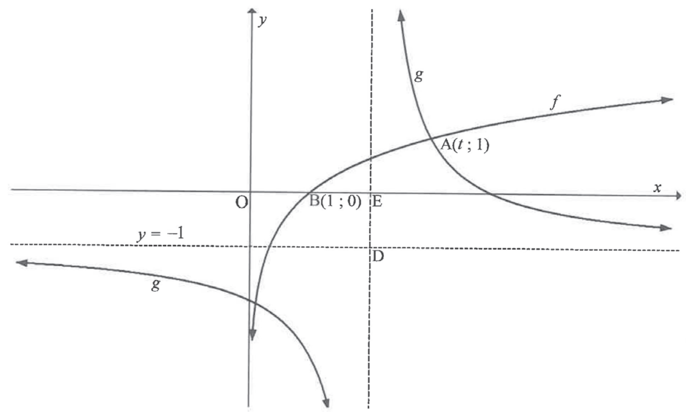
4.7 Write down the values of x for which $f(x).g(x) \leq 0$. (3)

[23]

QUESTION 5

The diagram below shows the graphs of $g(x) = \frac{2}{x+p} + q$ and $f(x) = \log_3 x$.

- $y = -1$ is the horizontal asymptote of g.
- B(1 ; 0) is the x-intercept of f.
- A(t ; 1) is a point of intersection between f and g.
- The vertical asymptote of g intersects the x-axis at E and the horizontal asymptote at D.
- OB = BE.



5.1 Write down the range of g. (2)

5.2 Determine the equation of g. (2)

5.3 Calculate the value of t. (3)

5.4 Write down the equation of f^{-1}, the inverse of f, in the form $y = \ldots$ (2)

5.5 For which values of x will $f^{-1}(x) < 3$? (2)

5.6 Determine the point of intersection of the graphs of f and the axis of symmetry of g that has a negative gradient. (3)

[14]

QUESTION 6

6.1 Mbali invested R10 000 for 3 years at an interest rate of r % p.a., compounded monthly. At the end of this period, she received R12 146,72. Calculate r, correct to ONE decimal place. (5)

6.2 Piet takes a loan from a bank to buy a car for R235 000. He agrees to repay the loan over a period of 54 months. The first instalment will be paid one month after the loan is granted. The bank charges interest at 11% p.a., compounded monthly.

6.2.1 Calculate Piet's monthly instalment. (4)

6.2.2 Calculate the total amount of interest that Piet will pay during the first year of the repayment of the loan. (6)
[15]

QUESTION 7

7.1 Given: $f(x) = 2x^2 - x$

Determine $f'(x)$ from first principles. (6)

7.2 Determine:

7.2.1 $D_x[(x+1)(3x-7)]$ (2)

7.2.2 $\frac{dy}{dx}$ if $y = \sqrt{x^3} - \frac{5}{x} + \frac{1}{2}\pi$ (4)
[12]

QUESTION 8

Given: $f(x) = x(x-3)^2$ with $f'(1) = f'(3) = 0$ and $f(1) = 4$

8.1 Show that f has a point of inflection at $x = 2$. (5)

8.2 Sketch the graph of f, clearly indicating the intercepts with the axes and the turning points. (4)

8.3 For which values of x will $y = -f(x)$ be concave down? (2)

8.4 Use your graph to answer the following questions:

8.4.1 Determine the coordinates of the local maximum of h if $h(x) = f(x-2)+3$. (2)

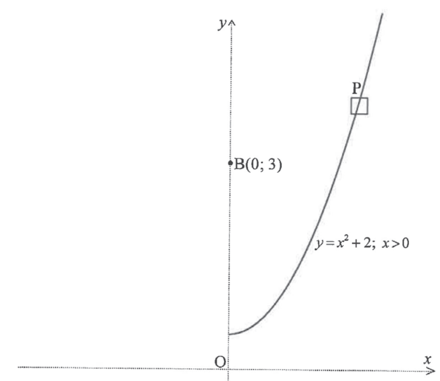
8.4.2 Claire claims that $f'(2) = 1$.

Do you agree with Claire? Justify your answer. (2)
[15]

QUESTION 9

An aerial view of a stretch of road is shown in the diagram below. The road can be described by the function $y = x^2 + 2$, $x \geq 0$ if the coordinate axes (dotted lines) are chosen as shown in the diagram.

Benny sits at a vantage point B(0 ; 3) and observes a car, P, travelling along the road.



Calculate the distance between Benny and the car, when the car is closest to Benny. **[7]**

QUESTION 10

A survey was conducted among 100 Grade 12 learners about their use of Instagram (I), Twitter (T) and WhatsApp (W) on their cell phones. The survey revealed the following:

- 8 use all three.
- 12 use Instagram and Twitter.
- 5 use Twitter and WhatsApp, but not Instagram.
- x use Instagram and WhatsApp, but not Twitter.
- 61 use Instagram.
- 19 use Twitter.
- 73 use WhatsApp.
- 14 use none of these applications.

10.1 Draw a Venn diagram to illustrate the information above. (4)

10.2 Calculate the value of x. (2)

10.3 Calculate the probability that a learner, chosen randomly, uses only ONE of these applications. (2)
[8]

QUESTION 11

A company uses a coding system to identify its clients. Each code is made up of two letters and a sequence of digits, for example AD108 or RR 45789.

The letters are chosen from A; D; R; S and U. Letters may be repeated in the code.

The digits 0 to 9 are used, but NO digit may be repeated in the code.

11.1 How many different clients can be identified with a coding system that is made up of TWO letters and TWO digits? (3)

11.2 Determine the least number of digits that is required for a company to uniquely identify 700 000 clients using their coding system. (3)
[6]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}\ ;\ r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ;\ -1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

RESOURCE 3

SUMMARY NOTES: Paper 2

Summary notes – Paper 2

ANALYTICAL GEOMETRY

All three formulae require 2 points: $(x1;y1)$ and $(x2;y2)$

Gradient	$m = \frac{y2 - y1}{x2 - x1}$
Distance	$d = \sqrt{(x_2 + x_1)^2 + (y_2 - y_1)^2}$
Midpoint	$\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

Useful information:

1. Collinear points: points that lie on a straight line. To prove three points (A, B & C) collinear, prove
   $_mAB = {_mBC} = {_mAC}$ (only two pairs required)
2. 2 lines are parallel if their gradients are equal.
3. 2 lines are perpendicular if the product of their gradients equals –1.
4. To find the y-intercept of any graph, let $x = 0$.
   To find the x-intercept of any graph, let $y = 0$
5. To show that 2 lines bisect each other – the midpoints of each line must be equal.
6. To show that a point lies on a graph: substituting the point should make LHS = RHS
7. To find where two graphs intersect, get both into standard form (y =…), solve simultaneously.
8. Properties of quadrilaterals (often needed):
   - Diagonals of rhombus bisect each other at 90º
   - Diagonals of a rectangle are equal in length.
9. To prove a quadrilateral is a parallelogram, prove one of the following:
   - diagonals bisect (same mid-point)
   - both pairs of opposite sides parallel (equal gradients)
   - both pairs of opposite sides equal (equal lengths)
   - one pair of opposite sides parallel and equal (equal lengths & equal gradients)

Finding the equation of a straight line

Examples: Determine the equation of a straight which:

a) is parallel to the line = $-3x + 4$; passing through the point A (4 ; 7).

$m = -3x$ → line is ∥

$y - y_1 = m(x - x_1)$

$y - 7 = -3(x - 4)$

$y = -3x + 19$

b) is perpendicular to the line = $\frac{-2}{3}x + 2$; with a y-intercept of –3.

$m = \frac{3}{2}$ x →line is ⊥ to $y = \frac{-2}{3}x + 2$

Sub $c = -3$

$\therefore y = \frac{3}{2}x - 3$

c) is parallel to the x-axis and passes through the point (-4 ; 3).

$y = 3$ (A line parallel to the x-axis is a horizontal line)

d) is parallel to the y-axis and passes through the point (-4 ; 3).

$x = -4$ (A line parallel to the y-axis is a vertical line)

e) passes through the points (–2 ; 4) and (3 ; –6)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-6-4}{3-(-2)} = \frac{-10}{5} = -2$$

$$y - y^1 = m(x - x^1)$$

$$y - 4 = -2(x - (-2))$$

$$y = -2x + 8$$

Angle of inclination

Angle of inclination is often shown as θ.

The gradient of a line (m) is equal to the tangent of the angle of inclination (θ).

$\tan\theta = m$ where $\theta \in (0°;180°)$

Circles

Circle with centre origin: $x^2 + y^2 = r^2$ (where r is the radius)

Circle with centre $(a ; b)$: $(x - a)^2 + (x - b)^2 = r^2$ (where r is the radius)

Knowledge required from other areas:

- Completing the square
- A tangent is always perpendicular to a radius
- Properties of quadrilaterals
- Basic geometry theorems from Gr 8 & 9

If a sketch is not given in the question ALWAYS draw one!!! (And update it as you proceed through the question)

Circles with the origin as the centre

$$x^2 + y^2 = r^2$$

- Given a point that the circle passes through
- Sub in the point to find r ; sub info back in to std equation
- Finding a missing value in co-ordinate but given r / r^2
- Sub in point given (with unknown value) and r ; Solve for unknown

Circles with centre at another point $(a ; b)$

$$(x - a)^2 + (x - b)^2 = r^2$$

represents a horizontal translation | represents a vertical translation

(NB for domain and range)

Sketching a circle.

- Determine centre: Find radius (square root if given r^2)
- Find y-int's ($x = 0$) and x-int's ($y = 0$)

Using completing the square to write the equation in std form (usually done to find the centre).

- Group x terms and y terms; Complete the square on both x and y terms (and remember to add chosen value to both sides!); Factorise and collect like terms on RHS

Finding the equation of a circle (Need to find centre and radius)

- If given 2 points that form diameter: find mid-point to find centre, then distance formula to find radius.

Tangents to circles

- To find the equation of any straight line we need a point and the gradient to use the formula: $y - y_1 = m(x - x_1)$
- Determine the gradient of the radius, then the gradient of the tangent
  ($m_1 m_2 = -1 \rightarrow$ perpendicular)

Point of intersection between a straight line and a circle

Make equations equal to each other and solve simultaneously

Other circle theorems that are often required in Analytical:

- The angle subtended from the diameter = 90º
- A tangent is always perpendicular to the radius
- The line from the centre of a circle to the mid-point of a chord is perpendicular to the chord
- 2 tangents from a common point are equal in length

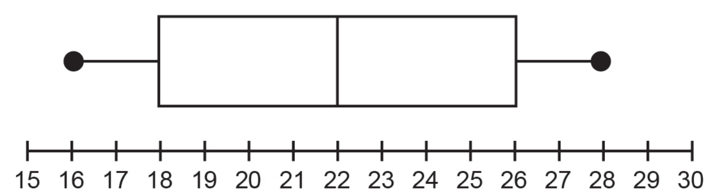
Application

- If two circles touch, then we know that the distance between the centres of the two circles is equal to the sum of their radii (using the distance formula)
- If we want to find whether two circles touch, check the distance apart of the two centres and check if it equals the sum of their radii. If the distance is less than the sum of the radii, the circles intersect in two places.

STATISTICS

Ungrouped data

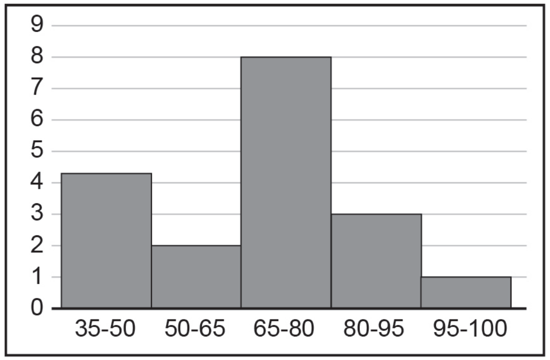
Representing ungrouped data graphically:

Bar Graph Bar graphs display discrete data.
Broken line graphs Broken line graphs represent changes in data over time. In such graphs, convention *dictates the independent variable is represented horizontally on the x-axis and the dependant variable is represented vertically on the y-axis.*
Box-and-whisker plot  This type of graph is used to show the shape of the distribution, its central value, and its variability. In a box and whisker plot: the ends of the box are the upper and lower quartiles, so the box spans the interquartile range. the median is marked by a vertical line inside the box.

Grouped data

Representing grouped data graphically:

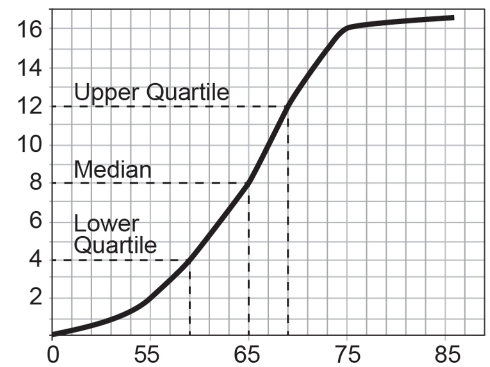
Histogram



Histograms display continuous data. Therefore, the bars are touching.

Discrete data has clear separation between the different possible values, while continuous data doesn't. We use <u>bar graphs</u> for displaying <u>discrete data</u>, and <u>histograms</u> for displaying <u>continuous</u> data.

Ogive (Cumulative frequency curve)



- A way of representing grouped data
- Never goes down and should form an S-shape
- The horizontal axis will represent the data
- Only the upper boundary numbers will be represented. These are the x-co-ordinates of the points found
- The vertical axis will always represent the cumulative frequency – no matter what the situation is being represented
- Remember to ground the ogive using the lower boundary number of the first class interval with zero
- Can be used to estimate median, quartiles and percentiles

Measures of Central Tendency

Ungrouped data:

Mean	Example: List of shoe sizes: 7, 9, 12, 9, 8, 6, 9, 12, 13, 17
Most commonly used measure of central tendency Add all data and divide by number of items in data set. The mean is distorted by outliers	$\frac{7+9+12+9+8+6+9+12+13+17}{10}$ $=\frac{102}{10}$ $=10,2$
Median	
Middlemost score (odd number) or average of the two middle scores (even number). Numbers need to be ordered	6 7 8 9 <u>9 9</u> 12 12 13 17 $\frac{9+9}{2}=\frac{18}{2}=9$
Mode	
The most frequently occurring score Can have more than one mode	9

Grouped data:

Estimate of the mean:

- Calculate the midpoint of each class
- Multiply each midpoint by the frequency for that interval
- Add up and divide by the total number of scores

The modal class:

- This is the interval in which the data occurs most frequently

The median:

- The best way to calculate the median is by drawing a cumulative frequency curve
- A way of representing grouped data
- Never goes down and should form an S-shape
- Can also be used to estimate median, quartiles and percentiles

Measures of Dispersion (spread of data)

Range:

The difference in the largest and the smallest value in the data set. The bigger the range the more spread out the data is.

Quartiles:

Measures of dispersion around the median. The median divides the data into two halves. The lower and upper quartiles divide the data further into quarters.

To find: Lower quartile - Q_1: $\frac{1}{4}(n+1)$

Median - Q_2: $\frac{1}{2}(n+1)$

Upper quartile - Q_3: $\frac{3}{4}(n+1)$

Remember: This gives the *position*!

Inter-quartile range (IQR)

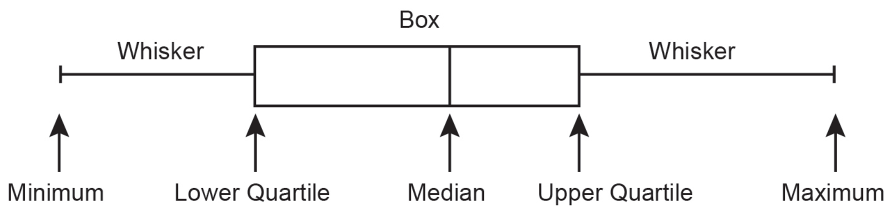
The difference between the upper quartile and lower quartile

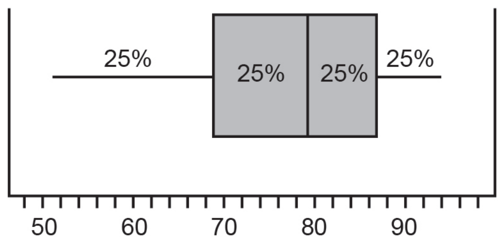
$(Q_3 - Q_1)$

Five number summary:

- Minimum: The smallest value in the set of data
- Lower quartile: The median of the lower half of the values
- Median: The value that divides the data into halves
- Upper quartile: The median of the upper half of the values
- Maximum: The largest value in the data.

The box and whisker plot is a graphical representation of the five number summary.





Variance and Standard deviation

- A way of measuring the spread of data
- Variance = average of squared differences of the mean
- Standard deviation = $\sqrt{variance}$

How standard deviation is found:

1. Work out the average (mean value) of your set of numbers
2. Work out the difference between each number and the mean
3. Square the differences
4. Add up the squares of all the differences
5. Divide this by the number of data in your set - this is called the **variance**
6. Take the square root of the variance – this is the standard deviation

If data is normally distributed

- Around 68% of data are within one standard deviation of the mean
- Around 95% of data are within two standard deviations of the mean
- Around 99% of data are within three standard deviations of the mean

If the data is grouped the middle value of the interval must be used as well as the frequency for the calculation as above.

Skewed data and outliers

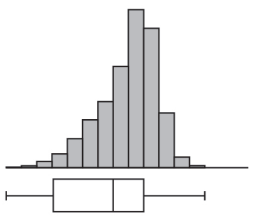
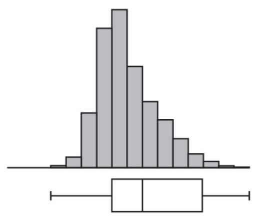
The mean is susceptible to the influence of outliers so if there are any outliers, the mean is not considered a good representation of the data.

If you have a normally distributed sample, the mean and median are both good measures of central tendency. (In perfectly symmetrical data the mean would equal the median)

If the data is skewed, the mean tends to be 'dragged' in the direction of the skewness. (In this case, the median is more likely to be a better representation of the data). Skewness exists if there are extreme scores or tail.

The more skewed the distribution, the greater the difference between the median and mean.

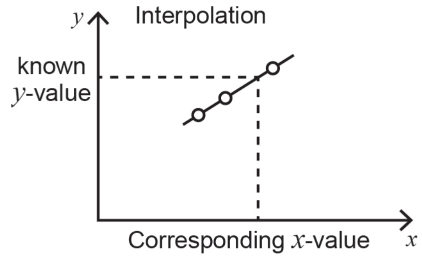
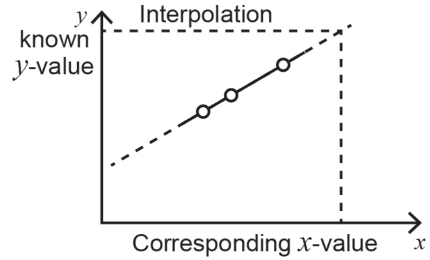
In most cases:

Negatively skewed (mean subtract median < 0)	Positively skewed (mean subtract median >0)
Skewed to the <u>left</u> – the data is more spread out on the left (Longer tail on left = skewed to left)	Skewed to the <u>right</u> – the data is more spread out on the right. (Longer tail on right = skewed to the right)
mean < median < mode	mode < median < mean
	

Scatter plots and line of best fit

- Two sets of data where a relationship may be visible
- Relationship = correlation (r)
- Relationship can be linear, quadratic or exponential
- Linear relationships can be positive or negative and strong or weak (correlation coefficient)
- Line of best fit: $y = a + bx$

Interpolation and Extrapolation

Interpolation: Predicting a value that lies within the domain or range of a given set of data	Extrapolation: Predicting a value that does not lie within the domain and range of a given set of data.
	

Correlation coefficient

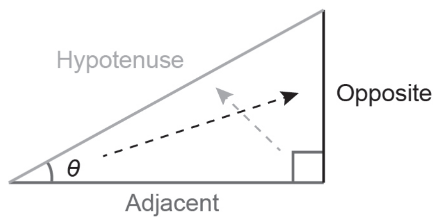
Value of r	**Interpretation**
$r = 0$	No relation between the data – it is randomly scattered
$r = 1$	Perfect linear correlation. All the data lie on a straight line. As x increases, y increases
$r = –1$	Perfect linear correlation. All the data lie on a straight line. As x increases, y decreases

$-1 < r < 0$	Negative linear correlation The closer to –1, the stronger the correlation Example: –0,3 → weak correlation
$0 < r < 1$	Positive linear correlation The closer to 1, the stronger the correlation Example: 0,85 → very strong correlation

TRIGONOMETRY

Right-angled triangles

SOHCAHTOA



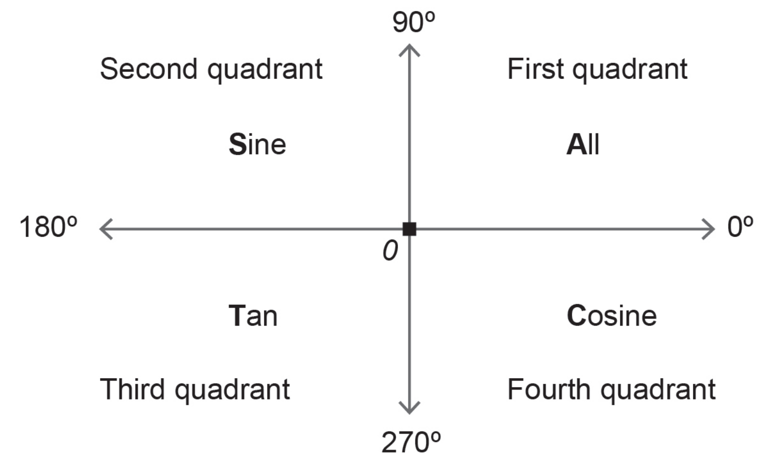
SOH **S**in $\theta = \frac{\textbf{O}\text{pposite}}{\textbf{H}\text{ypotenuse}}$

CAH **C**os $\theta = \frac{\textbf{A}\text{djacent}}{\textbf{H}\text{ypotenuse}}$

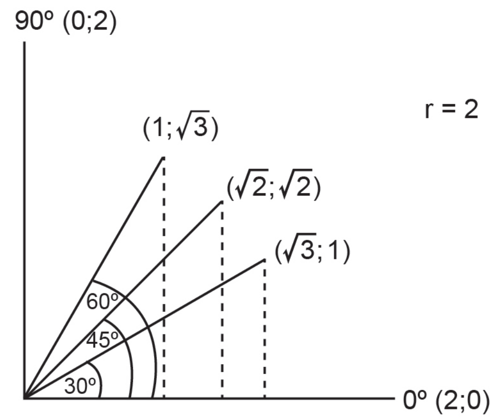
TOA **T**an $\theta = \frac{\textbf{O}\text{pposite}}{\textbf{A}\text{djacent}}$

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

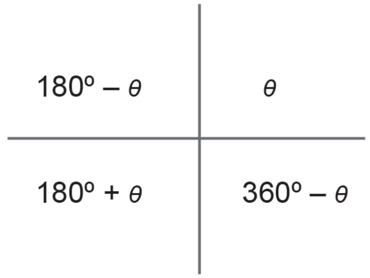
Quadrants



Special angles



Reductions



Step 1: re-name the angle
- The angle is in quadrant …
- Where we name angles..(according to above diagram)

Step 2: reduce to the acute angle
- This angle is in quadrant..
- Where the trig ratio is…(positive or negative)

NOTE: Adding 360° or subtracting 360° from an angle does not change the ratio of the angle.

For example: tan 500º

tan 500º	(subtract 360°)
= tan 140º	(quadrant 2 ∴ 180º–..)
= tan(180º – 40º)	(quadrant 2 ∴ negative)
= -tan 40º	

Complementary angles

$\sin(90° - \theta) = \cos\theta$

$\sin(90° + \theta) = \cos\theta$

$\cos(90° - \theta) = \sin\theta$

$\cos(90° + \theta) = -\sin\theta$

Compound angle formula

$$\cos(A - B) = \cos A.\cos B + \sin A.\sin B$$
$$\cos(A + B) = \cos A.\cos B - \sin A.\sin B$$
$$\sin(A + B) = \sin A.\cos B + \sin B.\cos A$$
$$\sin(A - B) = \sin A.\cos B - \sin B.\cos A$$

When simplifying:
- Reduce first (example, 169° = 180° – 11°)
- Use co-functions (example, sin 79° = cos 11°)
- Use compound angle formula
- Look for special angles to simplify further

[Do not use compound angle formula for: sin (90° – θ); sin (180° + θ); cos (θ – 180°)]

Double angle formula

$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 1 - 2\sin^2 A \qquad [\cos^2\theta = 1 - \sin^2\theta]$$
$$= 2\cos^2 A - 1 \qquad [\sin^2\theta = 1 - \cos^2\theta]$$
$$\sin 2A = 2\sin A.\cos A$$

Identities

$sin^2\theta + cos^2\theta = 1 \quad tan\ \theta = \frac{sin\theta}{cos\theta}$

Tips to prove identities:

- Change any tan function to $\frac{sin}{cos}$
- If there are any fractions to be added or subtracted, find LCD and simplify
- Consider the new numerator (after adding or subtracting) and check for factorising opportunities
- Watch out for double or compound angles

Proving trig identities using compound angles and double angles

TIPS:

- With cos $2A$, always consider carefully which would be the best of the 3 options.
  If there is a constant, aim to eliminate it. (If there is a '+1', use the 3rd option with the '–1' in)
- Work with LHS and RHS separately.
- Look for any compound angles and double angles and write them as single angles. (But if ALL angles are double angles this is not usually necessary)
- Write in terms of $sin\ A$ and $cos\ A$
- If you have addition/subtraction of fractions, find LCD and write as one fraction. Then simplify numerator using theory (compound angles/double angles/basic identities) or by factorising first.
- Keep referring back to the side you are trying to prove to make sure you are heading in the right direction
- Watch out for "1" – it may be useful to write it as $sin^2A + cos^2A$

Using diagrams to determine numerical values of ratios (Pythagoras questions)

Steps:

- Using BOTH pieces of info, decide which quadrant you need to work in
- Make a sketch, drawing the triangle in the correct quadrant.
- Fill in the two known sides from the given info
- Use Pythagoras to find the third side
- Summarise the info you now know regarding what x, y and r are all equal to
  (Be careful of signs here!)
- Use this information to complete the question using substitution
- Watch out for double or compound angles

NB: Need to know trig ratios in terms of x, y and r

Don't even consider the 'question' (find…) until the groundwork is done.

General solutions

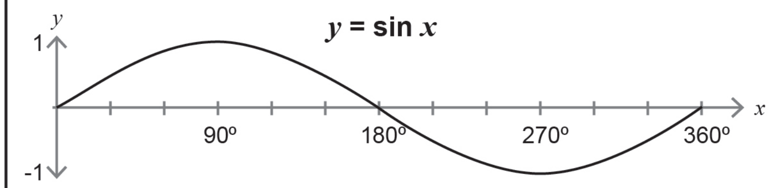
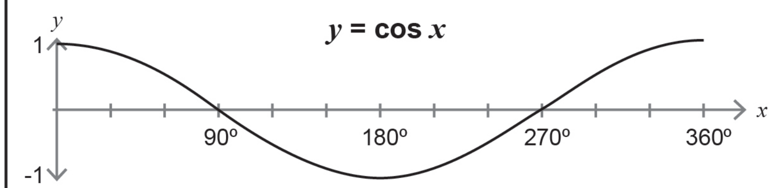
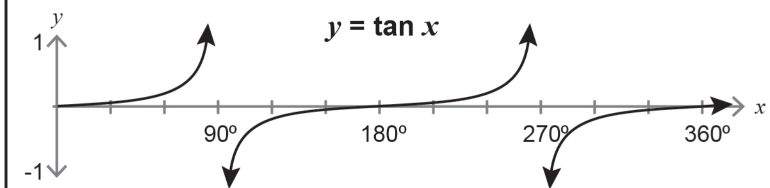
- Make the trig function the subject of the formula
- Use the 2nd function on the calculator: (shift ; trig function ; ratio) to find the reference angle
- Note whether the function is positive or negative
- Choose the quadrants accordingly and find the general solutions according to the quadrants
- Use the appropriate reductions to represent angles in the chosen quadrants.
- Use k to show that it is a general solution and if required, substitute integers to find specific solutions.

Examples:

Find the general solution: cos θ = 0,85 (shift ; cos ; 0,85) $RA = 31{,}79^{\circ}$ (ratio is positive - ∴ quadrant 1 and 4) $\theta = 31{,}79^{\circ} + k.360^{\circ}$ OR $\theta = 360^{\circ} - 31{,}79^{\circ} + k.360^{\circ}$ $\theta = 328{,}21^{\circ} + k.360^{\circ}$ $k \in Z$	Solve for $x \in (-180^{\circ};360^{\circ})$ $2\sin(x + 20^{\circ}) = -1{,}53$ $\sin(x + 20^{\circ}) = -\frac{1{,}53}{2}$ (shift ; sin ; $\frac{1{,}53}{2}$) [Do not use the negative sign] $\therefore RA = 49{,}91^{\circ}$ (ratio is negative ∴ quadrant 3 and 4) $x + 20^{\circ} = 180^{\circ} + 49{,}91^{\circ} + k.360^{\circ}$ $x = 209{,}91^{\circ} + k.360^{\circ}$ OR $x + 20^{\circ} = 360^{\circ} - 49{,}91^{\circ} + k.360^{\circ}$ $x = 290{,}09^{\circ} + k.360^{\circ}$ $k \in Z$ ∴ x ∈ {209,91°;290,09°}

- Watch out for special angles
- If there is a restriction on the unknown angle (eg $\theta \in [0°;720°]$), remember to check for all possibilities when finished solving them and list them.
- If there are 4 terms, group and factorise.
- If you have 2 trig ratios you may have to use identities (example, replace $cos^2\theta$ with $1 - sin^2\theta$) in order to make all the trig ratios in the equation the same.

Trig graphs

Sine graph	$y = sin\ x$ with a period of 360° 
Cosine graph	$x = cos\ x$ with a period of 360° 
Tangent graph	$y = tan\ x$ with a period of 180° 

Period: The number of degrees it takes for the graph to complete a pattern before it gets repeated

Amplitude: The maximum deviation from the x-axis.
Can be found by using: ½ (distance between maximum and minimum values)

<u>Vertical shifts of the sine, cosine and tangent graphs</u>

$y = \sin x + q \qquad y = \cos x + q \qquad y = \tan x + q$

'q' represents the units the basic graph shifts vertically (up or down)

It will change the maximum and minimum value and therefore the range.

It will NOT change the amplitude or period.

The vertical distance (size) remains the same.

<u>Amplitude shifts of the sine and cosine graphs:</u>

$y = a \sin x \qquad y = a \cos x$

The graph is stretched or squashed from its original position. The vertical distance (size) changes – it becomes longer or shorter.

The value of 'a':

- gives the new amplitude. If 'a' is negative, this affects the direction of the graph
- changes the maximum and minimum value and therefore the range.
- does NOT change the period. It remains 360°.

<u>Horizontal shifts</u>

$y = \sin(x + p) \qquad y = \cos(x + p) \qquad y = \tan(x + p)$

p represents the horizontal (left or right) shift of the basic graph

If $p < 0$, the graph shifts to the right

If $p > 0$, the graph shifts to the left

<u>Period changes</u>

$y = \sin bx \qquad y = \cos bx \qquad y = \tan bx$

b affects the period of the graph.

$$\frac{original\ period}{b} = new\ period$$

(b could also be seen as 'the number of full graphs that can be seen in the original period)

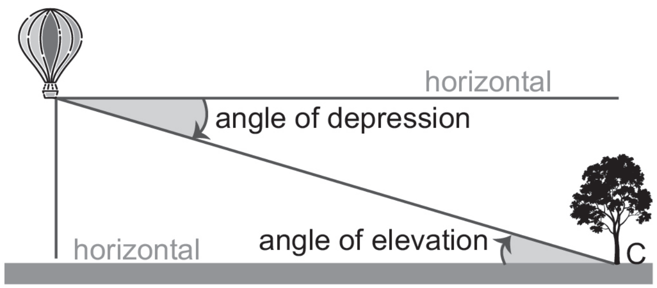
For example, for the function $y = \sin 3x$, the new period is $\frac{360°}{3} = 120°$ and if the function was drawn over 360°, there would be three sine curves visible.

<u>Summary</u>

$y = a\sin(b\theta + p) + q$
$y = a\cos(b\theta + p) + q$
$y = a\tan(b\theta + p) + q$

a	Amplitude	Stretches ($a > 1$) or squashes ($0 < a < 1$) or reflects in the x-axis (flips over) if $a < 0$
b	Period	Distance in degrees to complete a cycle. If $b = 1$, then period is 360° for sine & cosine graph & 180° for tangent graph To find 'new' period if b ≠ 1, divide regular period by b
p	Vertical shift	Number of units shifted up or down the y-axis
q	Horizontal shift	Number of degrees shifted left or right on the x-axis

2-dimensional problems



Example Find the height of the tree 	$tan\ 38º = \frac{tree}{4,2}$ $4,2 \times tan\ 38º = tree$ $3,28m = tree$

Sine, Cosine and Area rule

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$	Look for pairs with opposite side and angle. If you're missing only one of the 4 values when looking at 2 pairs, use Sine rule
$a^2 = b^2 + c^2 - 2bc \cos A$	Use if given 2 sides and included angle or 3 sides
$\frac{1}{2}ab \sin C$	Use to find area Need 2 sides and included angle

3-Dimensional problems

This mainly involves solving triangles, so the Sine, Cosine and Area rules will be used often.

If an object is 3-dimensional, then it has volume and it isn't always easy to represent it in 2-dimensional form. It is therefore essential that you understand the text that describes the situation.

A plane: A flat surface with 2 dimensions

The distance between a point and a plane is the shortest distance between them (ie perpendicular)

When approaching these questions:

- Fill in ALL possible information onto the diagram (even if it means calling an angle $(90° - x)$ or $(x + y)$)
- Shade at least one of the triangles (the one that is given as being in the same horizontal plane) to make the diagram look more 3-D-like
- Look for 'separate' triangles to see which ones present enough information to use the sin or cos rule to find more sides or angles.
- Make use of the sin rule wherever possible – it is the simplest one to use.
- Watch out for the ambiguous case (given 2 sides and included angle and the given angle is opposite the smallest side)

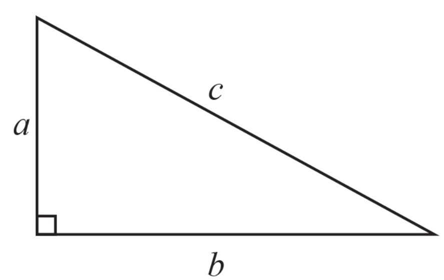
EUCLIDEAN GEOMETRY AND MEASUREMENT

Grade 8 theorems

Lines and angles: ● Vertically opposite angles are equal ● Adjacent angles on a staright line = 180º (supplementary) ● Angles around a point = 360º
Triangles: ● Angles of a triangle = 180º (supplementary) ● The exterior angle of a triangle is equal to the sum of opposite interior angles ● The angles opposite the equal sides in anisosceles triangle are equal
Parallel lines. When parallel lines are cut by a transversal the: ● Corresponding angles are equal ● The alternate angles are equal ● The co-interior angles are supplementary

The theorem of Pythagoras

In any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.



$c^2 = a^2 + b^2$

Congruency

4 conditions of congruency:

- SSS (3 sides equal)
- SAS (2 sides and the included angle equal)
- AAS (2 angles and a corresponding side equal)
- RHS (the right angle, the hypotenuse and a side equal)

Similar Triangles

Similar triangles have the same shape but are not the same size.

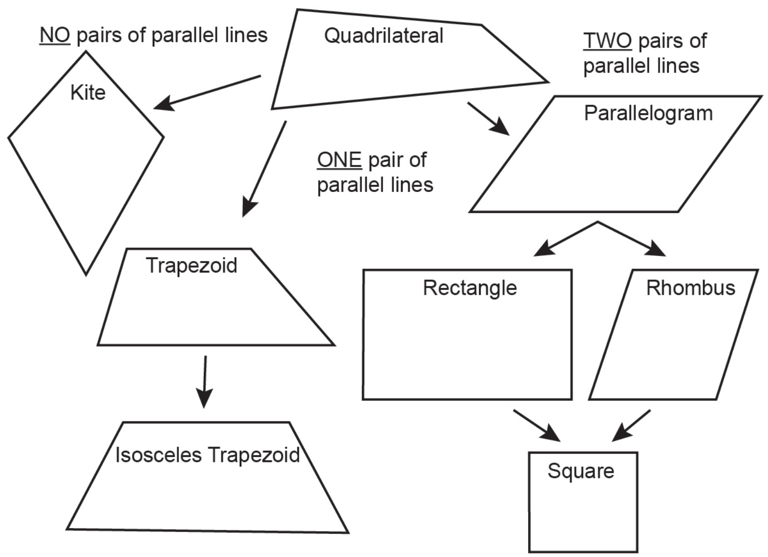
There are two conditions that make triangles similar:

- Sides are in proportion
- All three angles are equal

If $\Delta ABC \,///\, \Delta PQR$, then: $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

Remember when proving that two triangles are similar – 2 equal angles are required. The 3rd angles will be equal because of angles of a triangle adding up to 180º.

Family tree of quadrilaterals showing how they relate to each other



Definitions of the 6 quadrilaterals

Parallelogram	A quadrilateral with both pairs of opposite sides parallel
Rectangle	A parallelogram with 4 right angles
Rhombus	A parallelogram with 4 equal sides
Square	A parallelogram with 4 equal sides and 4 right angles
Kite	A quadrilateral with 2 pairs of adjacent sides equal and no opposite sides equal.
Trapezium	A quadrilateral with one pair of opposite sides parallel

Properties of quadrilaterals

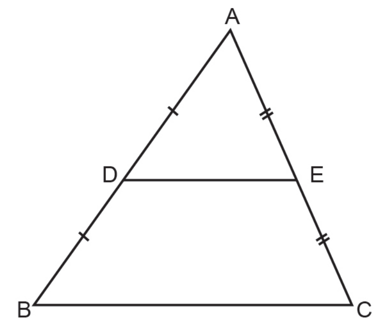
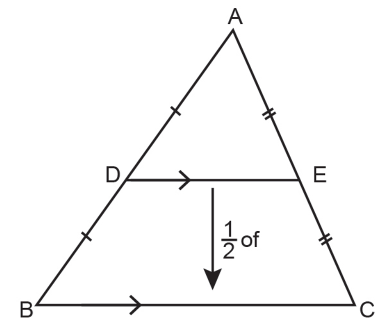
PROPERTY	PARALLELOGRAM	RECTANGLE	RHOMBUS	SQUARE
Opposite sides parallel	✓	✓	✓	✓
Opposite angles equal	✓	✓	✓	✓
Opposite sides equal	✓	✓	✓	✓
Diagonals bisect each other	✓	✓	✓	✓
Diagonals are equal		✓		✓
Diagonals are perpendicular			✓	✓
Diagonals bisect opposite angles			✓	✓
All sides equal			✓	✓
All angles right angles		✓		✓

How to prove a quadrilateral is a:

Parallelogram • both pairs of opposite sides parallel or • both pairs of opposite sides equal or • one pair of opposite sides equal and parallel or • diagonals bisect each other or • opposite angles equal	**Rectangle** It must be a parallelogram with: • equal diagonals or • one right angle
Rhombus It must be a parallelogram with: • 4 equal sides or • diagonals bisect at right angles	**Square** It must be a • rhombus with one right angle or • rectangle with 2 adjacent sides equal

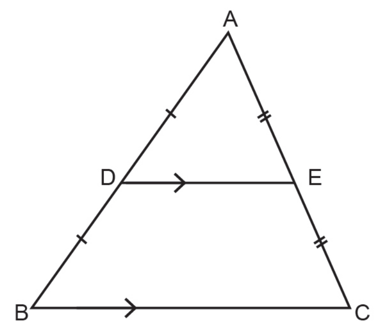
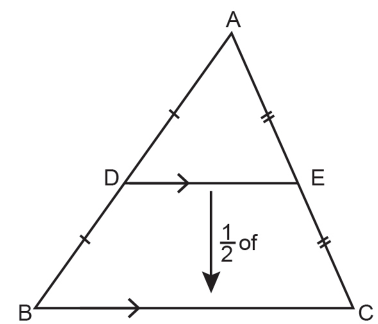
The midpoint theorem

The line joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side. (Abbreviated reason – midpt theorem)

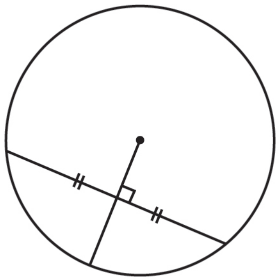
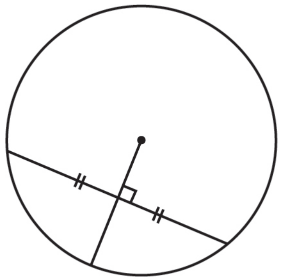
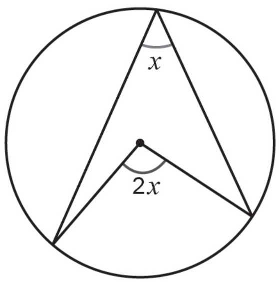
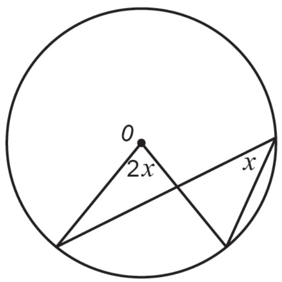
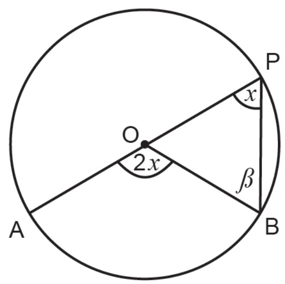
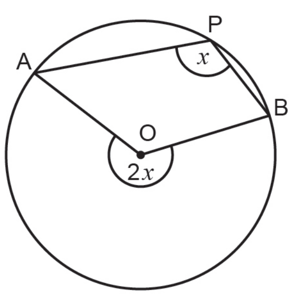
Given:	**Then:**
	

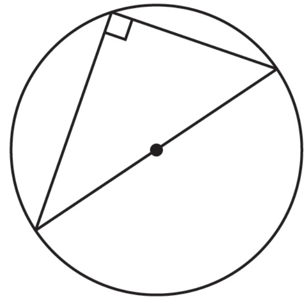
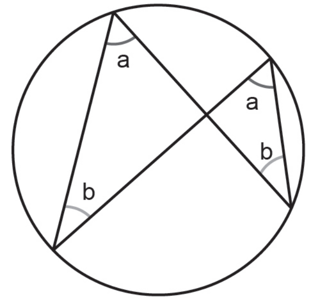
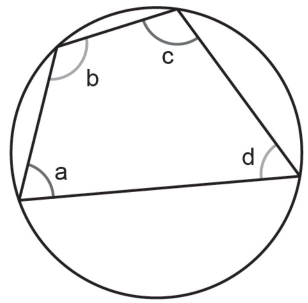
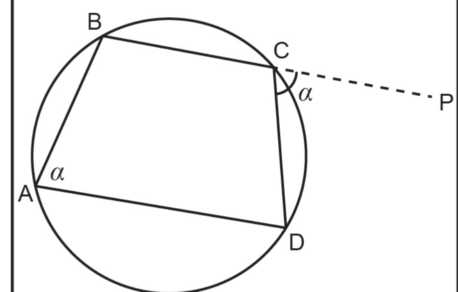
Converse of midpoint theorem

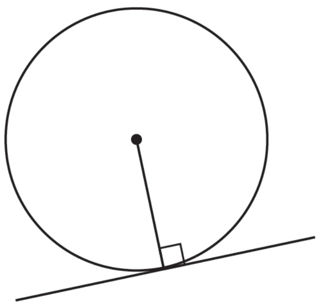
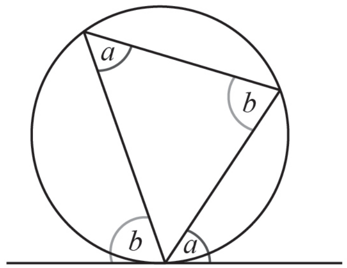
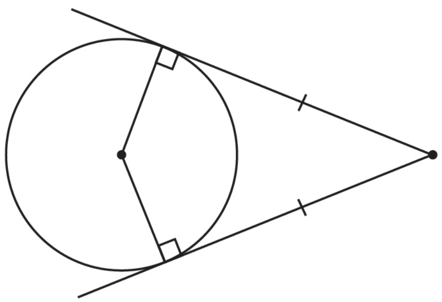
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side. (Abbreviated reason – line through midpt || to 2^{nd} side.

Given:	**Then:**
	

Circle Geometry

The perpendicular line from the centre of a chord bisects the cord	The line from the centre to the midpoint of a chord is perpendicular to the chord
	
The angle at the centre of a circle is twice the angle at the circumference of a circle	
	
	

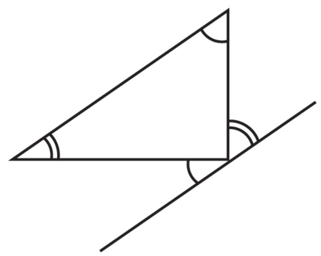
The angle in a semi-circle is always a right angle	Angles subtended by a chord (or arc) in the same segment are equal
	
Opposite angles of a cyclic quadrilateral are supplementary	The exterior angle of a cyclic quadrilateral is equal to the opposite interior angle
 $a + c = 180°$ $b + d = 180°$	

A tangent is perpendicular to the radius	The angle between a tangent and a chord is equal to the angle subtended by the chord in the opposite segment
	
Two tangents drawn from a common point to a circle are equal in length	
	

Tips to consider when you're stuck:

If you must prove:

- <u>sides equal</u>: look for the two angles that should be equal. If this doesn't seem possible, use congruency if the angles are in 2 different triangles.
- <u>that a quad is a cyclic quad</u>: look for:
  - (i) ext < = to opp int <
  - (ii) opp <'s = 180
  - (iii) line subtends equal <'s
- <u>2 lines parallel</u>: look for:
  - (i) alt <'s equal
  - (ii) corres <'s equal
  - (iii) co-int <'s = 180º
- <u>that a line is a tangent to an 'invisible' circle</u>:
  - (i) look for the tan-chord theorem 'diagram' and prove the appropriate angles equal



  - (ii) 90º angle where radius meets the line

Be careful of a quad in a circle with the centre as one of the points. It is NOT a cyclic quad. You will probably use the angle at the centre is twice the angle at the circumference, but with the REFLEX angle.

If you are given:

● Parallel lines: you WILL use either (i) alt <'s (ii) corres <'s (iii)co-int <'s	● The centre of a circle: Look for: (i) < in semi-circle (ii) < at centre = 2x < at circ (iii)radius / chord (perp)
● A cyclic quad: Look for: (i) ext < = int opp < (ii) opp <'s = 180 (iii)<'s in same segment	● Tangent: Look for: (i) tan/chord (ii) tan $\perp$ rad (or diameter)
● 2 tangents from same point: Mark them equal and look for equal angles from isosceles triangle formed.	

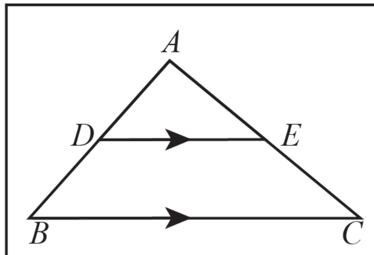
Similarity and proportion

Important: If $\Delta ABC\ |||\ \Delta PQR$, then: $\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}$

If two triangles are equiangular, their sides are in proportion (and therefore the triangles are similar)

Triangle proportionality theorem

A line drawn parallel to one side of a triangle divides the other two sides proportionally.



If $DE \| BC$ then:

$\frac{AD}{BD} = \frac{AE}{EC}$ or $\frac{BD}{AD} = \frac{CE}{AE}$

and $\frac{AD}{AB} = \frac{AE}{AC}$ or $\frac{AB}{AD} = \frac{AC}{AE}$

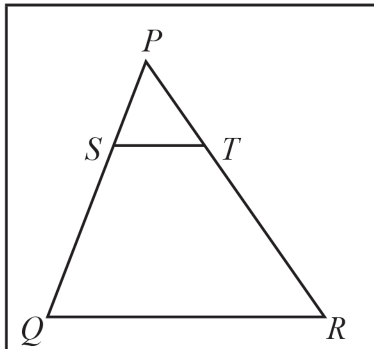
and $\frac{BD}{AB} = \frac{CE}{AC}$ or $\frac{AB}{BD} = \frac{AC}{CE}$

and $\frac{AB}{AD} = \frac{AC}{AE}$ or $\frac{AD}{AB} = \frac{AE}{AC}$

Reason: Line parallel one side of Δ

Converse of proportion theorem

If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.

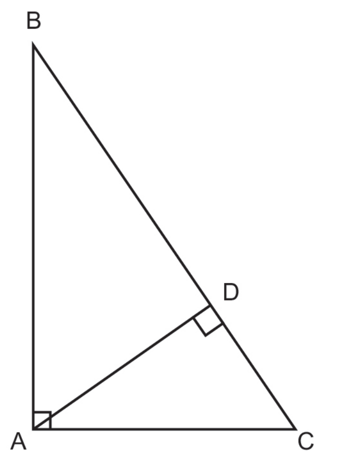


If $\frac{PS}{SQ} = \frac{PT}{TR}$ then ST||QR

Reason: Line divides 2 sides of Δ in proportion

Similar triangles in a right-angled triangle

The perpendicular drawn from the vertex of the right angle of a right-angled triangle to the hypotenuse divides the triangle into two right-angled triangles which are similar to each other and similar to the original triangle.



Given: $\hat{A} = 90°$ and $AD \perp BC$

$\therefore \Delta ABC\ /\!/\!/\ \Delta DBA\ /\!/\!/\ \Delta DAC$

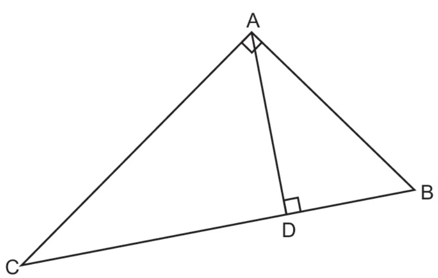
$\therefore AB^2 = BD.BC$

$\therefore AC^2 = CD.CB$

$\therefore AD^2 = BD.CD$

Proving the Theorem of Pythagoras using similarity

By using the above results, we can show that $BC^2 = AB^2 + AC^2$



$AB^2 = BD.BC$ and $AC^2 = CD.CB$

$\therefore AB^2 + AC^2 = BD.BC + CD.CB$

$\therefore AB^2 + AC^2 = BC(BD + CD)$

$\therefore AB^2 + AC^2 = BC(BC)$

$\therefore AB^2 + AC^2 = BC^2$

MEASUREMENT

Volume

The space taken up by a 3D object. To find volume, the area of the base is multiplied by the perpendicular height. This only works for right prisms

VOLUME OF:	AREA OF BASE x HEIGHT
Cube	$(l \times l) \times ht = l \times l \times l = l^3$
Rectangular prism	$(l \times b) \times h = lbh$
Triangular prism	$(\frac{1}{2} b \times h) \times H$ Note: h = height of Δ H = height of prism
Cylinder	$\pi r^2 \times ht = \pi r^2 h$

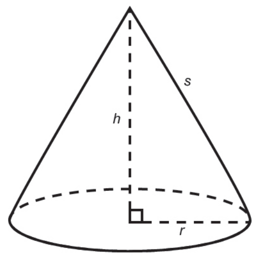
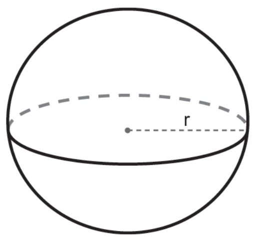
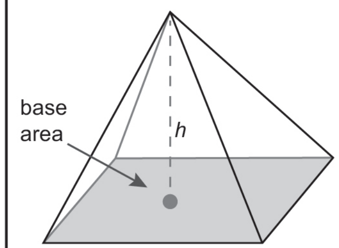
Surface Area

The area taken up by the net of a 3D solid. The sum of the area of all the faces. The following basic shape formulae are needed to find the area of the faces on any 3-dimensional object.

SHAPE	AREA FORMULA
Square	$l \times l = l^2$
Rectangle	$l \times b$
Triangle	$\frac{1}{2} b \times \perp$ height
Circle	πr^2

Cones, pyramids and spheres

(These formulae are given in an assessment)

3D object	Surface Area	Volume
Cone 	$\pi rs + \pi r^2$ (the slant height is sometimes named l)	$\frac{1}{3}\pi r^2 h$
Sphere 	$4\pi r^2$ (the slant height is sometimes named l)	$\frac{4}{3}\pi r^3$
Pyramid 	Sum of the areas of: ● the base and ● the triangles* * the number of triangles depends on the type of base	$\frac{1}{3}$(area of base) $\times h$ (remember that the base could be any polygon but generally the square, rectangle and triangle would be used)

The effect on volume when multiplying any dimension by a constant factor k:

- If only one dimension is changed by a value of k, the volume will be k times bigger
- If only two dimensions are changed by a value of k, the volume will be k^2 times bigger
- If all three dimensions are changed by a value of k, the volume will be k^3 times bigger

RESOURCE 4

PAST PAPER 2: Week 2

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

NOVEMBER 2017

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and an answer book of 28 pages.

Please turn over

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

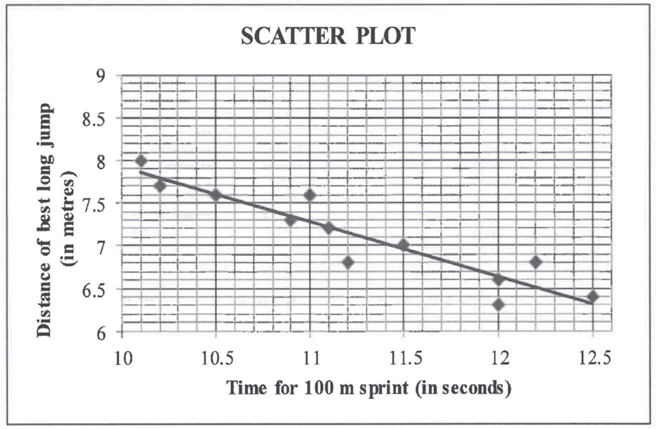
1. This question paper consists of 11 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. An information sheet with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The table below shows the time (in seconds, rounded to ONE decimal place) taken by 12 athletes to run the 100 metre sprint and the distance (in metres, rounded to ONE decimal place) of their best long jump.

Time for 100 m sprint (in seconds)	10,1	10,2	10,5	10,9	11	11,1	11,2	11,5	12	12	12,2	12,5
Distance of best long jump (in metres)	8	7,7	7,6	7,3	7,6	7,2	6,8	7	6,6	6,3	6,8	6,4

The scatter plot representing the data above is given below.



The equation of the least squares regression line is $\hat{y} = a + bx$.

1.1 Determine the values of a and b. (3)

1.2 An athlete runs the 100 metre sprint in 11,7 seconds. Use $\hat{y} = a + bx$ to predict the distance of the best long jump of this athlete. (2)

1.3 Another athlete completes the 100 metre sprint in 12,3 seconds and the distance of his best long jump is 7,6 metres. If this is included in the data, will the gradient of the least squares regression line increase or decrease? Motivate your answer without any further calculations. (2)
[7]

QUESTION 2

In an experiment, a group of 23 girls were presented with a page containing 30 coloured rectangles. They were asked to name the colours of the rectangles correctly as quickly as possible. The time, in seconds, taken by each of the girls is given in the table below.

12	13	13	14	14	16	17	18	18	18	19	20
21	21	22	22	23	24	25	27	29	30	36	

2.1 Calculate:

2.1.1 The mean of the data (2)

2.1.2 The interquartile range of the data (3)

2.2 The standard deviation of the times taken by the girls is 5,94. How many girls took longer than ONE standard deviation from the mean to name the colours? (2)

2.3 Draw a box and whisker diagram to represent the data on the number line provided in the ANSWER BOOK. (3)

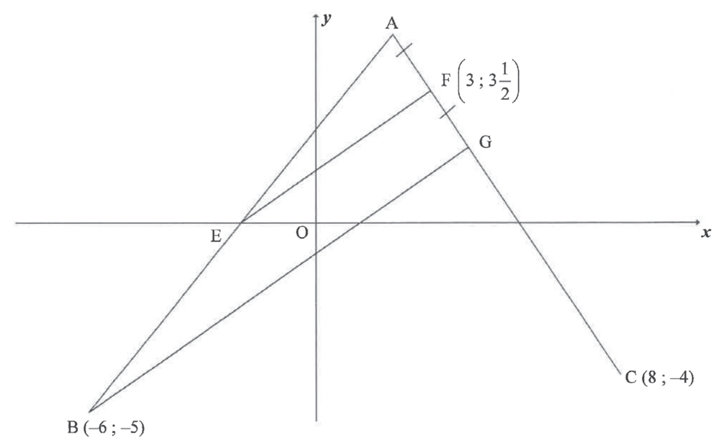
2.4 The five-number summary of the times taken by a group of 23 boys in naming the colours of the rectangles correctly is (15 ; 21 ; 23,5 ; 26 ; 38).

2.4.1 Which of the two groups, girls or boys, had the lower median time to correctly name the colours of the rectangles? (1)

2.4.2 The first three learners who named the colours of all 30 rectangles correctly in the shortest time will receive a prize. How many boys will be among these three prizewinners? Motivate your answer. (2)
[13]

QUESTION 3

In the diagram, A, B(–6 ; –5) and C(8 ; –4) are points in the Cartesian plane. $F\left(3;3\frac{1}{2}\right)$ and G are points on line AC such that AF = FG. E is the x-intercept of AB.



3.1 Calculate:

3.1.1 The equation of AC in the form $y = mx + c$ (4)

3.1.2 The coordinates of G if the equation of BG is $7x - 10y = 8$ (3)

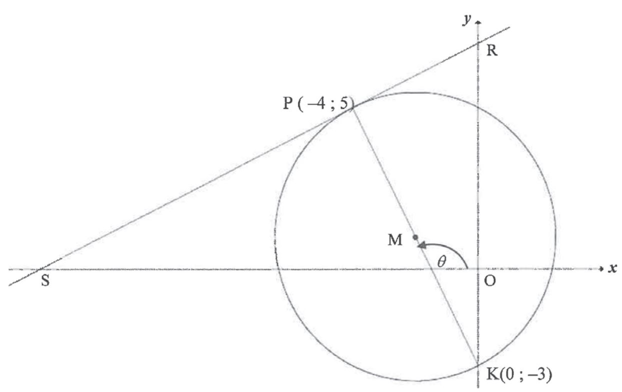
3.2 Show by calculation that the coordinates of A is (2 ; 5). (2)

3.3 Prove that EF || BG. (4)

3.4 ABCD is a parallelogram with D in the first quadrant. Calculate the coordinates of D. (4)
[17]

QUESTION 4

In the diagram, P(–4 ; 5) and K(0 ; –3) are the end points of the diameter of a circle with centre M. S and R are respectively the x- and y-intercept of the tangent to the circle at P. θ is the inclination of PK with the positive x-axis.



4.1 Determine:

4.1.1 The gradient of SR (4)

4.1.2 The equation of SR in the form $y = mx + c$ (2)

4.1.3 The equation of the circle in the form $(x-a)^2+(y-b)^2=r^2$ (4)

4.1.4 The size of $P\hat{K}R$ (3)

4.1.5 The equation of the tangent to the circle at K in the form $y = mx + c$ (2)

4.2 Determine the values of t such that the line $y=\frac{1}{2}x+t$ cuts the circle at two different points. (3)

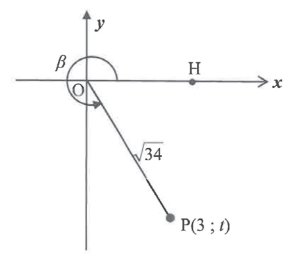
4.3 Calculate the area of ΔSMK. (5)
[23]

QUESTION 5

5.1 Given: $\dfrac{\sin(A-360°).\cos(90°+A)}{\cos(90°-A).\tan(-A)}$

Simplify the expression to a single trigonometric ratio. (6)

5.2 In the diagram, P(3 ; t) is a point in the Cartesian plane. OP $=\sqrt{34}$ and $H\hat{O}P=\beta$ is a reflex angle.



Without using a calculator, determine the value of:

5.2.1 t (2)

5.2.2 $\tan\beta$ (1)

5.2.3 $\cos 2\beta$ (4)

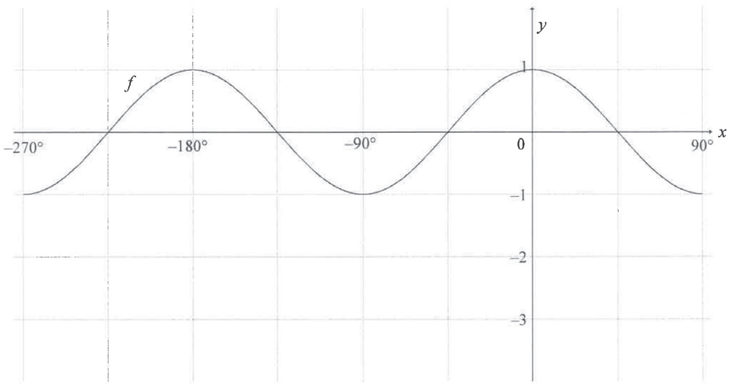
5.3 Prove:

5.3.1 $\sin(A+B)-\sin(A-B)=2\cos A.\sin B$ (2)

5.3.2 **Without using a calculator,** that $\sin 77°-\sin 43°=\sin 17°$ (4)
[19]

QUESTION 6

In the diagram, the graph of $f(x)=\cos 2x$ is drawn for the interval $x\in[-270°;90°]$.



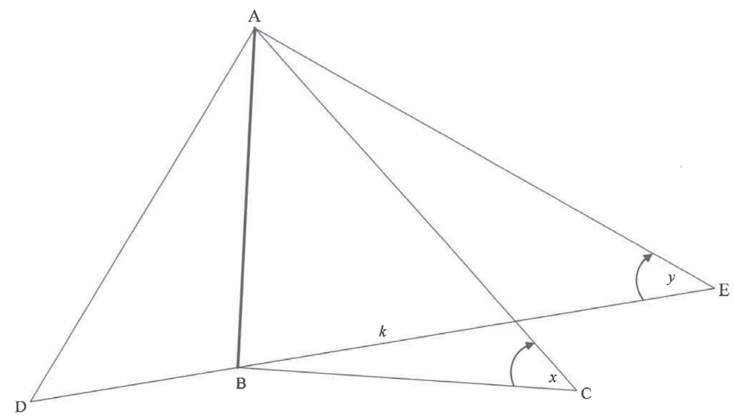
6.1 Draw the graph of $g(x)=2\sin x-1$ for the interval $x\in[-270°;90°]$ on the grid given in your ANSWER BOOK. Show ALL the intercepts with the axes, as well as the turning points. (4)

6.2 Let A be a point of intersection of the graphs of f and g. Show that the x-coordinate of A satisfies the equation $\sin x=\dfrac{-1+\sqrt{5}}{2}$. (4)

6.3 Hence, calculate the coordinates of the points of intersection of graphs of f and g for the interval $x\in[-270°;90°]$. (4)
[12]

QUESTION 7

AB represents a vertical netball pole. Two players are positioned on either side of the netball pole at points D and E such that D, B and E are on the same straight line. A third player is positioned at C. The points B, C, D and E are in the same horizontal plane. The angles of elevation from C to A and from E to A are x and y respectively. The distance from B to E is k.



7.1 Write down the size of $A\hat{B}C$. (1)

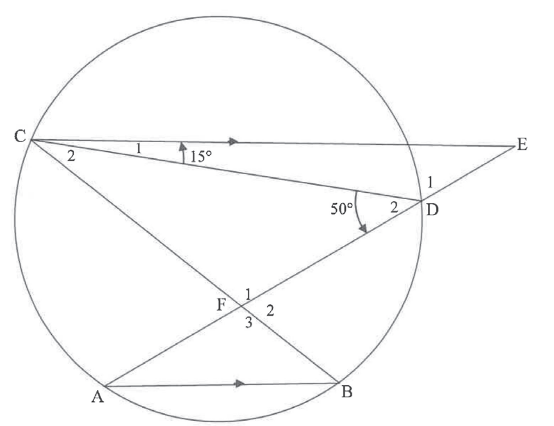
7.2 Show that $AC=\dfrac{k.\tan y}{\sin x}$ (4)

7.3 If it is further given that $D\hat{A}C=2x$ and AD = AC, show that the distance DC between the players at D and C is $2k\tan y$. (5)
[10]

Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram, points A, B, D and C lie on a circle. CE || AB with E on AD produced. Chords CB and AD intersect at F. $\hat{D}_2 = 50°$ and $\hat{C}_1 = 15°$.



8.1 Calculate, with reasons, the size of:

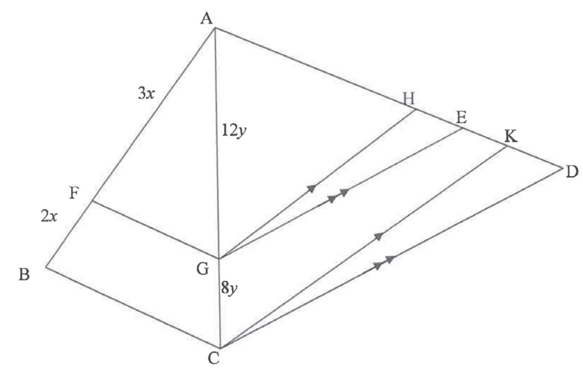
8.1.1 $\hat{A}$ (3)

8.1.2 $\hat{C}_2$ (2)

8.2 Prove, with a reason, that CF is a tangent to the circle passing through points C, D and E. (2)
[7]

QUESTION 9

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that AF = $3x$, FB = $2x$, AG = $12y$ and GC = $8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



9.1 Prove that:

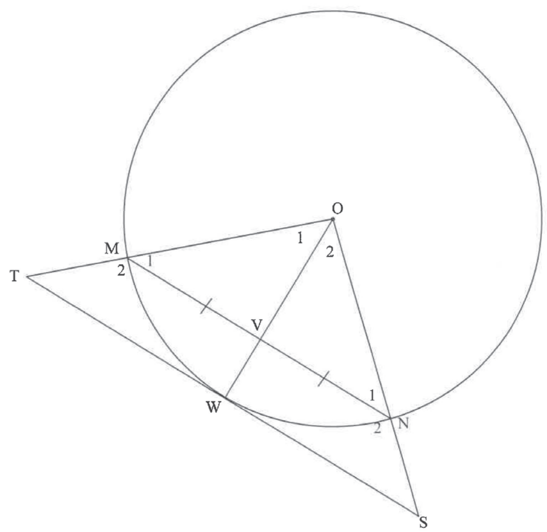
9.1.1 FG || BC (2)

9.1.2 $\frac{AH}{HK} = \frac{AE}{ED}$ (3)

9.2 If it is further given that AH = 15 and ED = 12, calculate the length of EK. (5)
[10]

QUESTION 10

In the diagram, W is a point on the circle with centre O. V is a point on OW. Chord MN is drawn such that MV = VN. The tangent at W meets OM produced at T and ON produced at S.



10.1 Give a reason why OV ⊥ MN. (1)

10.2 Prove that:

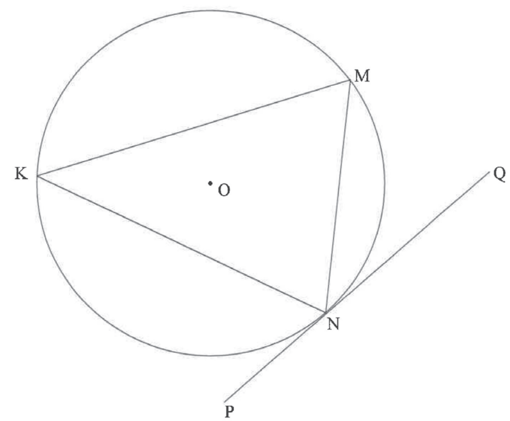
10.2.1 MN ∥ TS (2)

10.2.2 TMNS is a cyclic quadrilateral (4)

10.2.3 OS . MN = 2ON . WS (5)
[12]

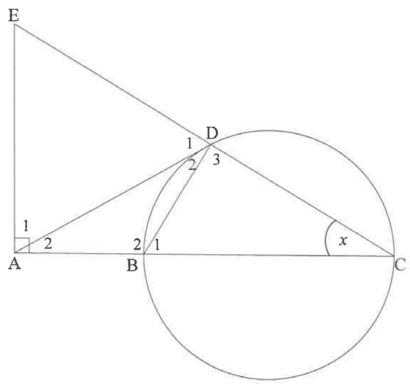
QUESTION 11

11.1 In the diagram, chords KM, MN and KN are drawn in the circle with centre O. PNQ is the tangent to the circle at N.



Prove the theorem which states that $M\hat{N}Q = \hat{K}$. (5)

11.2 In the diagram, BC is a diameter of the circle. The tangent at point D on the circle meets CB produced at A. CD is produced to E such that EA ⊥ AC. BD is drawn.
Let $\hat{C} = x$.



11.2.1 Give a reason why:

(a) $\hat{D}_3 = 90°$ (1)

(b) ABDE is a cyclic quadrilateral (1)

(c) $\hat{D}_2 = x$ (1)

11.2.2 Prove that:

(a) AD = AE (3)

(b) $\Delta ADB \,|||\, \Delta ACD$ (3)

11.2.3 It is further given that BC = 2AB = 2r.

(a) Prove that $AD^2 = 3r^2$ (2)

(b) Hence, prove that ΔADE is equilateral. (4)
[20]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

InΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\,\Delta ABC = \frac{1}{2} ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

RESOURCE 5

PAPER 1 EXEMPLAR: Revision Week 3

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P1

EXEMPLAR 2014

MARKS: 150

TIME: 3 hours

This question paper consists of 8 pages and 1 information sheet.

Please turn over

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. Answers only will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $3x^2 - 4x = 0$ (2)

1.1.2 $x - 6 + \frac{2}{x} = 0$; $x \neq 0$. (Leave your answer correct to TWO decimal places.) (4)

1.1.3 $x^{\frac{2}{3}} = 4$ (2)

1.1.4 $3^x(x-5) < 0$ (2)

1.2 Solve for x and y simultaneously:
$y = x^2 - x - 6$ **and** $2x - y = 2$ (6)

1.3 Simplify, without the use of a calculator:
$\sqrt{3}.\sqrt{48} - \frac{4^{x+1}}{2^{2x}}$ (3)

1.4 Given: $f(x) = 3(x-1)^2 + 5$ and $g(x) = 3$

1.4.1 Is it possible for $f(x) = g(x)$? Give a reason for your answer. (2)

1.4.2 Determine the value(s) of k for which $f(x) = g(x) + k$ has TWO unequal real roots. (2)
[23]

QUESTION 2

2.1 Given the arithmetic series: 18 + 24 + 30 + … + 300

2.1.1 Determine the number of terms in this series. (3)

2.1.2 Calculate the sum of this series. (2)

2.1.3 Calculate the sum of all the whole numbers up to and including 300 that are NOT divisible by 6. (4)

2.2 The first three terms of an infinite geometric sequence are 16, 8 and 4 respectively.

2.2.1 Determine the n^{th} term of the sequence. (2)

2.2.2 Determine all possible values of n for which the sum of the first n terms of this sequence is greater than 31. (3)

2.2.3 Calculate the sum to infinity of this sequence. (2)
[16]

QUESTION 3

3.1 A quadratic number pattern $T_n = an^2 + bn + c$ has a first term equal to 1. The general term of the first differences is given by $4n + 6$.

3.1.1 Determine the value of a. (2)

3.1.2 Determine the formula for T_n. (4)

3.2 Given the series: $(1\times 2)+(5\times 6)+(9\times 10)+(13\times 14)+...+(81\times 82)$

Write the series in sigma notation. (It is not necessary to calculate the value of the series.) (4)
[10]

QUESTION 4

4.1 Given: $f(x)=\dfrac{2}{x+1}-3$

4.1.1 Calculate the coordinates of the y-intercept of f. (2)

4.1.2 Calculate the coordinates of the x-intercept of f. (2)

4.1.3 Sketch the graph of f in your ANSWER BOOK, showing clearly the asymptotes and the intercepts with the axes. (3)

4.1.4 One of the axes of symmetry of f is a decreasing function. Write down the equation of this axis of symmetry. (2)

4.2 The graph of an increasing exponential function with equation $f(x)=a.b^x+q$ has the following properties:

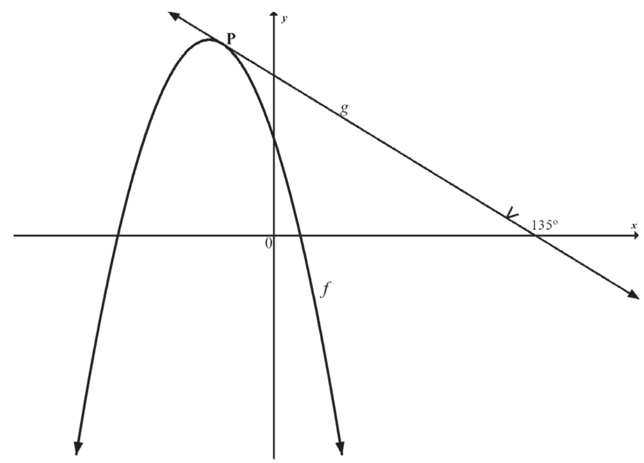
- Range: $y>-3$
- The points $(0\,;-2)$ and $(1;-1)$ lie on the graph of f.

4.2.1 Determine the equation that defines f. (4)

4.2.2 Describe the transformation from $f(x)$ to $h(x)=2.2^x+1$ (2)
[15]

QUESTION 5

The sketch below shows the graphs of $f(x)=-2x^2-5x+3$ and $g(x)=ax+q$. The angle of inclination of graph g is 135° in the direction of the positive x-axis. P is the point of intersection of f and g such that g is a tangent to the graph of f at P.



5.1 Calculate the coordinates of the turning point of the graph of f. (3)

5.2 Calculate the coordinates of P, the point of contact between f and g. (4)

5.3 Hence or otherwise, determine the equation of g. (2)

5.4 Determine the values of d for which the line $k(x)=-x+d$ will not intersect the graph of f. (1)
[10]

QUESTION 6

The graph of g is defined by the equation $g(x)=\sqrt{ax}$. The point (8 ; 4) lies on g.

6.1 Calculate the value of a. (2)

6.2 If $g(x)>0$, for what values of x will g be defined? (1)

6.3 Determine the range of g. (1)

6.4 Write down the equation of g^{-1}, the inverse of g, in the form $y=\dots$ (2)

6.5 If $h(x)=x-4$ is drawn, determine ALGEBRAICALLY the point(s) of intersection of h and g. (4)

6.6 Hence, or otherwise, determine the values of x for which $g(x)>h(x)$. (2)
[12]

QUESTION 7

Siphokazi bought a house. She paid a deposit of R102 000, which is equivalent to 12% of the selling price of the house. She obtained a loan from the bank to pay the balance of the selling price. The bank charges her interest of 9% per annum, compounded monthly.

7.1 Determine the selling price of the house. (1)

7.2 The period of the loan is 20 years and she starts repaying the loan one month after it was granted. Calculate her monthly instalment. (4)

7.3 How much interest will she pay over the period of 20 years? Round your answer correct to the nearest rand. (2)

7.4 Calculate the balance of her loan immediately after her 85^{th} instalment. (3)

7.5 She experienced financial difficulties after the 85^{th} instalment and did not pay any instalments for 4 months (that is months 86 to 89). Calculate how much Siphokazi owes on her bond at the end of the 89^{th} month. (2)

7.6 She decides to increase her payments to R8 500 per month from the end of the 90^{th} month. How many months will it take to repay her bond after the new payment of R8 500 per month? (4)
[16]

QUESTION 8

8.1 Determine $f'(x)$ from first principles if $f(x)=3x^2-2$. (5)

8.2 Determine $\frac{dy}{dx}$ if $y=2x^{-4}-\frac{x}{5}$. (2)
[7]

 Please turn over

QUESTION 9

Given: $f(x)=x^3-4x^2-11x+30$.

9.1 Use the fact that $f(2)=0$ to write down a factor of $f(x)$. (1)

9.2 Calculate the coordinates of the x-intercepts of f. (4)

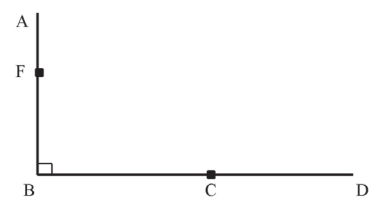
9.3 Calculate the coordinates of the stationary points of f. (5)

9.4 Sketch the curve of f in your ANSWER BOOK. Show all intercepts with the axes and turning points clearly. (3)

9.5 For which value(s) of x will $f'(x)<0$? (2)
[15]

QUESTION 10

Two cyclists start to cycle at the same time. One starts at point B and is heading due north to point A, whilst the other starts at point D and is heading due west to point B. The cyclist starting from B cycles at 30 km/h while the cyclist starting from D cycles at 40 km/h. The distance between B and D is 100 km. After time t (measured in hours), they reach points F and C respectively.



10.1 Determine the distance between F and C in terms of t. (4)

10.2 After how long will the two cyclists be closest to each other? (4)

10.3 What will the distance between the cyclists be at the time determined in QUESTION 10.2? (2)
[10]

 Please turn over

QUESTION 11

11.1 Events A and B are mutually exclusive. It is given that:

- P(B) = 2P(A)
- P(A or B) = 0,57

Calculate P(B). (3)

11.2 Two identical bags are filled with balls. Bag A contains 3 pink and 2 yellow balls. Bag B contains 5 pink and 4 yellow balls. It is equally likely that Bag A or Bag B is chosen. Each ball has an equal chance of being chosen from the bag. A bag is chosen at random and a ball is then chosen at random from the bag.

11.2.1 Represent the information by means of a tree diagram. Clearly indicate the probability associated with each branch of the tree diagram and write down all the outcomes. (4)

11.2.2 What is the probability that a yellow ball will be chosen from Bag A? (1)

11.2.3 What is the probability that a pink ball will be chosen? (3)
[11]

QUESTION 12

Consider the word M A T H S.

12.1 How many different 5-letter arrangements can be made using all the above letters? (2)

12.2 Determine the probability that the letters S and T will always be the first two letters of the arrangements in QUESTION 12.1. (3)
[5]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $\quad S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $\quad P = \frac{x[1-(1+i)^{-n}]}{i}$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

RESOURCE 6

PAPER 2 EXEMPLAR: Revision Week 3

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

EXEMPLAR 2014

MARKS: 150

TIME: 3 hours

This question paper consists of 12 pages, 3 diagram sheets and 1 information sheet.

Please turn over

INSTRUCTIONS AND INFORMATION

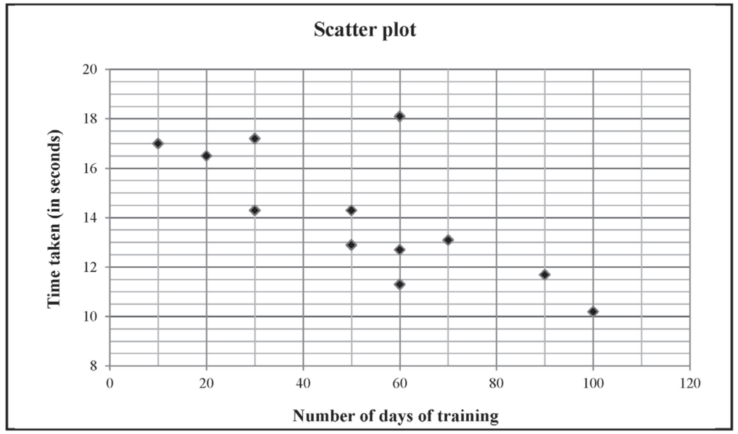
Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions.
3. Clearly show ALL calculations, diagrams, graphs, et cetera which you have used in determining your answers.
4. Answers only will NOT necessarily be awarded full marks.
5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
6. If necessary, round off answers to TWO decimal places, unless stated otherwise.
7. THREE diagram sheets for QUESTION 2.1, QUESTION 8.2, QUESTION 9, QUESTION 10.1, and QUESTION 10.2 are attached at the end of this question paper. Write your centre number and examination number on these sheets in the spaces provided and insert them inside the back cover of your ANSWER BOOK.
8. Number the answers correctly according to the numbering system used in this question paper.
9. Write neatly and legibly.

QUESTION 1

Twelve athletes trained to run the 100 m sprint event at the local athletics club trials. Some of them took their training more seriously than others. The following table and scatter plot shows the number of days that an athlete trained and the time taken to run the event. The time taken, in seconds, is rounded to one decimal place.

Number of days of training	50	70	10	60	60	20	50	90	100	60	30	30
Time taken (in seconds)	12,9	13,1	17,0	11,3	18,1	16,5	14,3	11,7	10,2	12,7	17,2	14,3



1.1 Discuss the trend of the data collected. (1)

1.2 Identify any outlier(s) in the data. (1)

1.3 Calculate the equation of the least squares regression line. (4)

1.4 Predict the time taken to run the 100 m sprint for an athlete training for 45 days. (2)

1.5 Calculate the correlation coefficient. (2)

1.6 Comment on the strength of the relationship between the variables. (1)
[11]

QUESTION 2

The table below shows the amount of time (in hours) that learners aged between 14 and 18 spent watching television during 3 weeks of the holiday.

Time (hours)	Cumulative frequency
$0 \leq t < 20$	25
$20 \leq t < 40$	69
$40 \leq t < 60$	129
$60 \leq t < 80$	157
$80 \leq t < 100$	166
$100 \leq t < 120$	172

2.1 Draw an ogive (cumulative frequency curve) on DIAGRAM SHEET 1 to represent the above data. (3)

2.2 Write down the modal class of the data. (1)

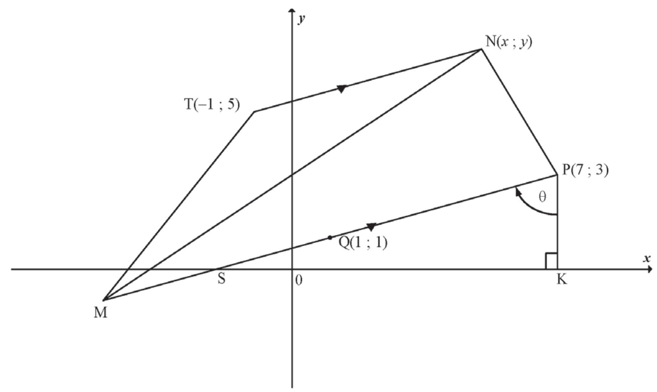
2.3 Use the ogive (cumulative frequency curve) to estimate the number of learners who watched television more than 80% of the time. (2)

2.4 Estimate the mean time (in hours) that learners spent watching television during 3 weeks of the holiday. (4)
[10]

 Please turn over

QUESTION 3

In the diagram below, M, T(–1 ; 5), N(x ; y) and P(7 ; 3) are vertices of trapezium MTNP having TN | | MP. Q(1 ; 1) is the midpoint of MP. PK is a vertical line and $S\hat{P}K = \theta$. The equation of NP is $y = -2x + 17$.



3.1 Write down the coordinates of K. (1)

3.2 Determine the coordinates of M. (2)

3.3 Determine the gradient of PM. (2)

3.4 Calculate the size of θ. (3)

3.5 Hence, or otherwise, determine the length of PS. (3)

3.6 Determine the coordinates of N. (5)

3.7 If A(a ; 5) lies in the Cartesian plane:

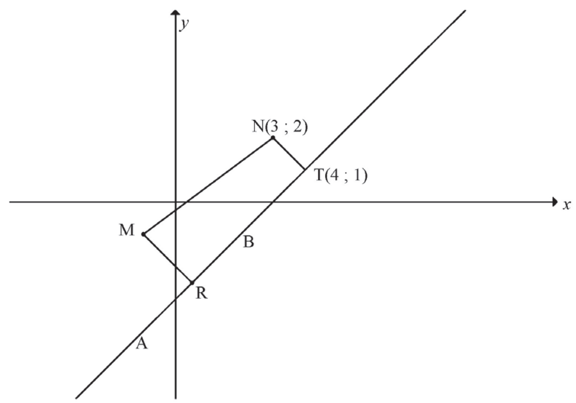
3.7.1 Write down the equation of the straight line representing the possible positions of A. (1)

3.7.2 Hence, or otherwise, calculate the value(s) of a for which $T\hat{A}Q = 45°$. (5)
[22]

 Please turn over

QUESTION 4

In the diagram below, the equation of the circle having centre M is $(x + 1)^2 + (y + 1)^2 = 9$. R is a point on chord AB such that MR bisects AB. ABT is a tangent to the circle having centre N(3 ; 2) at point T(4 ; 1).



4.1 Write down the coordinates of M. (1)

4.2 Determine the equation of AT in the form $y = mx + c$. (5)

4.3 If it is further given that $MR = \frac{\sqrt{10}}{2}$ units, calculate the length of AB.
Leave your answer in simplest surd form. (4)

4.4 Calculate the length of MN. (2)

4.5 Another circle having centre N touches the circle having centre M at point K. Determine the equation of the new circle. Write your answer in the form $x^2 + y^2 + Cx + Dy + E = 0$. (3)
[15]

QUESTION 5

5.1 Given that $\sin\alpha = -\frac{4}{5}$ and $90° < \alpha < 270°$.
WITHOUT using a calculator, determine the value of each of the following in its simplest form:

5.1.1 $\sin(-\alpha)$ (2)

5.1.2 $\cos\alpha$ (2)

5.1.3 $\sin(\alpha - 45°)$ (3)

5.2 Consider the identity: $\frac{8\sin(180° - x)\cos(x - 360°)}{\sin^2 x - \sin^2(90° + x)} = -4\tan 2x$

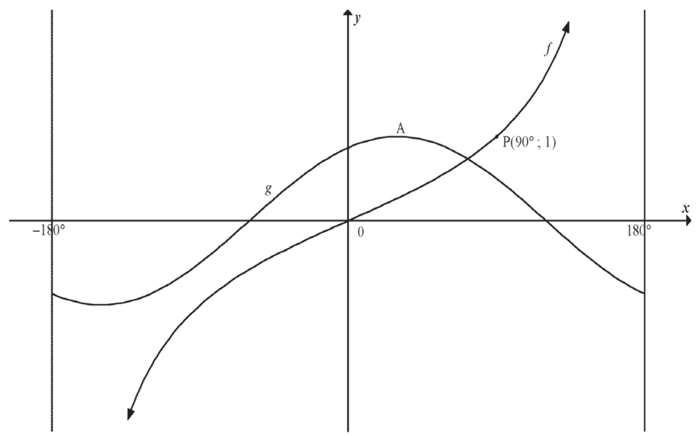
5.2.1 Prove the identity. (6)

5.2.2 For which value(s) of x in the interval $0° < x < 180°$ will the identity be undefined? (2)

5.3 Determine the general solution of $\cos 2\theta + 4\sin^2\theta - 5\sin\theta - 4 = 0$. (7)
[22]

QUESTION 6

In the diagram below, the graphs of $f(x) = \tan bx$ and $g(x) = \cos(x - 30°)$ are drawn on the same system of axes for $-180° \le x \le 180°$. The point $P(90°\,;\,1)$ lies on f. Use the diagram to answer the following questions.



6.1 Determine the value of b. (1)

6.2 Write down the coordinates of A, a turning point of g. (2)

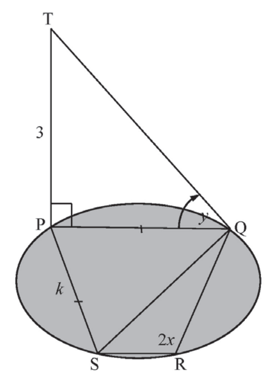
6.3 Write down the equation of the asymptote(s) of $y = \tan b(x + 20°)$ for $x \in [-180°; 180°]$. (1)

6.4 Determine the range of h if $h(x) = 2g(x) + 1$. (2)
[6]

QUESTION 7

7.1 Prove that in any acute-angled $\triangle ABC$, $\frac{\sin A}{a} = \frac{\sin B}{b}$. (5)

7.2 The framework for a construction consists of a cyclic quadrilateral PQRS in the horizontal plane and a vertical post TP as shown in the figure. From Q the angle of elevation of T is $y°$. $PQ = PS = k$ units, $TP = 3$ units and $S\hat{R}Q = 2x°$.



7.2.1 Show, giving reasons, that $P\hat{S}Q = x$. (2)

7.2.2 Prove that $SQ = 2k \cos x$. (4)

7.2.3 Hence, prove that $SQ = \frac{6\cos x}{\tan y}$. (2)
[13]

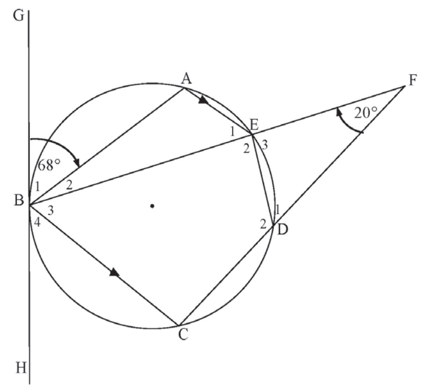
Give reasons for your statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 Complete the following statement:

The angle between the tangent and the chord at the point of contact is equal to ... (1)

8.2 In the diagram, A, B, C, D and E are points on the circumference of the circle such that AE | | BC. BE and CD produced meet in F. GBH is a tangent to the circle at B. $\hat{B}_1 = 68°$ and $\hat{F} = 20°$.



Determine the size of each of the following:

8.2.1 $\hat{E}_1$ (2)

8.2.2 $\hat{B}_3$ (1)

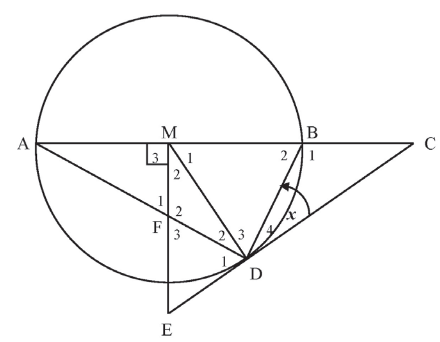
8.2.3 $\hat{D}_1$ (2)

8.2.4 $\hat{E}_2$ (1)

8.2.5 $\hat{C}$ (2)
[9]

QUESTION 9

In the diagram, M is the centre of the circle and diameter AB is produced to C. ME is drawn perpendicular to AC such that CDE is a tangent to the circle at D. ME and chord AD intersect at F. MB = 2BC.



9.1 If $\hat{D}_4 = x$, write down, with reasons, TWO other angles each equal to x. (3)

9.2 Prove that CM is a tangent at M to the circle passing through M, E and D. (4)

9.3 Prove that FMBD is a cyclic quadrilateral. (3)

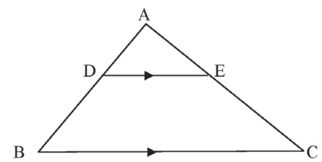
9.4 Prove that $DC^2 = 5BC^2$. (3)

9.5 Prove that $\Delta DBC \,|||\, \Delta DFM$. (4)

9.6 Hence, determine the value of $\frac{DM}{FM}$. (2)
[19]

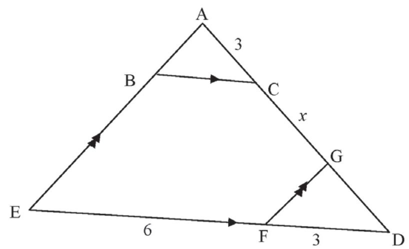
QUESTION 10

10.1 In the diagram, points D and E lie on sides AB and AC respectively of ΔABC such that DE | | BC. Use Euclidean Geometry methods to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

10.2 In the diagram, ADE is a triangle having BC | | ED and AE | | GF. It is also given that AB : BE = 1 : 3, AC = 3 units, EF = 6 units, FD = 3 units and CG = x units.



Calculate, giving reasons:

10.2.1 The length of CD (3)

10.2.2 The value of x (4)

10.2.3 The length of BC (5)

10.2.4 The value of $\frac{\text{area } \Delta ABC}{\text{area } \Delta GFD}$ (5)
[23]

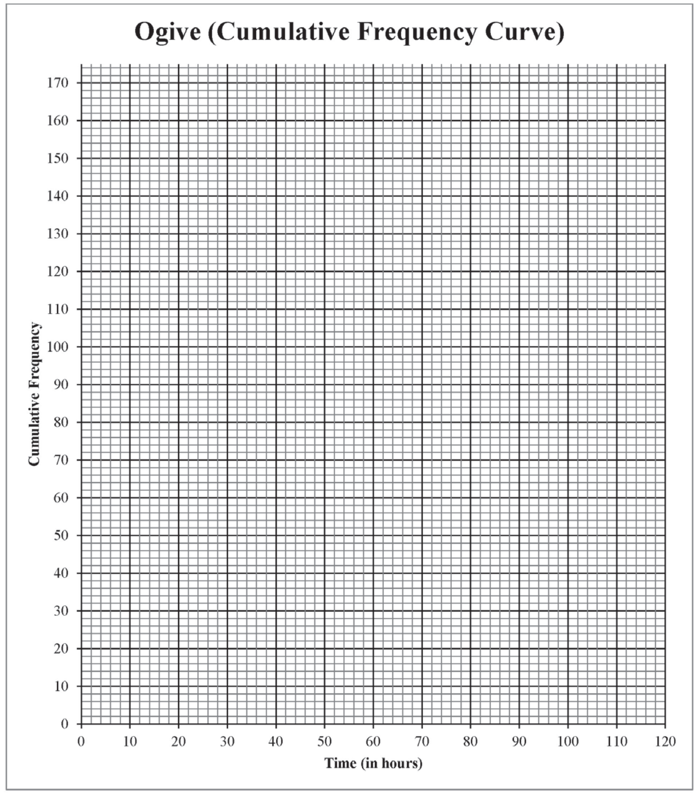
TOTAL: 150

NAME:

GRADE/CLASS:

DIAGRAM SHEET 1

QUESTION 2.1

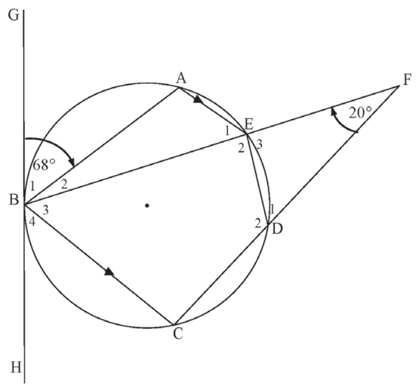


NAME:

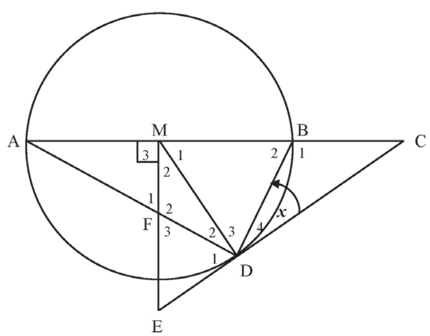
GRADE/CLASS:

DIAGRAM SHEET 2

QUESTION 8.2



QUESTION 9

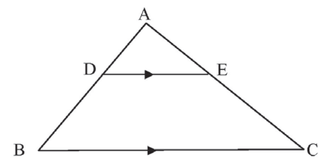


NAME:

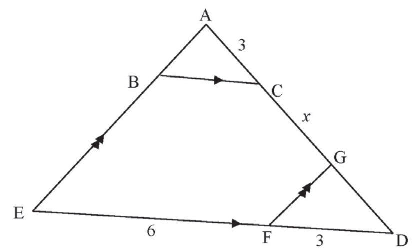
GRADE/CLASS:

DIAGRAM SHEET 3

QUESTION 10.1



QUESTION 10.2



INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $area\,\Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

RESOURCE 7

MEMORANDUM PAPER 1 EXEMPLAR: Revision Week 3

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL
SENIOR CERTIFICATE

GRADE/*GRAAD* 12

MATHEMATICS P1/*WISKUNDE V1*

EXEMPLAR 2014/MODEL 2014

MEMORANDUM

MARKS: 150
PUNTE: 150

This memorandum consists of 22 pages.
Hierdie memorandum bestaan uit 22 bladsye.

 Please turn over/*Blaai om asseblief*

NOTE:
- If a candidate answers a question/vraag TWICE, only mark the FIRST attempt.
- Consistent accuracy applies in all aspects of the marking memorandum.

LET WEL:
- *Indien 'n kandidaat 'n vraag TWEE keer beantwoord, merk slegs die EERSTE poging.*
- *Volgehoue akkuraatheid is DEURGAANS in ALLE aspekte van die memorandum van toepassing.*

QUESTION/*VRAAG* 1

1.1.1	$3x^2 - 4x = 0$ $x(3x-4) = 0$ $x = \frac{4}{3}$ or $x = 0$	✓factors ✓both answers (2)
1.1.2	$x - 6 + \frac{2}{x} = 0$ $x^2 - 6x + 2 = 0$ $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$ $= \frac{6 \pm \sqrt{28}}{2}$ $x = 0{,}35$ or $x = 5{,}65$ **OR** $x - 6 + \frac{2}{x} = 0$ $x^2 - 6x + 2 = 0$ $(x-3)^2 = -2 + 9$ $(x-3) = \pm\sqrt{7}$ $x = 3 \pm \sqrt{7}$ $x = 0{,}35$ or $x = 5{,}65$	✓ $x^2 - 6x + 2 = 0$ ✓ subs into correct formula ✓ $x = 0{,}35$ ✓ $x = 5{,}65$ (4) ✓ $x^2 - 6x + 2 = 0$ ✓ $(x-3)^2 = -2 + 9$ ✓ $x = 0{,}35$ ✓ $x = 5{,}65$ (4)
1.1.3	$x^{\frac{2}{3}} = 4;\ x > 0$ $x = (2^2)^{\frac{3}{2}}$ $x = 8$ **OR**	✓ $x = (2^2)^{\frac{3}{2}}$ ✓ $x = 8$ (2)

	$x^{\frac{2}{3}} = 4;\ x > 0$ $x = (4)^{\frac{3}{2}}$ $x = 8$ **OR** $x^{\frac{2}{3}} = 4$ $x^{\frac{2}{3}} - 4 = 0$ $\left(x^{\frac{1}{3}} - 2\right)\left(x^{\frac{1}{3}} + 2\right) = 0$ $x = (-2)^3$ or $x = 2^3$ $x = -8$ or $x = 8$ $x = 8\ (x > 0)$	✓ $x = (4)^{\frac{3}{2}}$ ✓ $x = 8$ (2) ✓factors ✓ $x = 8$ (2)
1.1.4	$3^x(x-5) < 0$ 3^x is always positive $x - 5 < 0$ $x < 5$ **Answer only full marks**	✓ $3^x > 0$ ✓ $x < 5$ (2)
1.2	$y = x^2 - x - 6$ and $2x - y = 2$ $2x - (x^2 - x - 6) = 2$ $-x^2 + 3x + 6 = 2$ $x^2 - 3x - 4 = 0$ $(x-4)(x+1) = 0$ $x = -1$ *or* $x = 4$ $y = -4$ *or* $y = 6$ **OR** $y = x^2 - x - 6$ and $2x - y = 2$ $y = 2x - 2$ $2x - 2 = x^2 - x - 6$ $x^2 - 3x - 4 = 0$ $(x-4)(x+1) = 0$ $x = -1$ *or* $x = 4$ $y = -4$ *or* $y = 6$	✓ subst $y = x^2 - x - 6$ ✓ standard form ✓ factors ✓ x-values ✓✓ y-values (6) ✓ $y = 2x - 2$ ✓ standard form ✓ factors ✓ x-values ✓✓ y-values

	OR $y = x^2 - x - 6$ and $2x - y = 2$ $x = \frac{y+2}{2}$ $y = \left(\frac{y+2}{2}\right)^2 - \left(\frac{y+2}{2}\right) - 6$ $y = \left(\frac{y^2+4y+4}{4}\right) - \left(\frac{2y+4}{4}\right) - 6$ $4y = y^2 + 2y - 24$ $y^2 - 2y - 24 = 0$ $(y-6)(y+4) = 0$ $y = -4$ *or* $y = 6$ $x = -1$ *or* $x = 4$	✓ $x = \frac{y+2}{2}$ ✓ standard form ✓ factors ✓ y - values ✓✓ x - values	(6) (6)
1.3	$\sqrt{3}.\sqrt{48} - \frac{4^{x+1}}{2^{2x}}$ $= \sqrt{3}.4\sqrt{3} - \frac{2^{2x+2}}{2^{2x}}$ $= 12 - 4$ $= 8$ **OR** $\sqrt{3}.\sqrt{48} - \frac{4^{x+1}}{2^{2x}}$ $= \sqrt{144} - \frac{2^{2x+2}}{2^{2x}}$ $= 12 - 4$ $= 8$	✓ 2^{2x+2} ✓ 4 ✓ answer ✓ 2^{2x+2} ✓ 4 ✓ answer	(3) (3)
1.4.1	No, there will be no intersection between the graphs. Min value of $3(x-1)^2 + 5$ is 5 *Nee, daar sal geen snyding tussen die grafieke wees nie.* *Min waarde van* $3(x-1)^2 + 5$ is 5 **OR** $3(x-1)^2 + 5 = 3$ $3(x-1)^2 = -2$ $(x-1)^2 \neq -\frac{2}{3}$ No, there will be no intersection between the graphs.	✓ answer ✓ reason ✓ reason ✓ answer	(2)

	Nee, daar sal geen snyding tussen die grafieke wees nie. **OR** $3(x-1)^2 + 5 = 3$ $3(x^2 - 2x + 1) + 2 = 0$ $3x^2 - 6x + 5 = 0$ $\Delta = (-6)^2 - 4(3)(5)$ $= -24$ < 0 No, there is no solution to the equation $f(x) = g(x)$ *Nee, daar is geen oplossing vir die vergelyking* $f(x) = g(x)$	✓ reason ✓ answer	(2) (2)
1.4.2	$3(x-1)^2 + 5 = 3 + k$ $3(x-1)^2 = k - 2$ $k - 2 > 0$ for all real values of x / *vir alle reëele waardes van* x $k > 2$ **Answer only full marks** **OR** $3x^2 - 6x + 3 + 5 = 3 + k$ $3x^2 - 6x + 5 - k = 0$ $\Delta = (-6)^2 - 4(3)(5-k)$ $= 36 - 60 + 12k$ $= 12k - 24$ For real unequal roots / *Vir reëele ongelyke wortels* $12k - 24 > 0$ $12k > 24$ $k > 2$	✓✓ answer ✓✓ answer	(2) (2) **[23]**

QUESTION/VRAAG 2

2.1.1	$T_n = a+(n-1)d$ $300 = 18+(n-1)6$ $300 = 18+6n-6$ $6n = 288$ $n = 48$	✓$a = 18$ and $d = 6$ ✓$T_n = 300$ ✓ answer (3)
2.1.2	$S_n = \frac{n}{2}[2a+(n-1)d]$ $= \frac{48}{2}[2(18)+47(6)]$ $= 7632$	✓ substitution in formula ✓ answer (2)
2.1.3	Sum of all numbers from 1 to 300 / *Som van alle getalle van 1 tot 300* $= \frac{300}{2}[2(1)+299(1)]$ $= \frac{300(301)}{2}$ $= 45150$ Sum of numbers not divisible by 6 / *Som van getalle wat nie deelbaar deur 6 is nie* $= 45150-(7632+6+12)$ $= 37500$	✓ substitution ✓ answer ✓ $(7632+6+12)$ ✓ answer (4)
2.2.1	16; 8; 4; …… $r = \frac{1}{2}$ $T_n = ar^{n-1}$ $= 16\left(\frac{1}{2}\right)^{n-1}$ $= 2^4\left(2^{-n+1}\right)$ $= 2^{5-n}$	✓ $r = \frac{1}{2}$ ✓ answer (in any format) (2)
2.2.2	$16+8+4+2+1+\frac{1}{2} = 31$ $S_5 = 31$ $n > 5$ or $n \geq 6$	✓$16+8+4+2+1+\frac{1}{2}$ ✓ $S_5 = 31$ ✓ $n > 5$ / $n \geq 6$ (3)

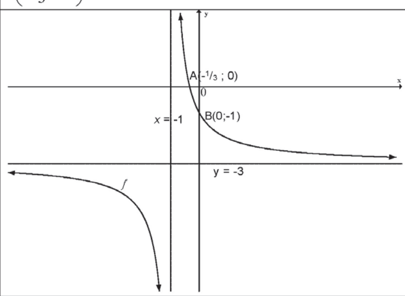
	OR $S_n = \frac{a(1-r^n)}{1-r}$ $31 < \frac{16\left(1-\frac{1}{2}^n\right)}{1-\frac{1}{2}}$ $31 < 32(1-2^{-n})$ $\frac{31}{32}-1 < -2^{-n}$ $\frac{1}{32} > 2^{-n}$ $2^{-5} > 2^{-n}$ $n > 5$ or $n \geq 6$	✓ $S_n > 31$ ✓ simplification ✓ $n > 5$ / $n \geq 6$ (3)
2.2.3	$S_\infty = \frac{a}{1-r}$ $= \frac{16}{1-\frac{1}{2}}$ $= 32$ **OR** $16+8+4+2+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}$…… Answer gets <u>closer and closer to 32</u> the more terms gets added together *Antwoord beweeg nader en nader aan 32 hoe meer terme bymekaar getel word*	✓substitution of a and r ✓ answer (2) ✓expanding the series ✓ answer (2) **[16]**

QUESTION/VRAAG 3

3.1.1	1; x; y; z….. $\quad T_n = 4n + 6$ 10; 14; 18…… 1 $\quad x \quad y$ 10 $\quad$ 14 4	2nd difference = 4
	$2a = 4$ $a = 2$	✓ $2a = 4$ ✓ $a = 2$
	OR	(2)
	$T_n = 4n + 6$ $d = 4$ $2a = 4$ $a = 2$	✓ $2a = 4$ ✓ $a = 2$ (2)
3.1.2	1 $\quad x \quad y$ 10 $\quad$ 14 4	✓1st differences 10; 14; 18……
	$3a + b = 10$ $6 + b = 10$ $b = 4$	✓ $3a + b = 10$
	$a + b + c = 1$ $2 + 4 + c = 1$ $c = -5$	✓ $a + b + c = 1$
	$T_n = 2n^2 + 4n - 5$	✓ $T_n = 2n^2 + 4n - 5$ (4)

3.2	Consider the sequence made up by the first factors of each term: *Beskou die ry wat deur die eerste faktore van elke term gevorm word:* 1; 5; 9; 13; … 81 An arithmetic sequence / *rekenkundige ry*: $T_n = a + (n-1)d$ $= 1 + (n-1)4$ $= 4n - 3$	✓ $T_n = 4n - 3$
	To find the no. of terms: $81 = 4n - 3$ *Aantal terme:* $4n = 84$ $\therefore n = 21$	✓no. of terms
	The second factor is 1 more than the first factor / *Tweede faktor is 1 meer as die eerste faktor:* $T_n = 4n - 3 + 1$ $= 4n - 2$	✓ $T_n = 4n - 2$
	OR	
	Consider the sequence made up by the second factors of each term: *Beskou die ry wat deur die tweede faktore van elke term gevorm word:* 2; 6; 10; 14; …82 Also an arithmetic sequence / *rekenkundige ry*: $T_n = a + (n-1)d$ $= 2 + (n-1)4$ $= 4n - 2$	✓ $T_n = 4n - 2$
	Answer only full marks	
	In sigma notation: $\sum_{n=1}^{21}(4n-3)(4n-2)$ **or** $\sum_{n=1}^{21}2(4n-3)(2n-1)$ **or** $\sum_{n=1}^{21}(16n^2 - 20n + 6)$	✓answer in sigma notation (4) **[10]**

QUESTION/VRAAG 4

4.1.1	$f(x)=\frac{2}{x+1}-3$ $y=f(0)$ $=\frac{2}{0+1}-3$ $=-1$ $(0\ ;-1)$	✓subst $x=0$ ✓$(0\ ;-1)$ (2)
4.1.2	$0=\frac{2}{x+1}-3$ $3=\frac{2}{x+1}$ $3x+3=2$ $x=-\frac{1}{3}$ $\left(-\frac{1}{3}\ ;\ 0\right)$	✓ subs $y=0$ ✓ $\left(-\frac{1}{3}\ ;\ 0\right)$ (2)
4.1.3		✓ shape ✓ both intercepts correct ✓ horizontal and vertical asymptote (3)
4.1.4	$y=-(x+1)-3$ $y=-x-4$ **OR** $y=-x+k$ $-3=-(-1)+k$ $k=-4$ $y=-x-4$	✓ $y=-(x+1)-3$ ✓ $y=-x-4$ (2) ✓$-3=-(-1)+k$ ✓ $y=-x-4$ (2)

4.2.1	$y=a.b^x+q$ $y=a.b^x-3$ $-2=a.b^0-3$ [subs $(0\ ;-2)$] $a=1$ $y=1.b^x-3$ [subs $(1;-1)$] $-1=b^1-3$ $b=2$ $f(x)=2^x-3$	✓subs $q=-3$ ✓ $a=1$ ✓$b=2$ ✓ $f(x)=2^x-3$ (4)
4.2.2	A translation of 4 units up and 1 unit to the left. *'n Translasie van 4 eenhede na bo en 1 eenheid na links.* **OR** Dilation by a factor of 2 and 7 units up. *Verkleining deur faktor van 2 en 7 eenhede na bo.*	✓4 units up ✓ 1 unit to the left (2) ✓ dilation by factor 2 ✓ 7 units up (2) **[15]**

QUESTION/VRAAG 5

5.1	$f(x) = -2x^2 - 5x + 3$ $x = -\frac{b}{2a}$ **or** $f'(x) = 0$ $x = -\left(\frac{-5}{2(-2)}\right)$ $\quad -4x - 5 = 0$ $x = -\frac{5}{4}$ $\quad x = -\frac{5}{4}$ $y = -2\left(-\frac{5}{4}\right)^2 - 5\left(-\frac{5}{4}\right) + 3$ $= \frac{49}{8}$ / 6,125 TP $(-\frac{5}{4}; \frac{49}{8})$	✓ $x = -\frac{b}{2a}$ / $f'(x) = 0$ ✓ $x = -\frac{5}{4}$ ✓ $y = \frac{49}{8}$ / 6,125 (3)
	OR $y = -2(x^2 + \frac{5}{2}x - \frac{3}{2})$ $= -2[(x + \frac{5}{4})^2 - \frac{25}{16} - \frac{3}{2}]$ $= -2[(x + \frac{5}{4})^2 - \frac{49}{16}]$ $\quad$ TP $(-\frac{5}{4}; \frac{49}{8})$ $= -2(x + \frac{5}{4})^2 + \frac{49}{8}$	✓ $-2[(x + \frac{5}{4})^2 - \frac{25}{16} - \frac{3}{2}]$ ✓ $x = -\frac{5}{4}$ ✓ $y = \frac{49}{8}$ / 6,125 (3)
5.2	$m_{tangent} = \tan 135^\circ$ $= -1$ $-4x - 5 = -1$ $-4x = 4$ $x = -1$ $y = -2(-1)^2 - 5(-1) + 3$ $= 6$ Point of contact: P(–1; 6)	✓ $\tan 135^\circ = -1$ ✓ $-4x - 5 = -1$ ✓ $x = -1$ ✓ $y = 6$ (4)
5.3	Eq of g: $y - y_1 = m(x - x_1)$ $y - 6 = -1(x + 1)$ $y = -x + 5$	✓ substitute in equation ✓ answer (2)
5.4	$d > 5$	✓ answer (1) **[10]**

QUESTION/VRAAG 6

6.1	$g(x) = \sqrt{ax}$ $4 = \sqrt{a(8)}$ $8a = 16$ $a = 2$	✓subst (8 ; 4) ✓ $a = 2$ (2)
6.2	$x \geq 0$	✓answer (1)
6.3	$y \geq 0$	✓answer (1)
6.4	$y = \sqrt{2x}$; $x \geq 0$ $x^2 = 2y$ $y = \frac{x^2}{2}$; $y \geq 0$	✓interchange x and y ✓answer (2)
6.5	$\sqrt{2x} = x - 4$ $2x = x^2 - 8x + 16$ $0 = x^2 - 10x + 16$ $0 = (x - 8)(x - 2)$ $x = 8$ or $x = 2$ when $x = 2$, LHS = 2 but RHS = –2 Hence $x = 8$ only	✓ $2x = x^2 - 8x + 16$ (squaring both sides) ✓factors ✓ $x = 8$ or $x = 2$ ✓ selects $x = 8$ (4)
6.6	$0 < x < 8$	✓ $x < 8$ ✓ $0 < x$ (2) **[12]**

QUESTION/VRAAG 7

7.1	Selling price / *Verkoopprys* $= \frac{102\ 000}{0,12}$ $= 850\ 000$	✓ 850 000 (1)
7.2	$P_v = \frac{x[1-(1+i)^{-n}]}{i}$ $748\ 000 = \frac{x\left[1-\left(1+\frac{0,09}{12}\right)^{-240}\right]}{\frac{0,09}{12}}$ $x = 6\ 729,95$ **OR** $F_v = \frac{x[(1+i)^n - 1]}{i}$ $748\ 000\left(1+\frac{0,09}{12}\right)^{240} = \frac{x\left[\left(1+\frac{0,09}{12}\right)^{240} - 1\right]}{\frac{0,09}{12}}$ $x = 6\ 729,95$	✓ $Pv = 748\ 000$ ✓ $i = \frac{0,09}{12}$ ✓ $n = -240$ ✓ $x = \text{R}6\ 729,95$ (4) ✓ $748000(1+\frac{0,09}{12})^{240}$ ✓ $i = \frac{0,09}{12}$ ✓ $n = 240$ ✓ $x = \text{R}6\ 729,95$ (4)
7.3	Total interest paid / *Totale rente betaal* = (6 729,95 x 240) –748 000 = R 867 188	✓ (6 729,95 x 240) ✓ 867 188 (2)
7.4	$\text{Balance} = \frac{x[1-(1+i)^{-n}]}{i}$ $= \frac{6729,95\left[1-\left(1+\frac{0,09}{12}\right)^{-155}\right]}{\frac{0,09}{12}}$ $x = 615\ 509,74$ **OR**	✓ 6729,95 ✓ $n = -155$ ✓ R615 509,74 (3)

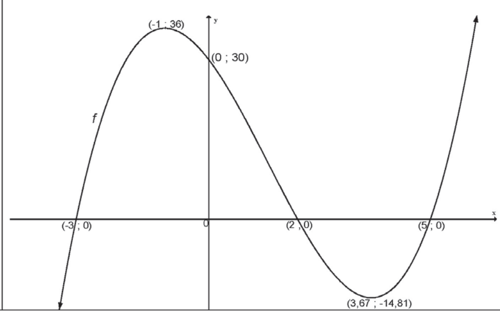
	$A = 748\ 000\left(1+\frac{0,09}{12}\right)^{85}$ $= 1\,411\,663,732$ $F_v = \frac{x[(1+i)^n - 1]}{i}$ $= \frac{6729,95\left[\left(1+\frac{0,09}{12}\right)^{85} - 1\right]}{\frac{0,09}{12}}$ $= 796\,153,962$ Balance of loan $= 1\,411\,663,732 - 796\,153,962$ $= 615\,509,77$ **OR** $\text{Balance} = 748\ 000\left(1+\frac{0,09}{12}\right)^{85} - \frac{6729,95\left[\left(1+\frac{0,09}{12}\right)^{85} - 1\right]}{\frac{0,09}{12}}$ $= 615\ 509,77$	✓ 1 411 663,732 ✓ $n = 85$ ✓ R615 509,77 (3) ✓ subs of 748 000 and 6729,95 ✓ $n = 85$ ✓ R615 509,77 (3)
7.5	New value of bond: $615\ 509,74\left(1+\frac{0,09}{12}\right)^4$ **or** $615\ 509,77\left(1+\frac{0,09}{12}\right)^4$ $= 634\ 183,81$ $= 634\ 183,84$	✓ $\text{R}615\ 509,74(1+\frac{0,09}{12})^4$ ✓ R634 183,81/ R634 183,84 (2)
7.6	$634\ 183,81 = \frac{8\ 500\left[1-\left(1+\frac{0,09}{12}\right)^{-n}\right]}{\frac{0,09}{12}}$ $\log(0,44042605) = -n\log\left(1+\frac{0,09}{12}\right)$ $n = 109,74$ $= 110$ months **OR**	✓ $x = 8\ 500$ ✓ subs into correct formula ✓ use of logs ✓ answer (4)

	$\mathbf{634\,183{,}81} = \dfrac{8\,500\left[1-\left(1+\frac{0{,}09}{12}\right)^{-n}\right]}{\frac{0{,}09}{12}}$ $-n = \log_{\left(1+\frac{0{,}09}{12}\right)}(0{,}44042605)$ $n = 109{,}74$ $= 110$ months	✓ $x = 8\,500$ ✓ subs into correct formula ✓ use of logs ✓ answer (4) **[16]**

QUESTION/VRAAG 8

8.1	$f(x) = 3x^2 - 2$ $f(x+h) = 3(x+h)^2 - 2$ $= 3x^2 + 6xh + 3h^2 - 2$ $f(x+h) - f(x) = 6xh + 3h^2$ $f'(x) = \lim\limits_{h\to 0}\dfrac{6xh+3h^2}{h}$ $= \lim\limits_{h\to 0}\dfrac{h(6x+3h)}{h}$ $= \lim\limits_{h\to 0}(6x+3h)$ $= 6x$	✓ substitution of of $x+h$ ✓ simplification to $6xh+3h^2$ ✓ formula ✓ taking out common factor ✓ answer (5)
	OR $f'(x) = \lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$ $= \lim\limits_{h\to 0}\dfrac{\left[3(x+h)^2-2\right]-\left(3x^2-2\right)}{h}$ $= \lim\limits_{h\to 0}\dfrac{\left[3\left(x^2+2xh+h^2\right)-2\right]-3x^2+2}{h}$ $= \lim\limits_{h\to 0}\dfrac{[3x^2+6xh+3h^2-2]-3x^2+2}{h}$ $= \lim\limits_{h\to 0}\dfrac{6xh+3h^2}{h}$ $= \lim\limits_{h\to 0}\dfrac{h(6x+3h)}{h}$ $= \lim\limits_{h\to 0}(6x+3h)$ $= 6x$	✓ formula ✓ substitution of $x+h$ ✓ simplification to $\dfrac{6xh+3h^2}{h}$ ✓ taking out common factor ✓ answer (5)
8.2	$y = 2x^{-4} - \dfrac{x}{5}$ $\dfrac{dy}{dx} = -8x^{-5} - \dfrac{1}{5}$	✓ $-8x^{-5}$ ✓ $-\dfrac{1}{5}$ (2) **[7]**

QUESTION/VRAAG 9

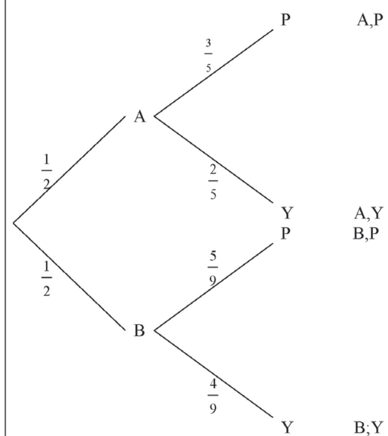
9.1	$(x-2)$ is a factor of f / *is 'n faktor van f.*	✓ answer (1)
9.2	$f(x)=x^3-4x^2-11x+30$ $=(x-2)(x^2-2x-15)$ $=(x-2)(x+3)(x-5)$ $f(x)=0$ $(x+3)(x-2)(x-5)=0$ $x=-3$ or $x=2$ or $x=5$ x-intercepts: (–3; 0); (2;0); (5;0)	✓ $(x^2-2x-15)$ ✓(–3; 0) ✓(2; 0) ✓(5; 0) (4)
9.3	$f(x)=x^3-4x^2-11x+30$ $f'(x)=3x^2-8x-11$ At turning points $f'(x)=0$ $(3x-11)(x+1)=0$ $x=-1$ or $x=\frac{11}{3}$ $y=36$ $y=-\frac{400}{27}$ $(-14{,}81)$ TP's are $(-1;36)$ and $\left(\frac{11}{3};-14{,}81\right)$	✓ $f'(x)=3x^2-8x-11$ ✓ $f'(x)=0$ ✓x - value ✓x - value ✓y - values (5)
9.4		✓y and x - intercepts ✓shape ✓turning points (3)

9.5	$f'(x)<0$ if $-1<x<3{,}67$ **OR** (-1 ; 3,67)	✓extreme values ✓notation (2) ✓extreme values ✓notation (2) **[15]**

QUESTION/VRAAG 10

10.1	After t hours : $BF = 30t$ km and $CD = 40t$ km $\therefore BC = 100 - 40t$ $FC = \sqrt{(30t)^2 + (100-40t)^2}$ $= \sqrt{900t^2 + 10000 - 8000t + 1600t^2}$ $= \sqrt{2500t^2 - 8000t + 10000}$	✓ $BF = 30t$ ✓ $BC = 100 - 40t$ ✓Pythagoras ✓answer (4)
10.2	FC is a minimum when FC^2 is a minimum. $FC^2 = 2500t^2 - 8000t + 10000$ $\frac{dFC^2}{dt} = 5000t - 8000 = 0$ $t = \frac{8000}{5000} = 1{,}6$hrs (96 minutes)	✓$FC^2 =$ $2500t^2 - 8000t + 10000$ ✓ $\frac{dFC^2}{dt} = 5000t - 8000$ ✓ $\frac{dFC^2}{dt} = 0$ ✓answer (4)
10.3	$FC = \sqrt{2500t^2 - 8000t + 10000}$ $= \sqrt{2500(1.6)^2 - 8000(1.6) + 10000}$ $= 60$ They will be 60km apart.	✓subs into equation ✓answer (2) **[10]**

QUESTION/VRAAG 11

11.1	P(A or B) = P(A) + P(B) 0,57 = P(A) + 2P(A) 0,57 = 3P(A) P(A) = 0,19 $\therefore$ P(B) = 2(0,19) = 0,38	✓ P(A or B) = P(A) + P(B) ✓ P(A) = 0,19 ✓answer (3)
11.2.1		✓ first tier ✓ second tier ✓ probabilities ✓outcomes (4)
11.2.2	$P(AY) = \left(\frac{1}{2}\right)\left(\frac{2}{5}\right)$ $= \frac{1}{5}$	✓ answer (1)
11.2.3	$P(P) = \left(\frac{1}{2}\right)\left(\frac{3}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{5}{9}\right)$ $= \frac{3}{10} + \frac{5}{18}$ $= \frac{26}{45}$	✓ $\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)$ ✓ $\left(\frac{1}{2}\right)\left(\frac{5}{9}\right)$ ✓ answer (3) **[11]**

QUESTION/VRAAG 12

12.1	| 5 | 4 | 3 | 2 | 1 | Number of different letter arrangements: *Aantal verskillende letter rangskikkings wat gevorm kan word:* 5! = 5x4x3x2x1 = 120	✓5! ✓120 (2)
12.2	S and T can be arranged in 2! different ways. The remaining three letters can be arranged in 3! different ways $\therefore$ Total number of different letter arrangements having S and T as the first two letters = 2!.3! S *en* T *kan op* 2! *verskillende maniere rangskik word.* *Die 3 letters wat oorbly kan op* 3! *verskillende maniere rangskik word* $\therefore$ *Totale aantal letterrangskikkings waarin* S *en* T *die eerste twee letters van die rangskikking sal wees* = 2!.3! P(having S and T as first two letters) $= \frac{2!.\ 3!}{120}$ $= \frac{2.6}{120}$ $= \frac{1}{10}$	✓2! ✓3! ✓answer (3) **[5]**

TOTAL/*TOTAAL*: 150

RESOURCE 8

MEMORANDUM PAPER 2 EXEMPLAR: Revision Week 3

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P2

EXEMPLAR 2014

MEMORANDUM

MARKS: 150

This memorandum consists of 13 pages.

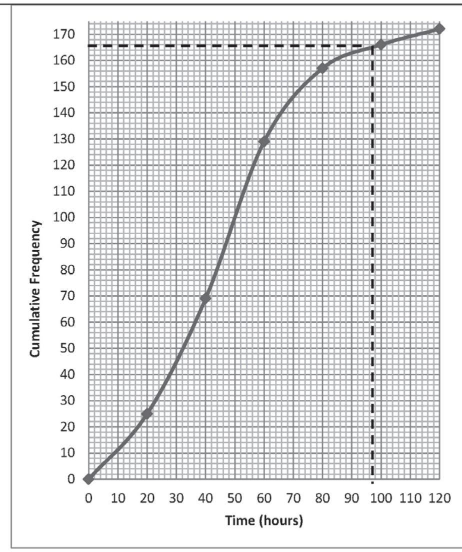
Please turn over

NOTE:
- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum.
- Assuming answers/values in order to solve a problem is NOT acceptable.

QUESTION 1

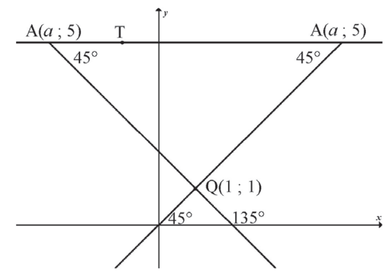
1.1	As the number of days that an athlete trained increased, the time taken to run the 100m event decreased. **OR** The fewer number of days an athlete trained, the longer the time he took to complete the 100m sprint. **OR** The greater number of days an athlete trained, the shorter the time he ran the 100m sprint.	✓ explanation (1)
1.2	(60 ; 18,1)	✓ (1)
1.3	$a = 17{,}81931464\ldots$ $b = -0{,}070685358\ldots$ $\therefore \hat{y} = -0{,}07x + 17{,}82$	✓✓ a ✓ b ✓ equation (4)
1.4	$\therefore \hat{y} \approx -0{,}07(45) + 17{,}82$ $\approx 14{,}67$ seconds	✓ substitution ✓ answer (2)
1.5	$r = -0{,}74$ $(-0{,}740772594\ldots)$	✓✓ r (2)
1.6	There is a moderately strong relationship between the variables.	✓ moderately strong (1) **[11]**

QUESTION 2

2.1		✓grounding at 0 ✓plotting at upper limits ✓ smooth shape of curve (3)
2.2	$40 \le t < 60$	✓ class (1)
2.3	(96 ; 164) $\therefore$ 172 – 164 = 8 learners	✓164 ✓ 8 (2)
2.4	Frequency: 25; 44; 60; 28; 9; 6 $\text{Mean} = \frac{25\times10+44\times30+60\times50+28\times70+9\times90+6\times110}{172}$ $= \frac{8000}{172}$ $= 46{,}51$ hours	✓frequency ✓ midpoints ✓ $\frac{8000}{172}$ ✓ answer (4) **[10]**

QUESTION 3

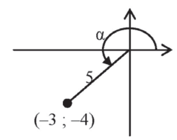
3.1	K(7 ; 0)	✓ answer (1)
3.2	$1=\frac{x_M+7}{2}$ *and* $1=\frac{y_M+3}{2}$ $\therefore$ M(– 5 ; – 1)	✓ x ✓ y (2)
3.3	$m_{PM}=\frac{3-1}{7-1}$ $=\frac{1}{3}$	✓ substitution ✓answer (2)
3.4	$\tan \text{P}\hat{\text{S}}\text{K}=m_{PM}=\frac{1}{3}$ $\text{P}\hat{\text{S}}\text{K}=\tan^{-1}\left(\frac{1}{3}\right)=18{,}43°$ $\therefore\ \theta=180°-90°-18{,}43°=71{,}57°$	✓ $\tan \text{P}\hat{\text{S}}\text{K}=m_{PM}$ ✓ $\text{P}\hat{\text{S}}\text{K}$ ✓ θ (3)
3.5	$\cos 71{,}57°=\frac{\text{PK}}{\text{PS}}=\frac{3}{\text{PS}}$ $\text{PS}=\frac{3}{\cos 71{,}57°}$ = 9,49 units **OR** $\sin 18{,}43°=\frac{\text{PK}}{\text{PS}}=\frac{3}{\text{PS}}$ $\text{PS}=\frac{3}{\sin 18{,}43°}$ = 9,49 units	✓ correct ratio ✓ PS as subject ✓ answer (3) ✓ correct ratio ✓ PS as subject ✓ answer (3)
3.6	N(x ; $-2x+17$) $m_{TN}=m_{PM}$ (TN | | PM) $\frac{-2x+17-5}{x-(-1)}=\frac{1}{3}$ $-6x+36=x+1$ $-7x=-35$ $x=5$ $\therefore\ y=-2(5)+17=7$ $\therefore$ N(5 ; 7) **OR**	✓ N in terms of x ✓ equal gradients ✓ substitution ✓ x -value ✓ y -value (5)

	$m_{TM}=\frac{1}{3}$ (TN | | PM) equation of TM: $y-y_1=\frac{1}{3}(x-x_1)$ $y-5=\frac{1}{3}(x-(-1))$ $y-5=\frac{1}{3}x+\frac{1}{3}$ $y=\frac{1}{3}x+5\frac{1}{3}$ **OR** $y=\frac{1}{3}x+c$ $5=\frac{1}{3}(-1)+c$ $5\frac{1}{3}=c$ $y=\frac{1}{3}x+5\frac{1}{3}$ $-2x+17=\frac{1}{3}x+5\frac{1}{3}$ $-2\frac{1}{3}x=-11\frac{2}{3}$ $x=5$ $\therefore\ y=-2(5)+17=7$ $\therefore$ N(5 ; 7)	✓ m_{TM} ✓ equation of TM ✓ equating ✓ x -value ✓ y -value (5)
3.7.1	$y=5$	✓ equation (1)
3.7.2	 gradient of AQ = tan 45° or tan 135° = 1 or –1 $m_{AQ}=\frac{5-1}{a-1}=\pm 1$ $\therefore\ a-1=4$ or -4 $\therefore\ a=5$ or -3	✓ $m_{AQ}=1$ or ✓ $m_{AQ}=-1$ ✓ substitution into gradient formula ✓ x-value ✓ y-value (5) **[22]**

QUESTION 4

4.1	M(–1 ; –1)	✓ answer (1)
4.2	$m_{NT}=\frac{2-1}{3-4}=-1$ $\therefore m_{AT}=1$ (radius ⊥ tangent) $y-1=1(x-4)$ $y=x-3$	✓ m_{NT} ✓ m_{AT} ✓ reason ✓ substitution of m and (4 ; 1) ✓ equation (5)
4.3	MR ⊥ AB (line from centre to midpt of chord) $MB^2=MR^2+RB^2$ (Theorem of Pythagoras) $9=(\frac{\sqrt{10}}{2})^2+RB^2$ $RB^2=\frac{13}{2}$ $RB=\sqrt{\frac{13}{2}}$ $AB=2\left(\sqrt{\frac{13}{2}}\right)=\sqrt{26}$ *units*	✓ MR ⊥ AB ✓MB = 3 ✓ substitution into Theorem of Pythagoras ✓ AB in surd form (4)
4.4	$MN^2=(-1-3)^2+(-1-2)^2$ $=16+9$ $=25$ MN = 5 units	✓ substitution into distance formula ✓ answer (2)
4.5	$r=5-3=2$ units $\therefore (x-3)^2+(y-2)^2=4$ $\therefore x^2+y^2-6x-4y+9=0$	✓r ✓substitution into circle equation ✓equation (3) **[15]**

QUESTION 5

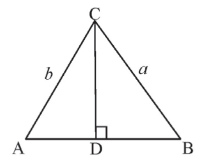
5.1.1	$-\sin\alpha$ $=-(-\frac{4}{5})=\frac{4}{5}$	✓ reduction ✓ answer (2)
5.1.2	$(-4)^2+b^2=5^2$ $b^2=25-16=9$ $b=-3$ $\cos\alpha=\frac{-3}{5}$ 	✓ $b=-3$ ✓answer (2)
5.1.3	$\sin(\alpha-45°)$ $=\sin\alpha\cos45°-\cos\alpha\sin45°$ $=-\frac{4}{5}\cdot\frac{1}{\sqrt{2}}-(-\frac{3}{5})\cdot\frac{1}{\sqrt{2}}$ $=-\frac{1}{5\sqrt{2}}$ **OR** $\sin(\alpha-45°)$ $=\sin\alpha\cos45°-\cos\alpha\sin45°$ $=-\frac{4}{5}\cdot\frac{\sqrt{2}}{2}-(-\frac{3}{5})\cdot\frac{\sqrt{2}}{2}$ $=-\frac{\sqrt{2}}{10}$	✓ expansion ✓ $\frac{1}{\sqrt{2}}$ ✓ answer in simplest form (3) ✓ expansion ✓ $\frac{\sqrt{2}}{2}$ ✓ answer in simplest form (3)
5.2.1	$LHS=\frac{8\sin x.\cos x}{\sin^2 x-\cos^2 x}$ $=\frac{4(2\sin x.\cos x)}{\sin^2 x-\cos^2 x}$ $=\frac{4\sin 2x}{-(\cos^2 x-\sin^2 x)}$ $=\frac{4\sin 2x}{-\cos 2x}$ $=-4\tan 2x$	✓ sin x ✓ cos x ✓ $\cos^2 x$ ✓ 4 sin 2x ✓ factorise ✓ – cos 2x (6)
5.2.2	Undefined when cos 2x = 0 or tan 2x = ∞: x = 45° and x = 135°	✓ 45° ✓ 135° (2)

5.3	$1-2\sin^2\theta+4\sin^2\theta-5\sin\theta-4=0$ $2\sin^2\theta-5\sin\theta-3=0$ $(2\sin\theta+1)(\sin\theta-3)=0$ $\therefore\ \sin\theta=-\frac{1}{2}$ or $\sin\theta=3$ (no solution) $\therefore\theta=210°+360°k$ or $\theta=330°+360°k$; $k\in Z$ **OR** $\therefore\theta=210°+360°k$ of $\theta=30°+360°k$; $k\in Z$	✓ $1-2\sin^2\theta$ ✓ standard form ✓ factors ✓ no solution ✓ 210° ✓ 330° ✓ $+360°k$; $k\in Z$ (7) **[22]**

QUESTION 6

6.1	$b=\frac{1}{2}$	✓ value of b (1)
6.2	A(30° ; 1)	✓ 30° ✓ 1 (2)
6.3	$x=160°$	✓ $x=160°$ (1)
6.4	$h(x)=2\cos(x-30°)+1$ $y\in[-1;3]$ **OR** $-1\le y\le 3$	✓ critical values ✓ notation (2) **[6]**

QUESTION 7

7.1	Draw CD ⊥ AB In ΔACD: $\sin A=\frac{CD}{b}$ $\therefore$ CD = b. sin A In ΔCBD: $\sin B=\frac{CD}{a}$ $\therefore$ CD = a. sin B $\therefore\ b$. sin A = a. sin B $\therefore\ \frac{\sin A}{a}=\frac{\sin B}{b}$ 	✓ construction ✓ sin A ✓ making CD the subject ✓ sin B ✓ b. sin A = a. sin B (5)
7.2.1	$S\hat{P}Q=180°-2x$ (opp ∠s of cyclic quad) $P\hat{S}Q+P\hat{Q}S=2x$ (sum of ∠s in Δ) $P\hat{S}Q=P\hat{Q}S=x$ (∠s opp equal sides)	✓ $S\hat{P}Q=180°-2x$ (S/R) ✓ reason (2)
7.2.2	$\frac{\sin S\hat{P}Q}{SQ}=\frac{\sin P\hat{S}Q}{PQ}$ $\frac{\sin(180°-2x)}{SQ}=\frac{\sin x}{k}$ $SQ=\frac{k\sin 2x}{\sin x}$ $SQ=\frac{k(2\sin x.\cos x)}{\sin x}=2k\cos x$ **OR** $SQ^2=PQ^2+PS^2-2PQ.PS.\cos S\hat{P}Q$ $=k^2+k^2-2.k.k.\cos(180°-2x)$ $=2k^2+2k^2\cos 2x$ $=2k^2+2k^2(2\cos^2 x-1)$ $=4k^2\cos^2 x$ $SQ=2k\cos x$	✓ substitution into correct formula ✓ sin $2x$ ✓ SQ subject ✓ $2\sin x.\cos x$ (4) ✓ substitution into correct formula ✓ $-\cos 2x$ ✓ $2\cos^2 x-1$ ✓ simplification (4)
7.2.3	$\tan y=\frac{3}{k}$ $k=\frac{3}{\tan y}$ $\therefore$ $SQ=2\cos x\left(\frac{3}{\tan y}\right)$ $=\frac{6\cos x}{\tan y}$	✓ tan ratio ✓ k subject and substitution (2) **[13]**

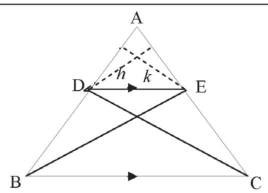
QUESTION 8

8.1	the angle subtended by the chord in the alternate segment		✓correct theorem (1)
8.2.1	$\hat{B}_1 = \hat{E}_1 = 68°$	(tan chord theorem)	✓ $\hat{E}_1 = 68°$ ✓ reason (2)
8.2.2	$\hat{E}_1 = \hat{B}_3 = 68°$	(alt ∠s; AE | | BC)	✓ $\hat{B}_3 = 68°$ (S/R) (1)
8.2.3	$\hat{D}_1 = \hat{B}_3 = 68°$	(ext ∠ of cyclic quad)	✓ $\hat{D}_1 = 68°$ ✓ reason (2)
8.2.4	$\hat{E}_2 = 20° + 68°$ $= 88°$	(ext ∠ of Δ)	✓ $\hat{E}_2 = 88°$ (S/R) (1)
8.2.5	$\hat{C} = 180° - 88°$ $= 92°$	(opp ∠s of cyclic quad)	✓ $\hat{C} = 92°$ ✓ reason (2) **[9]**

QUESTION 9

9.1	$\hat{D}_4 = \hat{A} = x$ $\hat{A} = \hat{D}_2 = x$	(tan chord theorem) (∠s opp equal sides)	✓ $\hat{A} = x$ ✓ reason ✓ $\hat{A} = \hat{D}_2 = x$ (S/R) (3)
9.2	$\hat{M}_1 = 2x$ $M\hat{D}E = 90°$ $\hat{M}_2 = 90° - 2x$ $\therefore \hat{E} = 180° - (90° + 90° - 2x)$ $= 2x$ ∴ CM is a tangent	(ext ∠ of Δ) or (∠ at centre = 2∠ at circum) (radius ⊥ tan) (sum of ∠s in ΔMDE) (converse tan chord theorem)	✓ $\hat{M}_1 = 2x$ (S/R) ✓ $M\hat{D}E = 90°$ (S/R) ✓ $\hat{E} = 2x$ ✓ reason (4)
9.3	$\hat{M}_3 = 90°$ $A\hat{D}B = 90°$ ∴ FMBD a cyclic quad **OR** $E\hat{M}C = 90°$ $A\hat{D}B = 90°$ ∴ FMBD a cyclic quad	(EM ⊥ AC) (∠ in semi-circle) (ext ∠ of quad = int opp ∠) (EM ⊥ AC) (∠ in semi-circle) (opp ∠s of quad supp)	✓ $\hat{M}_3 = 90°$ ✓ $A\hat{D}B = 90°$ (S/R) ✓ reason (3) ✓ $E\hat{M}C = 90°$ ✓ $A\hat{D}B = 90°$ (S/R) ✓ reason (3)
9.4	$DC^2 = MC^2 - MD^2$ $= (3BC)^2 - (2BC)^2$ $= 9BC^2 - 4BC^2$ $= 5BC^2$	(Theorem of Pythagoras) (MB = MD = radii)	✓ Th of Pythagoras ✓ substitution ✓ $9BC^2 - 4BC^2$ (3)
9.5	In ΔDBC and ΔDFM: $\hat{D}_4 = \hat{D}_2 = x$ $\hat{B}_1 = \hat{F}_2$ $\hat{C} = \hat{M}_2$ ∴ ΔDBC | | | ΔDFM (∠; ∠; ∠)	 (proven in 9.1) (ext ∠ of cyclic quad)	✓ $\hat{D}_4 = \hat{D}_2$ ✓ $\hat{B}_1 = \hat{F}_2$ ✓ reason ✓ $\hat{C} = \hat{M}_2$ or (∠; ∠; ∠) (4)
9.6	$\frac{DM}{FM} = \frac{DC}{BC}$ $= \frac{\sqrt{5}BC}{BC}$ $= \sqrt{5}$	(ΔDBC | | | ΔDFM)	✓ S ✓ answer (2) **[19]**

QUESTION 10

10.1	 Construction: Join DC and BE and heights k and h $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\frac{1}{2}.AD.k}{\frac{1}{2}.DB.k} = \frac{AD}{DB}$ (equal heights) $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{\frac{1}{2}.AE.h}{\frac{1}{2}.EC.h} = \frac{AE}{EC}$ (equal heights) But Area ΔDEB = Area ΔDEC (same base, same height) $\therefore \frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓ construction ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{AD}{DB}$ ✓ reason ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEC} = \frac{AE}{EC}$ ✓ Area ΔDEB = Area ΔDEC (S/R) ✓ $\frac{\text{area } \Delta ADE}{\text{area } \Delta DEB} = \frac{\text{area } \Delta ADE}{\text{area } \Delta DEC}$ (6)

10.2.1	$\frac{AB}{BE} = \frac{AC}{CD}$ (Prop Th; BC | | ED) $\frac{1}{3} = \frac{3}{CD}$ $\therefore$ CD = 9 units	✓ $\frac{AB}{BE} = \frac{AC}{CD}$ (S/R) ✓ substitution ✓ answer (3)
10.2.2	$\frac{DG}{GA} = \frac{FD}{FE}$ (Prop Th; FG | | EA) $\frac{9-x}{3+x} = \frac{3}{6}$ $54 - 6x = 9 + 3x$ $-9x = -45$ $x = 5$	✓ $\frac{DG}{GA} = \frac{FD}{FE}$ (S/R) ✓ substitution ✓ simplification ✓ answer (4)
10.2.3	In ΔABC and ΔAED: $\hat{A}$ is common $A\hat{B}C = \hat{E}$ (corres ∠s; BC | | ED) $A\hat{C}B = \hat{D}$ (corres ∠s; BC | | ED) ΔABC | | | ΔAED (∠, ∠, ∠) $\therefore \frac{BC}{ED} = \frac{AC}{AD}$ $\frac{BC}{9} = \frac{3}{12}$ $BC = 2\frac{1}{4}$ units	✓ $\hat{A}$ is common ✓ $A\hat{B}C = \hat{E}$ (S/R) ✓ $A\hat{C}B = \hat{D}$ (S/R) or (∠; ∠; ∠) ✓ $\frac{BC}{ED} = \frac{AC}{AD}$ ✓ answer (5)
10.2.4	$\frac{\text{area } \Delta ABC}{\text{area } \Delta GFD} = \frac{\frac{1}{2}AC.BC.\sin A\hat{C}B}{\frac{1}{2}GD.FD.\sin \hat{D}}$ $= \frac{\frac{1}{2}(3)(2\frac{1}{4})\sin \hat{D}}{\frac{1}{2}(4)(3)\sin \hat{D}}$ (corres ∠s; BC | | ED) $= \frac{9}{16}$	✓ use of area rule ✓ correct sides and angles ✓ substitution of values ✓ $\sin A\hat{C}B = \sin \hat{D}$ (S/R) ✓ answer (5) **[23]**

TOTAL: 150